Gravitation & orbits questions

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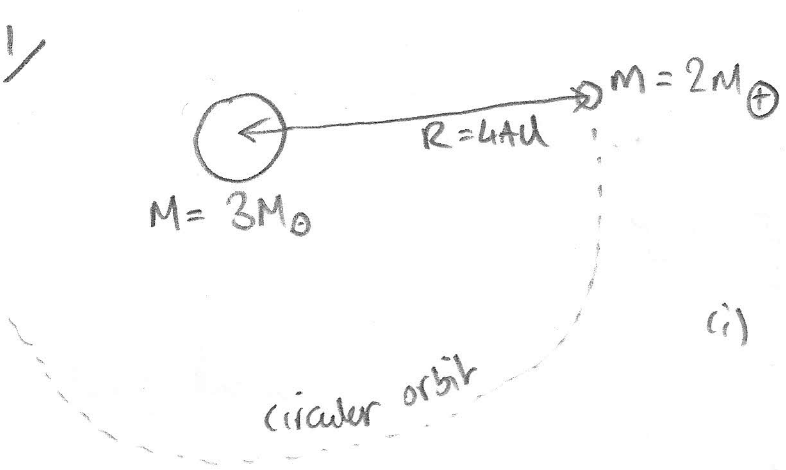


Kepler's Third Law

$$T^2 = \frac{4\pi^2}{G(m+M)} R^3$$

(i) For Earth - Sun

$$T_\oplus^{\,2} = \frac{4\pi^2}{G(M_\oplus + M_\odot)} R_\oplus^3$$

$$T_\oplus = 1 \text{ year}$$

$$R_\oplus = 1 \text{ AU}$$

So $$\left(\frac{T}{T_\oplus}\right)^2 = \frac{M_\oplus + M_\odot}{2M_\oplus + 3M_\odot}\left(\frac{R}{R_\oplus}\right)^3$$

Now since $M_\odot \simeq 332{,}837 \, M_\oplus$ $\qquad M_\odot \gg M_\oplus$

$M_\odot$ ← solar mass

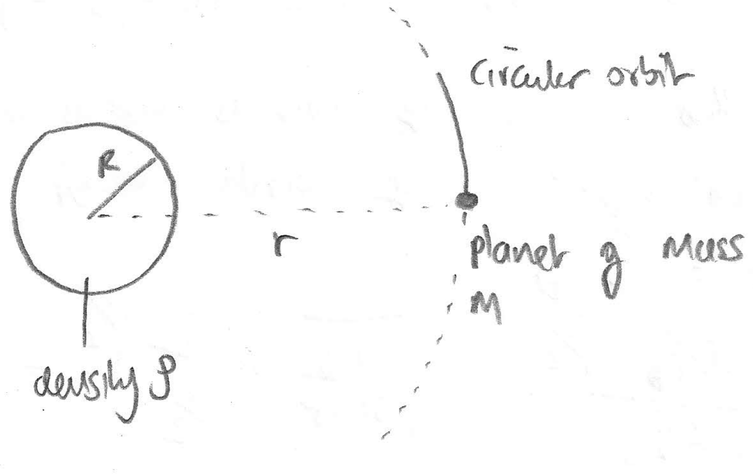
$$\therefore \left(\frac{T}{T_\oplus}\right)^2 \simeq \frac{1}{3}\left(\frac{R}{R_\oplus}\right)^3$$

ie mass of planet is largely irrelevant to the calculation.

$$\therefore \frac{T}{T_\oplus} = \frac{1}{\sqrt{3}} 4^{3/2} = \boxed{\frac{8}{\sqrt{3}}}$$

$$\simeq \boxed{4.62 \text{ years}}$$

2/



Newton II

$$\frac{Mv^2}{r} = \frac{GmM}{r^2}$$

$$v = \frac{2\pi r}{T}$$

period

$$\therefore \quad \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

Now $\quad M = \frac{4}{3}\pi R^3 \rho$

$$\therefore \quad \frac{\cancel{4}\pi^2 r^3}{T^2} = \frac{\cancel{4\pi} G}{3} R^3 \rho$$

$$\therefore \quad T^2 = \frac{3\pi r^3}{G\rho R^3}$$

$$\boxed{T = \sqrt{\frac{3\pi}{G}} \; \frac{1}{\sqrt{\rho}} \left(\frac{r}{R}\right)^{3/2}}$$

orbital period

So $T \propto \frac{1}{\sqrt{\rho}}$

and $\propto \left(\frac{r}{R}\right)^{3/2}$

Now for Sun $\rho_\odot \simeq 1406 \text{ kg m}^{-3}$

{Solar mass $M_\odot = 1.989\text{e}30$ kg}

$R_\odot \simeq 109.123 \, R_\oplus$

$r_\oplus = 1.496\text{e}11$ m

$R_\oplus = 6.38\text{e}6$ m $\qquad G = 6.67\text{e-}11 \text{ N kg}^{-2} \text{m}^2$

[So for Earth orbiting about the Sun

$$T_\oplus = \sqrt{\frac{3\pi}{6.67\text{e-}11}} \times \frac{1}{\sqrt{1406}} \left(\frac{1.496\text{e}11}{109.123 \times 6.38\text{e}6}\right)^{3/2} \quad \text{(seconds)}$$

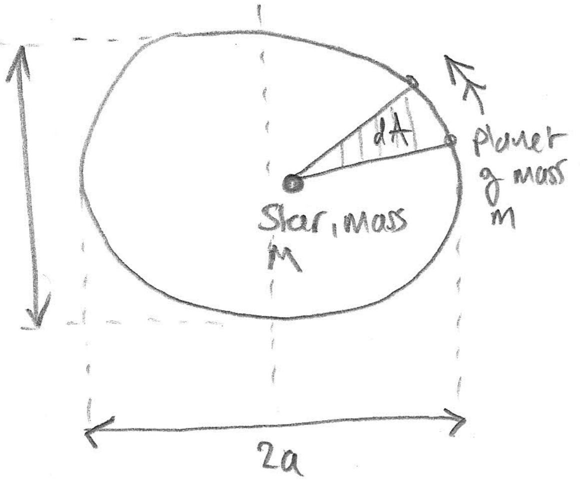
$= 1.001$ years $\quad (3.158\text{e}7 \text{ s})$]

So if $\quad R = \frac{1}{2} \times R_\odot \quad$ (ie half as large as our Sun)

$\rho = 5488 \text{ kg m}^{-3} \quad$ (ie Earth density)

$r = 5 \text{AU} = 5 r_\oplus$

$$T/T_\oplus = \sqrt{\frac{\rho_\odot}{\rho}} \left(\frac{r/r_\oplus}{R/R_\odot}\right)^{3/2} = \sqrt{\frac{1406}{5488}} \left(\frac{5}{\frac{1}{2}}\right)^{3/2} = \boxed{16 \text{ years}}$$



$$\boxed{\frac{dA}{dt} = \frac{1}{2}\sqrt{G(m+M)(1-\varepsilon^2)a}}$$

Kepler II ie area swept out is the same in any given time interval by a line joining a planet to a star

m	mass of planet
M	mass of star
ε	Eccentricity of orbit
a	Semi-major axis of orbit

$$\varepsilon = \sqrt{1 - b^2/a^2}$$

$$\varepsilon^2 = 1 - b^2/a^2$$

$$a = b\sqrt{1-\varepsilon^2}$$

$$G = 6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2}$$

For Earth, Sun system

$$M_\oplus = 5.97\times10^{24}\ \mathrm{kg}$$

$$M_\odot = 1.99\times10^{30}\ \mathrm{kg}$$

$$\varepsilon = 0.02$$

$$a = 1\,\mathrm{AU} = 1.496\times10^{11}\ \mathrm{m}$$

$$\therefore \frac{dA}{dt} = \frac{1}{2}\sqrt{6.67\times10^{-11}\left(5.97\times10^{24} + 1.99\times10^{30}\right)\left(1-0.02^2\right)\times 1.496\times10^{11}}$$

$$= 2.23\times10^{15}\ \mathrm{m^2/s}$$

$$= \boxed{1.92\times10^{20}\ \mathrm{m^2/day}}\ (*) \quad (1\,\mathrm{day} = 24\times3600\,\mathrm{s})$$

Now area of orbit is $\pi a^2(1-\varepsilon^2)$ (ie area of ellipse is πab and $b = (1-\varepsilon^2)^{\frac{1}{2}}a$)

so expect $\dfrac{\pi a^2(1-\varepsilon^2)^{\frac{1}{2}}}{(*)}$ to be $\simeq 365$

$$\simeq \frac{\pi\times\left(1.496\times10^{11}\right)^2}{1.92\times10^{20}} = 365.3\ ✓$$

Note $\dfrac{\pi a^2 (1-\varepsilon^2)^{\frac{1}{2}}}{dA/dt}$

$$= \frac{2\pi a^{3/2}\sqrt{1-\varepsilon^2}}{\sqrt{G(m+M)(1-\varepsilon^2)}} = \frac{2\pi a^{3/2}}{\sqrt{G(m+M)}}$$

Now from Kepler III

$$T^2 = \frac{4\pi^2}{G(M+m)} a^3$$

$$T_\oplus^2 = \frac{4\pi^2}{G(M_\odot+M_\oplus)} a_\oplus^3$$

$$T_\oplus = \frac{2\pi}{\sqrt{(M_\odot+M_\oplus)}} a_\oplus^{3/2}$$

which ends up with the same formula.

For Pluto: $M_p = 0.003$ (so $\ll M_\odot$)

$\varepsilon = 0.25$ $a = 39.509$ AU

So
$$\frac{dA}{dt} \cong \frac{1}{2}\sqrt{GM_\odot(1-\varepsilon^2)a}$$

$$= \frac{1}{2}\sqrt{6.67\times10^{-11} \times 1.99\times10^{30} \times (1-0.25^2) \times \underbrace{39.509 \times}_{1.496\times10^{11}}}$$

$$= 1.36 \times 10^{16} \text{ m}^2/\text{s}$$

$$= \boxed{1.17 \times 10^{21} \text{ m}^2/\text{day}}$$

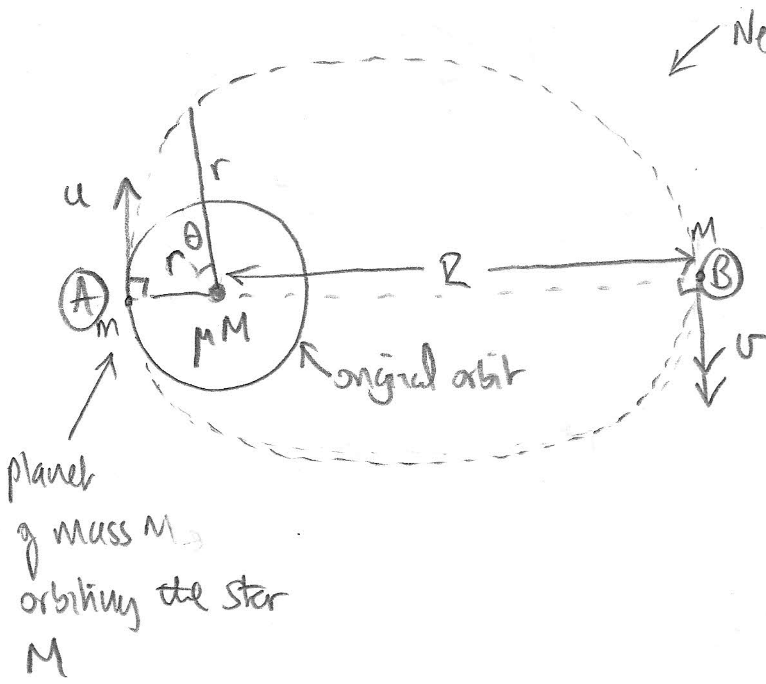
NOTE

$$\frac{dA}{dt} \Big/ \left(\frac{dA}{dt}\right)_\oplus \cong \sqrt{(1-\varepsilon)^2\, a/a_\oplus}$$ so for Pluto this is

$$\sqrt{(1-0.25^2) \times 39.509} = \boxed{6.19}$$

$$\frac{1.17\times10^{21}}{1.92\times10^{20}} = 6.09$$, so agreement (to numerical precision!)

4/



let Star lose mass fraction $\boxed{1-\mu}$ from original mass M

ie $\boxed{\Delta m = (1-\mu)M}$

and $\boxed{M' = \mu M}$ new mass

Angular momentum is conserved so $\boxed{ru = Rv}$

New orbit has polar equation $r = \dfrac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta} = \dfrac{a(1+\varepsilon)(1-\varepsilon)}{1+\varepsilon\cos\theta}$

where $\boxed{2a = r + R}$. Now when $\theta = 0$, $\boxed{r = a(1-\varepsilon)}$

and when $\theta = 180°$, $\boxed{R = a(1+\varepsilon)}$

$$\therefore \boxed{\frac{R}{r} = \frac{1+\varepsilon}{1-\varepsilon}} \qquad \text{Now } \frac{R}{r} = \frac{u}{v}$$

$$\text{so} \quad \boxed{\frac{1+\varepsilon}{1-\varepsilon} = \frac{u}{v}}$$

Newton II

Now for original circular orbit $\dfrac{u^2}{r} = \dfrac{GM}{r^2}$

$$\therefore \boxed{u^2 = \frac{GM}{r}}$$

Conserving energy between points A and B

$$\frac{1}{2}u^2 - \frac{G\mu M}{r} = \frac{1}{2}v^2 - \frac{G\mu M}{R}$$

$$\therefore \quad u^2 - \frac{2G\mu M}{r} = v^2 - \frac{2G\mu M}{R}$$

$$\text{So} \quad \frac{GM}{r} - \frac{2G\mu M}{r} + \frac{2G\mu M}{R} = v^2$$

$$\Rightarrow \quad v^2 = \frac{GM}{r}\left(1 - 2\mu + 2\mu\frac{r}{R}\right)$$

Now using $ru = Rv$

$$\Rightarrow \quad v^2 = \left(\frac{r}{R}\right)^2 u^2$$

$$v^2 = \left(\frac{r}{R}\right)^2 \frac{GM}{r}$$

$$\therefore \quad \left(\frac{r}{R}\right)^2 = 1 - 2\mu + 2\mu\frac{r}{R}$$

$$\text{So} \quad \left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)(2\mu) + 2\mu - 1 = 0$$

$$\left(\frac{r}{R} - \mu\right)^2 - \mu^2 + 2\mu - 1 = 0$$

$$\left(\frac{r}{R} - \mu\right)^2 - \left(\mu^2 - 2\mu + 1\right) = 0$$

$$\left(\frac{r}{R} - \mu\right)^2 - (\mu - 1)^2 = 0$$

$$\left(\frac{r}{R} - \mu + \mu - 1\right)\left(\frac{r}{R} - \mu - \mu + 1\right) = 0$$

$$\left(\frac{r}{R} - 1\right)\left(\frac{r}{R} + 1 - 2\mu\right) = 0$$

Now $r = R$ is not a solution since $\mu \neq 0$ and orbit clearly must become more eccentric as the gravitational force on the planet must decrease initially as $M \to \mu M$

So $\frac{r}{R} + 1 - 2\mu = 0 \Rightarrow \frac{r}{R} = 2\mu - 1$

Since $\frac{r}{R} > 0 \Rightarrow 2\mu - 1 > 0$

$$\boxed{\mu > \frac{1}{2}}$$

ie we cannot lose more than 50% of the star mass otherwise the orbit will become unbound (ie planet will escape).

↑ via a hyperbola or parabolic orbit.

So $$\boxed{\frac{r}{R} = 2\mu - 1 \qquad \mu > \frac{1}{2}}$$

$\therefore \frac{R}{r} = \frac{1}{2\mu - 1} = \frac{1+\varepsilon}{1-\varepsilon}$

$$\therefore \quad 1 - \varepsilon = (2\mu - 1)(1+\varepsilon)$$

$$1 = \varepsilon(2\mu - 1 + 1) + 2\mu - 1$$

$$2 = 2\mu\varepsilon + 2\mu$$

$$1 = \mu\varepsilon + \mu$$

$$\boxed{\varepsilon = \frac{1-\mu}{\mu}}$$

$\frac{\mu}{\mu} + \frac{1-\mu}{\mu}$

$\therefore \quad 1 + \varepsilon = \frac{1}{\mu}$

$1 - \varepsilon = \frac{\mu}{\mu} - \frac{1-\mu}{\mu}$

$= \frac{2\mu - 1}{\mu}$

$$\therefore \quad R = \frac{1+\varepsilon}{1-\varepsilon} r$$

$$= \frac{1/\mu}{\frac{2\mu-1}{\mu}} r$$

$$\boxed{R = \frac{1}{2\mu-1} r}$$

$$\therefore \quad a = \frac{r+R}{2}$$

$$= \frac{r}{2}\left(1 + \frac{1}{2\mu-1}\right)$$

$$= \frac{r}{2}\left(\frac{2\mu-1+1}{2\mu-1}\right)$$

$$\boxed{a = \frac{\mu}{2\mu-1} r}$$

So using Kepler III and $T_0^2 = \frac{4\pi^2}{GM} r^3$

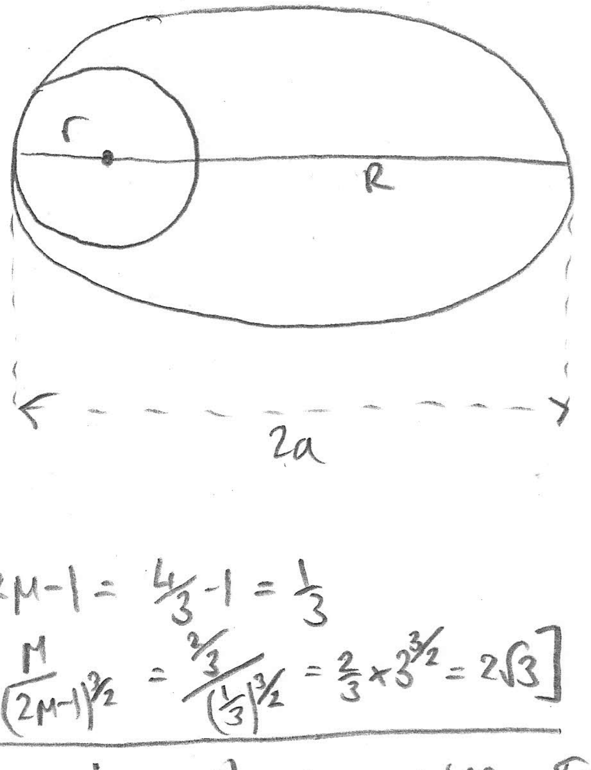
$$T^2 = \frac{4\pi^2}{\mu GM} a^3 \qquad \therefore \quad \left(\frac{T}{T_0}\right)^2 = \frac{1}{\mu}\left(\frac{a}{r}\right)^3$$

$\mu M + M_\oplus \simeq \mu M$

$$\therefore \left(\frac{T}{T_0}\right)^2 = \frac{1}{\mu}\left(\frac{\mu}{2\mu-1}\right)^3 = \frac{\mu^2}{(2\mu-1)^3}$$

$$\boxed{\frac{T}{T_0} = \frac{\mu}{(2\mu-1)^{3/2}}}$$

Summary of new orbit: $M \to \mu M \qquad \mu > \frac{1}{2}$



$$a = \frac{\mu}{2\mu-1}r$$

$$R = \frac{1}{2\mu-1}r$$

$$\varepsilon = \frac{1-\mu}{\mu}$$

$$\frac{T}{T_0} = \frac{\mu}{(2\mu-1)^{3/2}}$$

$\left[2\mu - 1 = \frac{4}{3} - 1 = \frac{1}{3}\right.$

so $\frac{\mu}{(2\mu-1)^{3/2}} = \frac{2/3}{\left(\frac{1}{3}\right)^{3/2}} = \frac{2}{3} \times 3^{3/2} = 2\sqrt{3}\left.\right]$

So if planet was Earth and star the Sun
if $\mu = \frac{2}{3}$ and $T_0 = 1$ year and $r = 1\text{AU}$

$\boxed{\varepsilon = \frac{1}{2} \qquad R = 3 \qquad a = 2} \qquad \frac{T}{T_0} = \boxed{3.46} = 2\sqrt{3}$

By Kepler II, planet will move faster when it is nearer the star. So long, very cold winters followed by a short summer similar to Earth

The winters will be longer as the planet will spend more of the year > r away, and much colder

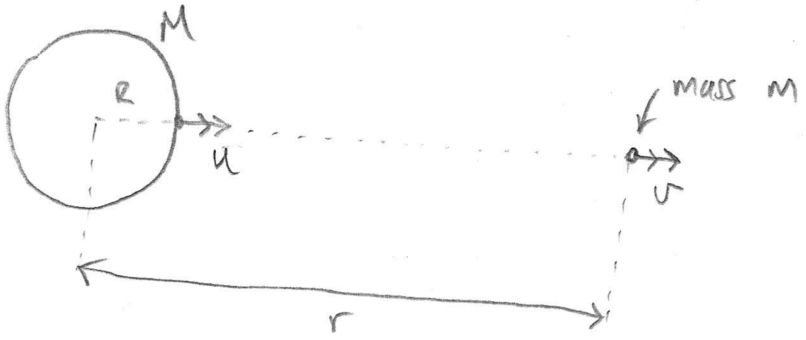
since the heat flux from the star $\propto \frac{1}{\text{distance}^2}$

At the furthest point it recieves $\frac{1}{3^2}$ = nine times

less heat / m^2 than at the closest point.

5/

(i)



let particle of mass m leave the surface of a body of mass M and radius R with radial velocity u

Conservation of Energy

$$\frac{1}{2}mu^2 - \frac{GmM}{R} = \frac{1}{2}mv^2 - \frac{GmM}{r}$$

Particle will escape if v is positive as $r \to \infty$

Now this means KE left as $r \to \infty$

So $\boxed{\frac{1}{2}u^2 - \frac{GM}{R} > 0}$ implies u is sufficient for particle to escape the gravitational pull of M

$\therefore$ Escape velocity $\boxed{u > \sqrt{\frac{2GM}{R}}}$

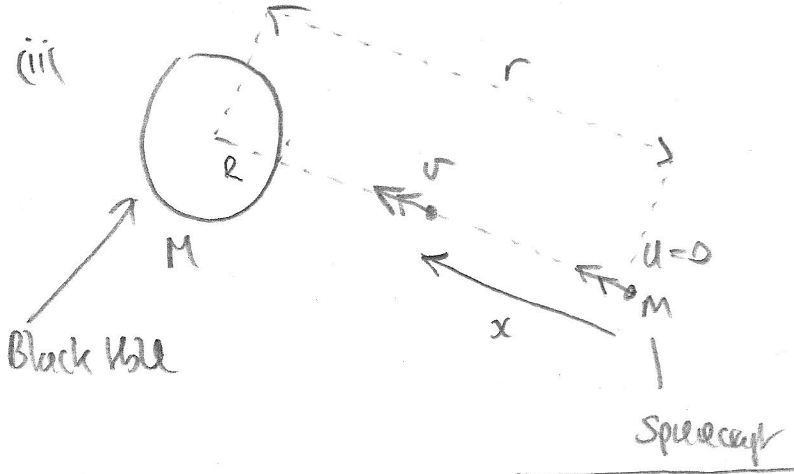
If $u = c$ then object of mass M is a Black Hole

let $c = \sqrt{\frac{2GM}{R}}$ $\therefore$ $\boxed{R = \frac{2GM}{c^2}}$

This is the maximum radius of a Black Hole of mass M.

"Event Horizon"

(iii)



Conservation of Energy

$$-\frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{r-x}$$

So $v^2 = 2GM\left(\frac{1}{r-x} - \frac{1}{r}\right)$

$\therefore$ when $x = r-R$ $\boxed{v^2 = 2GM\left(\frac{1}{R} - \frac{1}{r}\right)}$

Now let $R = \frac{2GM}{c^2}$

$$\therefore \quad v^2 = 2GM\left(\frac{c^2}{2GM} - \frac{1}{r}\right)$$

$$v^2 = c^2 - \frac{2GM}{r}$$

$$\boxed{v = \sqrt{c^2 - 2GM/r}} \qquad r > 2GM/c^2$$

let $r = kR$ where $k > 1$

$$\therefore \quad r = k \times \frac{2GM}{c^2}$$

$$\therefore \quad \frac{2GM}{r} = 2GM \times \frac{c^2}{2GMk} = c^2/k$$

$$\therefore \quad \boxed{v = c\sqrt{1 - 1/k}}$$

So if spacecraft starts from $k \gg 1$ $v \to c$

Now $v = \frac{dx}{dt}$ $\qquad v^2 = 2GM\left(\frac{1}{r-x} - \frac{1}{r}\right)$

$$\therefore \quad \frac{dx}{dt} = \sqrt{2GM\left(\frac{1}{r-x} - \frac{1}{r}\right)}$$

$$\therefore \quad t = \frac{1}{\sqrt{2GM}} \int_0^{r-R} \frac{1}{\sqrt{\frac{1}{r-x} - \frac{1}{r}}}\, dx$$

$$\frac{1}{r-x} - \frac{1}{r} = \frac{r-(r-x)}{r(r-x)} = \frac{x}{r(r-x)}$$

$$\therefore \quad \frac{1}{\sqrt{\frac{1}{r-x} - \frac{1}{r}}} = \frac{1}{\sqrt{\frac{x}{r(r-x)}}} = \sqrt{\frac{r-x}{x}} \times \sqrt{r}$$

$$\therefore \quad t = \sqrt{\frac{r}{2GM}} \int_0^{r-R} \sqrt{\frac{r-x}{x}}\, dx$$

consider a substitution $u = \sqrt{\frac{r-x}{x}}$

$$\therefore \quad u^2 = \frac{r-x}{x} \Rightarrow xu^2 = r-x \Rightarrow xu^2 + x = r$$

$$\Rightarrow \boxed{x = \frac{r}{1+u^2}}$$

$$\therefore \quad \frac{dx}{du} = \frac{-2ur}{(1+u^2)^2}$$

$$\therefore \quad \int \sqrt{\frac{r-x}{x}}\, dx = -2r \int \frac{u^2}{(1+u^2)^2}\, du$$

let $u = \tan\theta$ Note $1 + \tan^2\theta = \sec^2\theta$

$$\frac{d}{d\theta} \tan\theta = \sec^2\theta$$

So $\frac{du}{d\theta} = \sec^2\theta$

$$\therefore \quad \int \frac{u^2}{(1+u^2)^2}\, du = \int \frac{\tan^2\theta \sec^2\theta\, d\theta}{\sec^2\theta \times \sec^2\theta} = \int \frac{\tan^2\theta}{\sec^2\theta}\, d\theta$$

$$= \int \frac{\sec^2\theta - 1}{\sec^2\theta}\, d\theta = \int (1 - \cos^2\theta)\, d\theta$$

$\left[\frac{1}{\sec^2\theta} = \cos^2\theta \right]$ Now $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$$\text{So} \quad \int \frac{\tan^2\theta}{\sec^2\theta}\, d\theta = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta \right) d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + C$$

$$\therefore \int \frac{u^2}{(1+u^2)^2}\,du = \frac{1}{2}\tan^{-1}u - \frac{1}{4}\sin\left(2\tan^{-1}u\right) + c$$

Now $\quad \sin 2\theta = 2\sin\theta\cos\theta = 2\sqrt{1-\cos^2\theta}\,\cos\theta$

$$= 2\sqrt{\cos^2\theta - \cos^4\theta}$$

Also $\quad \dfrac{1}{\cos^2\theta} = 1 + \tan^2\theta \qquad \therefore \cos^2\theta = \dfrac{1}{1+\tan^2\theta}$

Hence $\quad \cos^2\theta - \cos^4\theta = \dfrac{1}{1+\tan^2\theta} - \dfrac{1}{(1+\tan^2\theta)^2}$

$$= \frac{1}{(1+\tan^2\theta)^2}\left(1+\tan^2\theta - 1\right)$$

$$= \frac{\tan^2\theta}{(1+\tan^2\theta)^2}$$

$\therefore \boxed{\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}} \qquad$ So $\quad \dfrac{1}{4}\sin 2\theta = \dfrac{1}{2}\dfrac{\tan\theta}{1+\tan^2\theta}$

$$\therefore \int \frac{u^2}{(1+u^2)^2}\,du = \frac{1}{2}\tan^{-1}u - \frac{1}{2}\frac{u}{1+u^2} \qquad \text{Since } u = \tan\theta$$

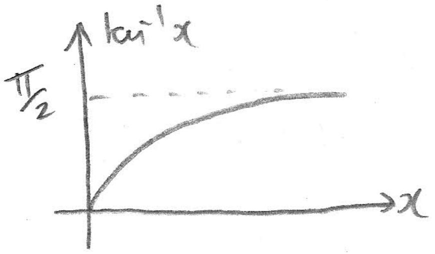
$\therefore$ Since $\quad u = \sqrt{\dfrac{r-x}{x}}$

$$\therefore t = \sqrt{\frac{r}{2GM}}\left[-2r\left(\frac{1}{2}\tan^{-1}\sqrt{\frac{r-x}{x}} - \frac{1}{2}\frac{\sqrt{\frac{r-x}{x}}}{1+\frac{r-x}{x}}\right)\right]_0^{r-R}$$

$$t = \sqrt{\frac{r^3}{2GM}}\left[\frac{1}{r}\sqrt{x(r-x)} - \tan^{-1}\sqrt{\frac{r-x}{x}}\right]_0^{r-R}$$

$\left[\text{Since } \dfrac{\sqrt{\frac{r-x}{x}}}{1+\frac{r-x}{x}} = \dfrac{x\sqrt{\frac{r-x}{x}}}{r} = \dfrac{1}{r}\sqrt{x(r-x)}\right]$

Now :



So as $x \to \infty$
$\tan^{-1}x \to \pi/2$

$$\therefore \quad t = \sqrt{\frac{r^3}{2GM}}\left(\left(\frac{1}{r}\sqrt{(r-R)(r-r+R)} - \tan^{-1}\sqrt{\frac{r-r+R}{r-R}}\right) - \left(-\frac{\pi}{2}\right)\right)$$

$$t = \sqrt{\frac{r^3}{2GM}}\left(\frac{1}{r}\sqrt{R(r-R)} - \tan^{-1}\sqrt{\frac{R}{r-R}} + \frac{\pi}{2}\right)$$

let $r = kR$ as before, where $R = 2GM/c^2$, $k > 1$

$$\sqrt{\frac{r^3}{2GM}} = \sqrt{\frac{k^3 \times 8G^3M^3}{2GM \times c^6}} = \sqrt{\frac{4k^3G^2M^2}{c^6}} = \boxed{\frac{2GM}{c^3}k^{3/2}}$$

$$\therefore \quad \frac{1}{r}\sqrt{R(r-R)} = \frac{R}{r}\sqrt{k-1} = \frac{1}{k}\sqrt{k-1} = \sqrt{\frac{k-1}{k^2}}$$

$$\frac{R}{r-R} = \frac{R}{kR-R} = \frac{1}{k-1}$$

$$\text{So} \quad t = \frac{2GM}{c^2}\frac{k^{3/2}}{c}\left(\sqrt{\frac{k-1}{k^2}} - \tan^{-1}\left(\frac{1}{k-1}\right) + \frac{\pi}{2}\right)$$

$$\boxed{t = \frac{R}{c}k^{3/2}\left(\sqrt{\frac{k-1}{k^2}} - \tan^{-1}\left(\frac{1}{k-1}\right) + \frac{\pi}{2}\right) \qquad R = 2GM/c^2}$$

Now for our Sun $R = \frac{2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(2.998 \times 10^{8})^2}$

$= \boxed{2,954 \text{m}}$

So $R/c = \boxed{9.85 \times 10^{-6} \text{ s}}$ $(\simeq 9.8 \mu s)$

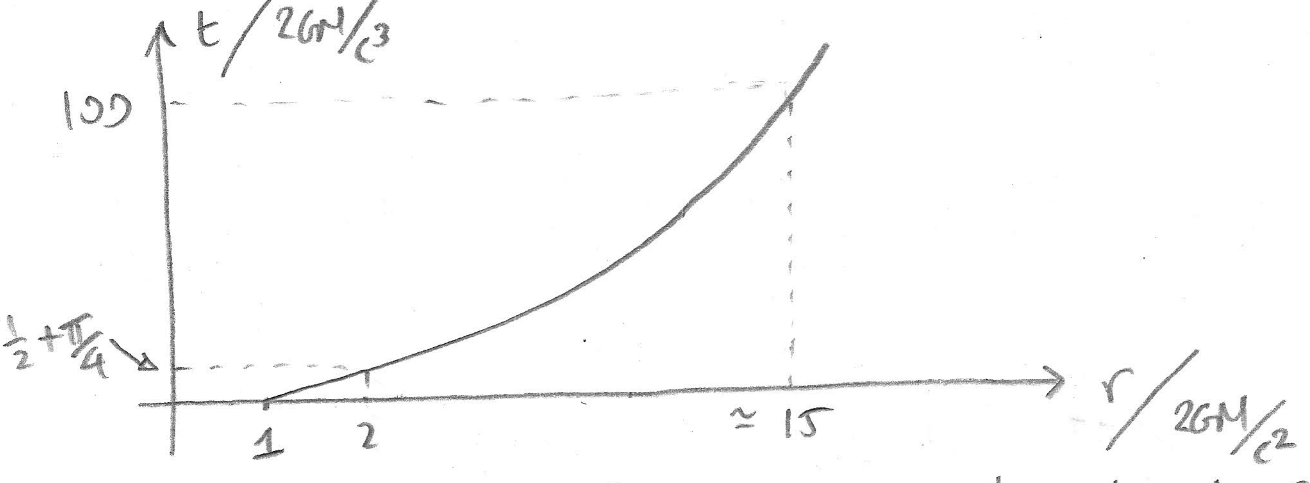
In the movie Interstellar, the Black Hole "Gargantua"
has $M = 100 \times 10^{6} M_{\odot}$

$\therefore$ $R = \frac{2,954 \times 10^{8}}{1.496 \times 10^{11}}$ AU

$R \simeq \boxed{1.97 \text{ AU}}$

(orbit of Mars is 1.523 AU, so Gargantua would contain the inner solar system)

For Gargantua R/c = 985.2 seconds ie $\simeq$ 16.4 minutes



Miller's planet in the film is very close to the Event Horizon of Gargantua, so let $h=2$.

In this case $t = 985.2 \left(\sqrt{\frac{1}{4}} - \tan^{-1}\left(\frac{1}{1}\right) + \frac{\pi}{2} \right) \times 2^{3/2}$ ← $\tan^{-1} 1 = \pi/4$

$t = 985.2 \times 2\sqrt{2} \left(\frac{1}{2} + \frac{\pi}{4} \right) \simeq 3582 s \simeq \boxed{1 \text{ hour}}$