Cosmology

Age of the Universe
- Future?
- The cosmic calendar
- Age of the Earth
- Red shift

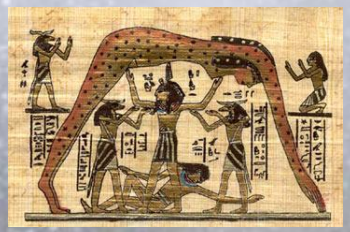
Structure and evolution of stars

Relativity
- Special Relativity
- Gravitational lensing
- Ideas of General Relativity
- Precession of Mercury

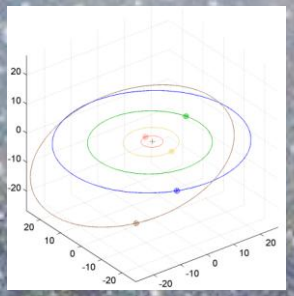
Size of the Universe
- Parallax
- Luminosity
- Measurement
- Earth
- Solar System
- Galaxy ...

Exotic case studies
- Neutron stars
- Black holes, quasars

A human history of cosmology



Gravity
- Newton/Kepler
- Einstein
- Computer Simulation

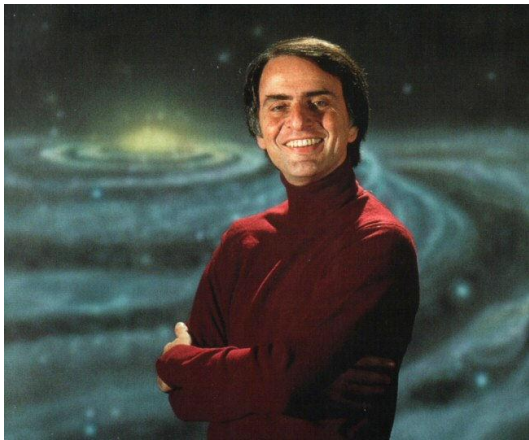


Planets & moons
- Detecting planets orbiting other stars
- The Solar System
- The Earth

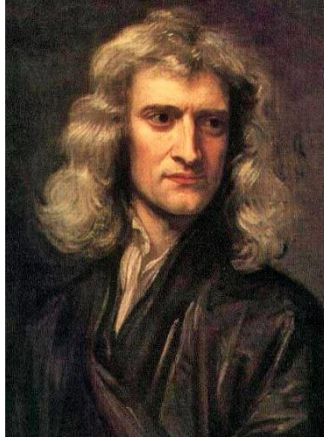
The Cosmos is all that is or ever was or ever will be.

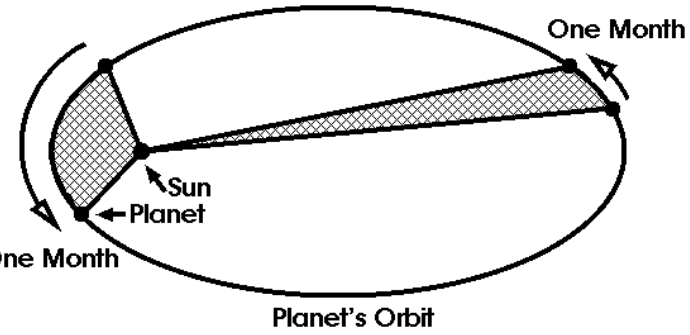
In the last few millennia we have made the most astonishing and unexpected discoveries about the Cosmos and our place within it, explorations that are exhilarating to consider. They remind us that humans have evolved to wonder, that understanding is a joy, that knowledge is prerequisite to survival.

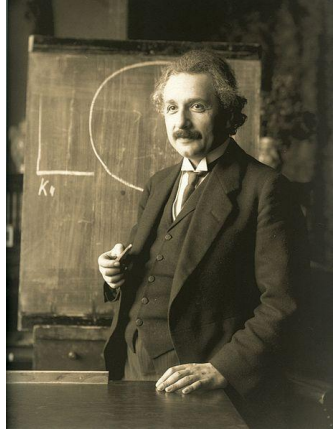
I believe our future depends on how well we know this Cosmos in which we float like a mote of dust in the morning sky.



Carl Sagan (1934-1996)
Cosmos pp20



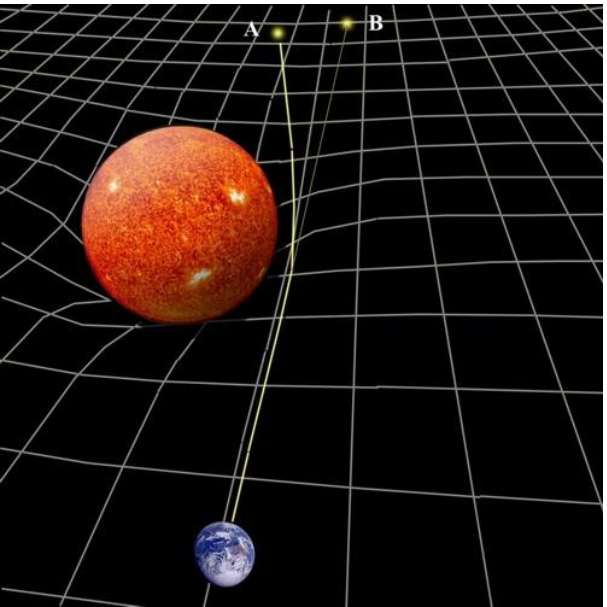


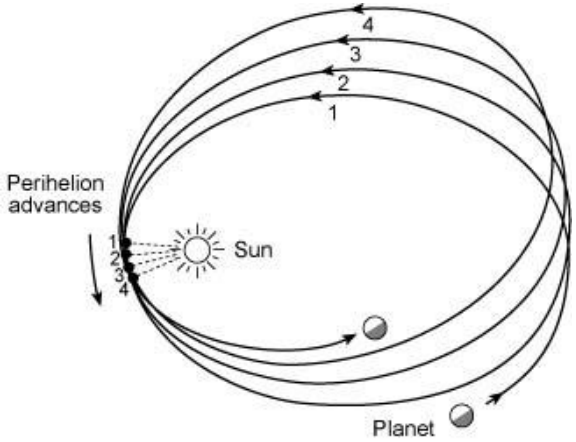


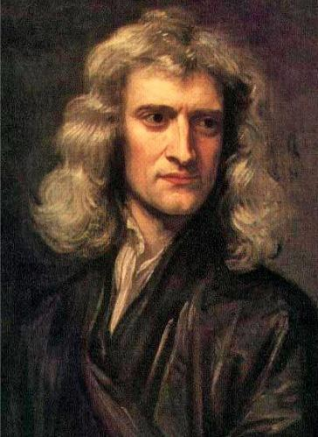
$$F = \frac{GMM_{\odot}}{r^2}$$

Gravity

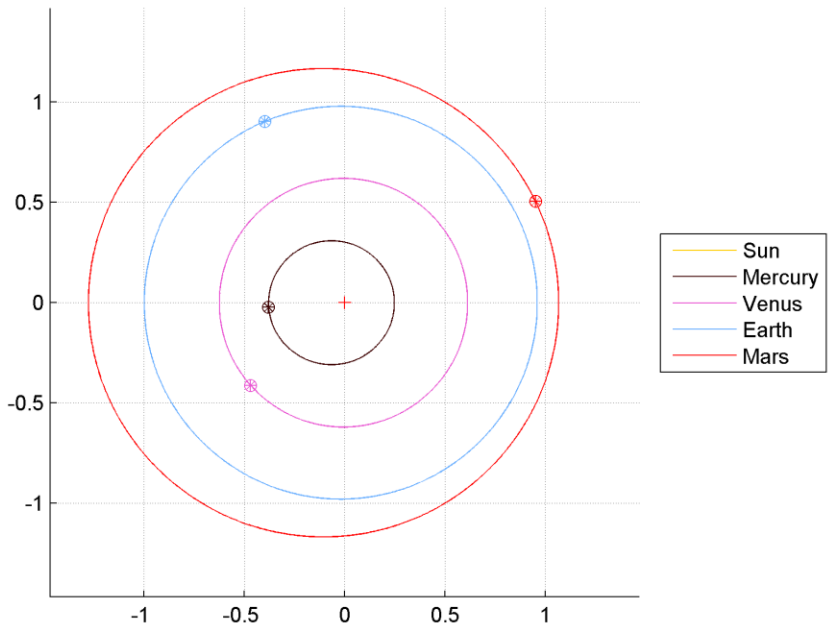
$$P^2 = \frac{4\pi^2}{G\left(M + M_{\odot}\right)} a^3$$







Isaac Newton
(1642-1727) developed a mathematical model of Gravity which predicted the elliptical orbits proposed by Kepler



Planet and Solar masses

Force of gravity

$$F = \frac{GMM_\odot}{r^2}$$

$$G = 6.67 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$$

$$r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\theta}$$ *Polar* equation of ellipse

$$\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$$ *Eccentricity* of ellipse

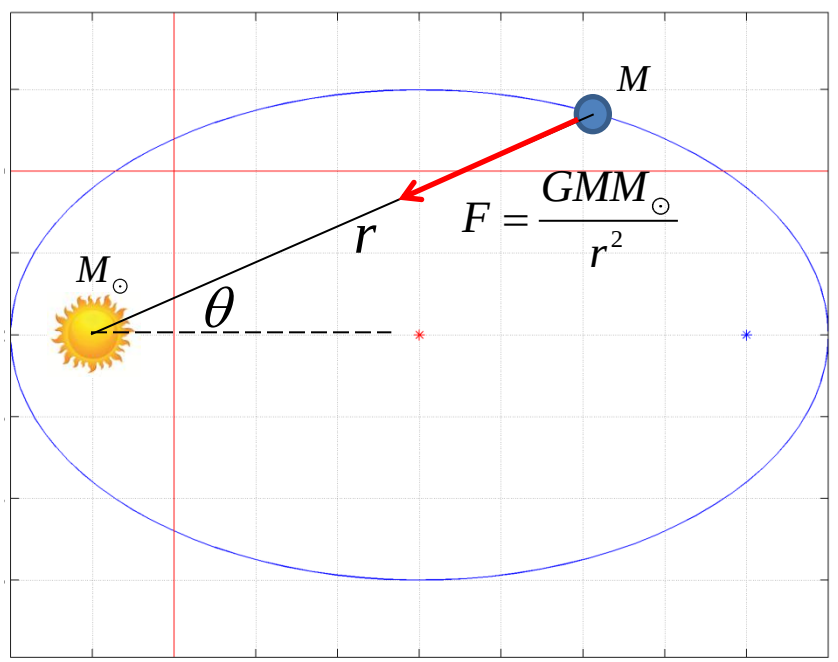
$$P^2 = \frac{4\pi^2}{G(M+M_\odot)}a^3$$ Orbital period *P*

Semi-major axis

$2a$

Semi-minor axis

$2b$



Kepler's three laws are:

1. *The orbit of every planet in the solar system is an ellipse with the Sun at one of the two foci.*
2. *A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.*
3. *The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.*

The wording of Kepler's laws implies a specific application to the solar system. However, the laws are more generally applicable to any system of two masses whose mutual attraction is an inverse-square law.

$$r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\theta}$$ *Polar* equation of ellipse

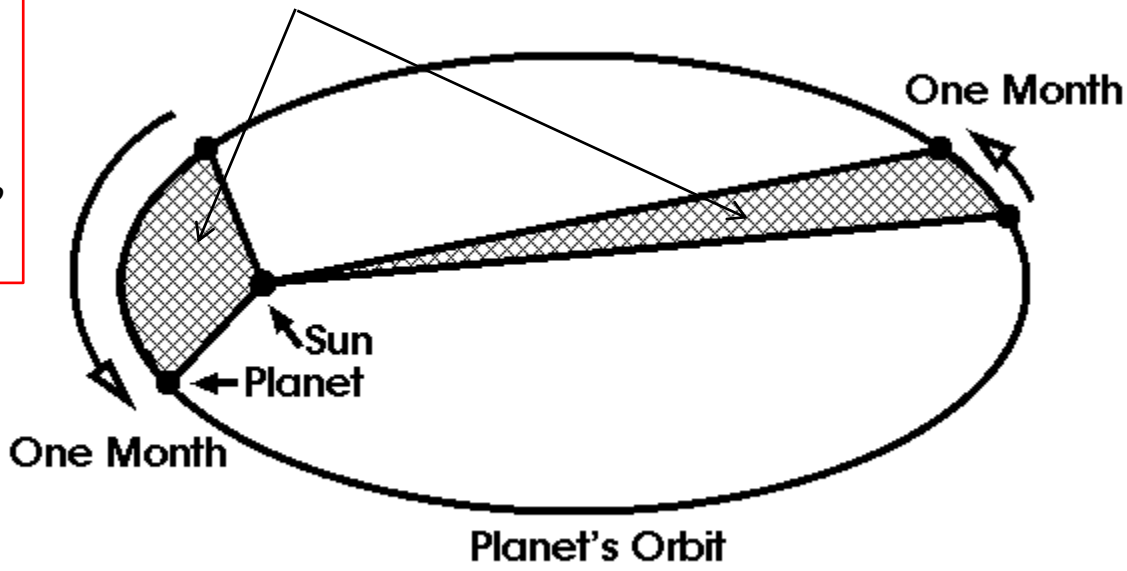
$$\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$$ *Eccentricity* of ellipse

$$P^2 = \frac{4\pi^2}{G(M+M_\odot)}a^3$$ Orbital period *P*

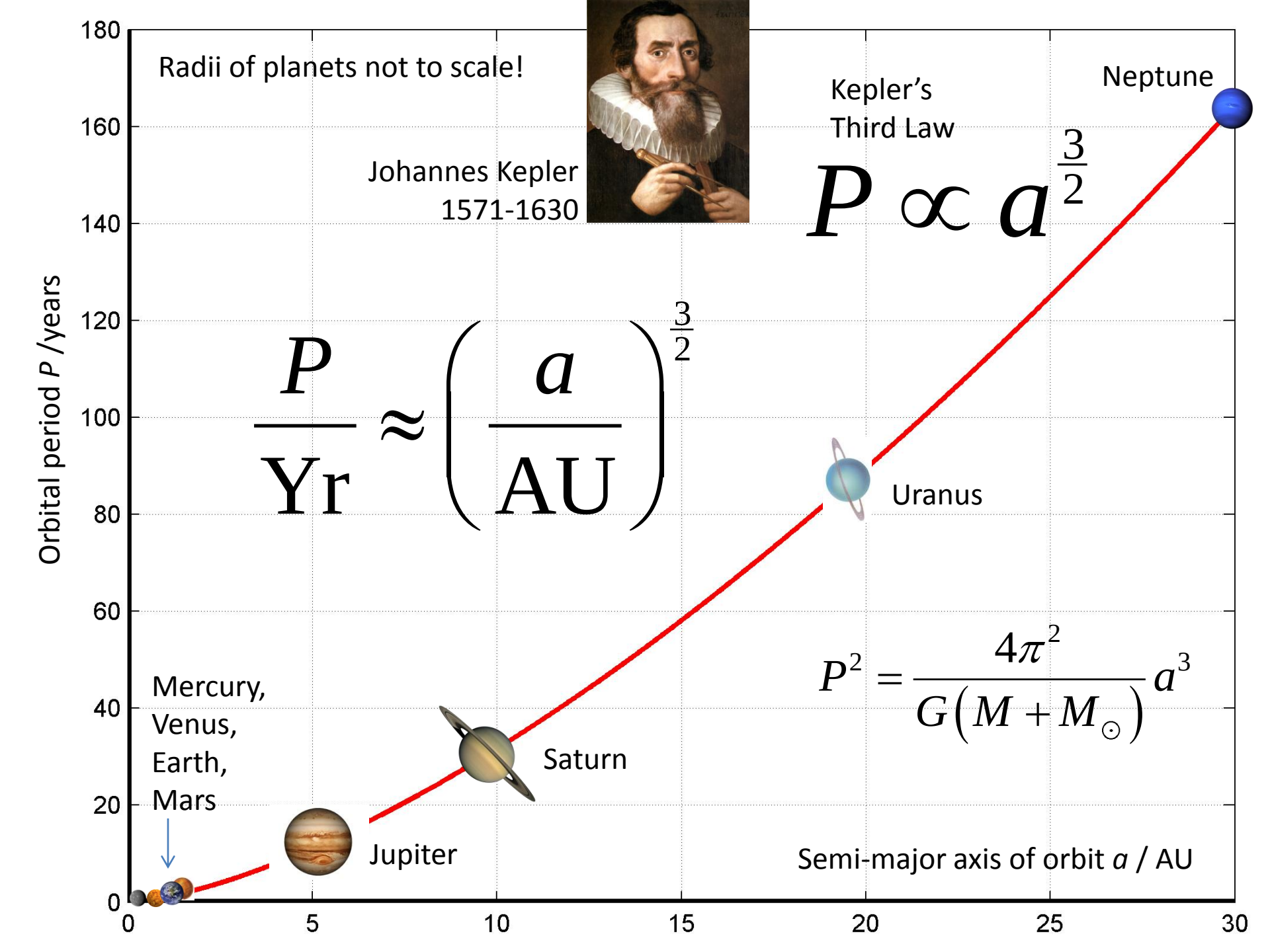
$$\frac{dA}{dt} = \tfrac{1}{2}\sqrt{G(M+M_\odot)\left(1-\varepsilon^2\right)a}$$

This is a constant

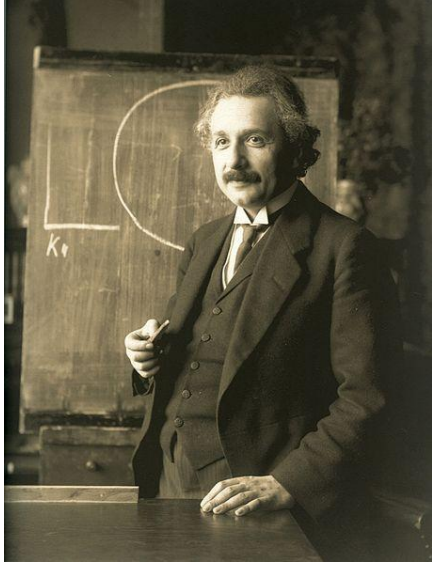
Equal areas swept out in equal times



Johannes Kepler
1571-1630

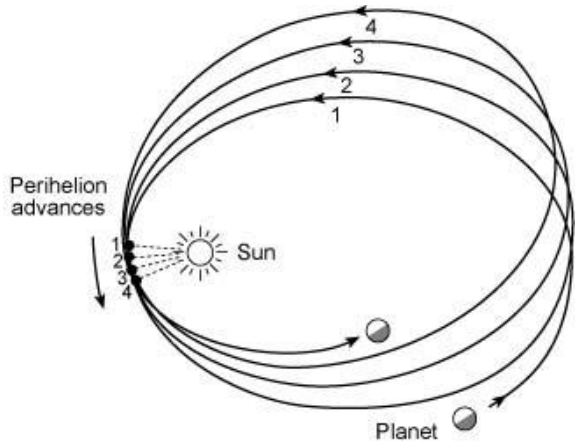

Radii of planets not to scale!
Johannes Kepler
1571-1630
Kepler's
Third Law
$P \propto a^{\frac{3}{2}}$
$\frac{P}{\mathrm{Yr}} \approx \left(\frac{a}{\mathrm{AU}}\right)^{\frac{3}{2}}$
$P^2 = \frac{4\pi^2}{G\left(M + M_\odot\right)} a^3$
Orbital period P /years
Semi-major axis of orbit a / AU
Neptune
Uranus
Saturn
Jupiter
Mercury,
Venus,
Earth,
Mars
0
20
40
60
80
100
120
140
160
180
0
5
10
15
20
25
30

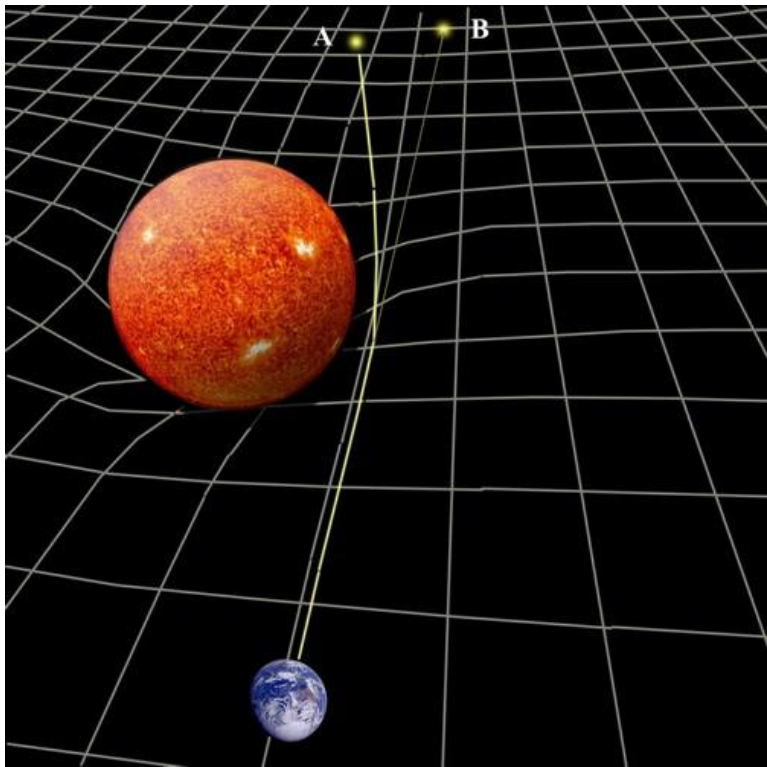
Albert Einstein (1879-1955) proposed a radical new theory of gravity, General Relativity, in which both space & time ('*spacetime*') are *curved* by the presence of mass. This helped to explain *anomalies* in the Newtonian model such as the *precession of the orbit Mercury* and the amount that light is bent by massive objects (*Gravitational lensing*). Note General Relativity predicts the *same* planetary dynamics as Newton's model when gravity is fairly weak. i.e. Newton's model can be thought of as an *approximation*.



Sources of the precession of perihelion for Mercury

Amount (arcsec/Julian century)	Cause
531.63 ±0.69[4]	Gravitational tugs of the other planets
0.0254	Oblateness of the Sun (quadrupole moment)
42.98 ±0.04[5]	General relativity
574.64±0.69	Total
574.10±0.65[4]	Observed





Escape velocity

To escape the gravity of a spherical astronomical body of mass *M* and radius *R* the total energy of the system must be positive at an infinite distance from the body.

In other words, it will have some kinetic energy and will never be gravitationally attracted back towards the body.

For a mass *m* blasting off with velocity *v,* it will escape the gravitational influence of *M* if:

$$\frac{1}{2}mv^2 - \frac{GMm}{R} > 0$$

$$\therefore \quad v > \sqrt{\frac{2GM}{R}}$$

For Earth, the escape velocity is:

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

$$v_{escape} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.38 \times 10^{6}}} \approx 11.2 \text{kms}^{-1}$$

It is interesting to work out the radius of a star of mass *M* such that the escape velocity exceeds that of the speed of light. Since this is not possible, the star becomes a ***Black Hole***.

This inequality defines the maximum radius of a Black Hole, which is called the ***Schwarzschild radius***. This is the *event horizon*, or 'point of no return' from the centre of a Black Hole.

$$\sqrt{\frac{2GM}{R}} > c$$

$$\frac{2GM}{R} > c^2$$

$$R < \frac{2GM}{c^2}$$

For the Sun to become a Black Hole ($M = 2 \times 10^{30}$ kg, $R = 6.96 \times 10^{8}$ m) its radius would have to **shrink to less than 2.97 km.**

$$\rho_{Black\ hole} > \frac{M}{\frac{4}{3}\pi\left(\frac{2GM}{c^2}\right)^3}$$

$$\rho_{Black\ hole} > \frac{3c^6}{32\pi G^3 M^2}$$

This is a mindblowing density of 1.8×10^{19} kgm^{-3} !