

Cosmology problems

1/ One physics hour (Ph) = 35×60 s

$1.5 = \frac{1 \text{ Ph}}{35 \times 60}$

Age of universe (T) = 13.8×10^9 yr

1 yr = $365 \times 24 \times 3600 \times (\pi \times 10^7)$ s

$\therefore T = 13.8 \times 10^9 \times 365 \times 24 \times 3600 \times \frac{1}{35 \times 60}$ Ph

$T = 2.07 \times 10^{14}$ Ph

2/	<u>Real time</u>	<u>Cosmic calendar</u>
	Big Bang	Midnight on January 1 st
	Now	Midnight on January 1 st } + 1 year

(i) Earth birth

$$\frac{4.5 \times 10^9}{13.8 \times 10^9} \times 365 = 119 \text{ days}$$

So April 29th

(ii) Homo Sapiens emerge from Africa

$$\frac{60,000}{13.8 \times 10^9} \times 365 = 1.59 \times 10^{-3} \text{ days}$$

$$= 137 \text{ seconds before end of year}$$

$= 2 \text{ mins } 17 \text{ seconds " " " "}$

		Cumulative
Sa	31	31
Feb	28	59
Mar	31	90
Apr	30	120
May	31	151
Jun	30	181
Jul	31	212
Aug	31	243
Sept	30	273
Oct	31	304
Nov	30	334
Dec	31	365
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	365	

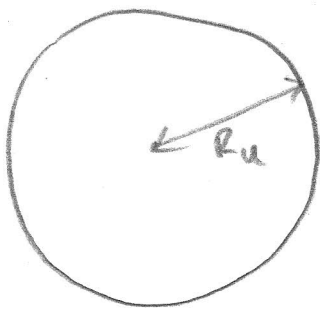
(iii) $2015 - 1384 = 631$ years
(Age of Winchester Gilgel)

$$\frac{631}{13.8 \times 10^9} \times 365 = 1.67 \times 10^{-5} \text{ days}$$

$= 1.44 \text{ seconds}$

before the end of the year.

3/



" Universe "

$$2R_u = 93 \times 10^9 \text{ light years}$$

$$1 \text{ light year} = 2.998 \times 10^8 \times 365 \times 24 \times 3600 \\ = \boxed{9.45 \times 10^{15} \text{ m}}$$

$$\therefore \text{(i)} \quad R_u = \frac{93 \times 10^9}{2} \times 9.45 \times 10^{15} \\ = \boxed{4.40 \times 10^{26} \text{ m}}$$

$$\text{(ii)} \quad V_u = \frac{4}{3} \pi R_u^3 \\ = \frac{4}{3} \pi (4.40 \times 10^{26})^3 = \boxed{3.56 \times 10^{80} \text{ m}^3}$$

$$\text{(iii)} \quad \text{Total mass of universe} \approx 10^{53} \text{ kg} \\ \therefore \text{if proton mass is } m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$\# \text{ protons in universe} \approx \frac{10^{53}}{1.673 \times 10^{-27}}$$

$$\therefore \# \text{ protons / cubic metre} \approx \frac{10^{53}}{1.673 \times 10^{-27} \times 3.56 \times 10^{80}} \\ \approx \boxed{0.17} \text{ protons per cubic metre}$$

[According to WMAP*, actual density is $\approx \boxed{0.25 \text{ protons per cubic metre}}$. BUT mean 'energy density' \Rightarrow mass density of $\approx \boxed{6 \text{ protons / m}^3}$. $\uparrow E=mc^2$ \therefore 95% of mass/energy in the universe is 'dark matter' or 'dark energy' ... or our models of gravity on this scale are wrong!]

* Accurate measurements of the Cosmic Microwave Background Radiation

(2)

$$4/ \quad M_{\text{ordinary}} \approx 10^{53} \text{ kg}$$

$$a) \quad M_{\text{Tot}} \times 0.049 = M_{\text{ordinary}}$$

$$M_{\text{Tot}} \approx \frac{10^{53}}{0.049} = \boxed{2.04 \times 10^{54}} \text{ kg}$$

$$\therefore E_{\text{Tot}} = M_{\text{Tot}} c^2 = 2.04 \times 10^{54} \times (2.998 \times 10^8)^2$$

$$\boxed{E_{\text{Tot}} = 1.83 \times 10^{71} \text{ J}}$$

b) If Big Bang 'released' $1.83 \times 10^{71} \text{ J}$

$$\text{Tsar Bomba energy } E_{\text{TB}} = 210,000 \times 10^{12} \text{ J}$$

$$\therefore \text{Big Bang is equivalent to } \frac{1.83 \times 10^{71}}{210,000 \times 10^{12}}$$

$$= \boxed{8.73 \times 10^{53}} \text{ Tsar Bombas!}$$

$$\text{Note } 1 \text{ kg of matter has energy} = 1 \text{ kg} \times (2.998 \times 10^8)^2 \\ = 8.99 \times 10^{16} \text{ J}$$

$$\text{This is } \frac{8.99 \times 10^{16}}{210,000 \times 10^{12}} = \boxed{0.43} \text{ Tsar Bombas}$$

So in $\boxed{\text{every } 2\frac{1}{3} \text{ kg}}$ of matter, we have sufficient energy to equal the most powerful explosion detonated by humans. If only this energy could be easily released!

5/ $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$

$c = 2.998 \times 10^8 \text{ ms}^{-1}$

(i) $1 \text{ light-minute} = 2.998 \times 10^8 \times 60$
 $= 1.80 \times 10^{10} \text{ m}$

(ii) $1 \text{ light-year} = 2.998 \times 10^8 \times 365 \times 24 \times 3600$
 $= 9.45 \times 10^{15} \text{ m}$

(iii) $1 \text{ AU} = \frac{1.496 \times 10^{11}}{1.80 \times 10^{10}} \text{ light minutes}$
 $= 8.3 \text{ light minutes}$

(iv) Sirius A is 8.6 light years away

$8.6 \text{ light years} = 8.6 \times 9.45 \times 10^{15} \text{ m}$
 $= \frac{8.6 \times 9.45 \times 10^{15}}{1.496 \times 10^{11}} \text{ AU}$
 $= 5.43 \times 10^5 \text{ AU}$

i.e. $\approx 543,000 \times$ distance of Earth to Sun

	<u>New Horizons</u>	Spacecraft	speed	distance/m	time/yr
	Earth - Jupiter	↑ Assume closest approach	16.26 km/s	$4.202 \times 1.496 \times 10^{11}$	t_1
	Jupiter - Pluto		20.26 km/s	$\approx 1 \text{ AU}$	t_2

Furthest $x = 7.5 \times 10^9 + 10^3 - 4.202 \times 1.496 \times 10^{11} = 6.87 \times 10^{12} \text{ m}$
 closest $x = 4.28 \times 10^9 + 10^3 - 4.202 \times 1.496 \times 10^{11} = 3.65 \times 10^{12} \text{ m}$

$t_1 = \frac{4.202 \times 1.496 \times 10^{11}}{16.26 \times 10^3} + \frac{1}{365 \times 24 \times 3600} = 1.23 \text{ years}$ to reach Jupiter.

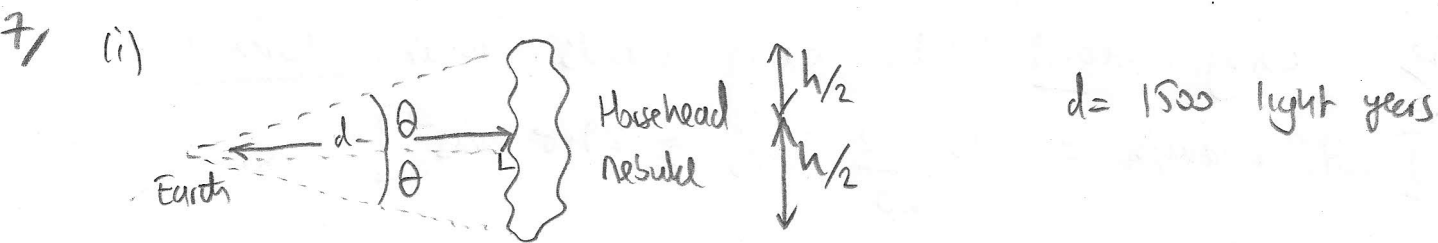
$$\min t_2 = \frac{3.65 \times 6^{12}}{20.26 \times 6^3} + \frac{1}{365 + 24 + 3600} = \boxed{5.71 \text{ years}}$$

$$\max t_2 = \frac{6.87 \times 6^{12}}{20.26 \times 6^3} + \frac{1}{365 + 24 + 3600} = \boxed{10.75 \text{ years}}$$

So total time to reach Pluto will be between 6.94 and 11.98 years.

[In 2015 it hasn't reached Pluto yet! so perhaps a rendezvous anytime 2016 - 2018]

→ NASA mission website predicts July 14, 2015 as the encounter date. It was launched on Sep 19, 2005



$$2\theta = \frac{8}{60} \text{ degrees}$$

let height be h

$$\tan \theta = \frac{h/2}{d}$$

$$h = 2d \tan \theta$$

$$= 2 \times 1500 \times \underbrace{2.998 \times 10^8 \times 365 \times 24 \times 3600}_{1 \text{ light-year in m}} \times \tan\left(\frac{8}{120}\right)$$

$$= \boxed{3.30 \times 10^{16} \text{ m}}$$

$$= \frac{3.30 \times 10^{16}}{1.496 \times 10^{11}} \text{ AU} = \boxed{220,604 \text{ AU}}$$

$$= 2 \times 1500 + \tan\left(\frac{8}{120}\right) \text{ light years}$$

$$= \boxed{3.49 \text{ light years}}$$

(ii) If a Spacecraft travels at $0.01c$
 it will take 349 years to traverse the
 Nebula, since it takes light 3.49 years to
 cover the same distance.

8/ The Earth's orbit is \approx circular about the Sun. In one
year it covers $2\pi \times 1.496 \times 10^{11}$ m

\therefore orbital speed (in km/hour) is

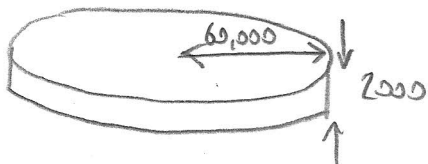
$$v = \frac{2\pi \times 1.496 \times 10^{11} \times \frac{1}{1000} \text{ km}}{365 \times 24 \text{ hours}} = \boxed{107,302 \text{ km/h}}$$

\therefore every second the Earth travels about $\boxed{30 \text{ km}}$

$$\left[107,302 \text{ km/h} = \frac{107,302}{3600} \text{ km/s} = 29.8 \text{ km s}^{-1} \right]$$

9/ $\boxed{\text{Milky Way}}$ (distances in light years)

contains $\approx 400 \times 10^9$ stars



$$\begin{aligned} \text{(i) Volume } V &\approx 2000 \times \pi \times (60,000)^2 \text{ cubic light years} \\ &= 2.26 \times 10^{13} \text{ (light-year)}^3 \\ &= 2.26 \times 10^{13} \times (2.998 \times 10^8 \times 365 \times 24 \times 3600)^3 \\ &= \boxed{1.91 \times 10^{61} \text{ m}^3} \end{aligned}$$

$$\text{(ii) } 1 \text{ AU} = 1.496 \times 10^{11} \text{ m} \quad \therefore 1 \text{ m}^3 = \left(\frac{1}{1.496 \times 10^{11}} \right)^3 \text{ AU}^3$$

$$\therefore V = \frac{1.91 \times 10^{61}}{(1.496 \times 10^{11})^3} = \boxed{5.70 \times 10^{27} \text{ AU}^3}$$

(iii) let each star of the 400×10^9 in the Milky Way correspond to a volume $\frac{4}{3}\pi \times 50^3 \text{ AU}^3$

fraction of empty space is

$$1 - \frac{\frac{4}{3}\pi \times 50^3 \times 400 \times 10^9}{5.75 \times 10^{27}} = \boxed{1 - 3.67 \times 10^{-11}}$$

ie $\boxed{99.9999999963 \%}$

(iv) orbital speed of the Sun about the galactic centre is

$$v = \frac{2\pi \times 27,000 \times 2.998 \times 10^8 \times 365 \times 24 \times 3600 \times \frac{1}{1000} \text{ km}}{240 \times 10^6 \times 365 \times 24 \times 3600 \text{ s}}$$

$$= \boxed{212 \text{ km s}^{-1}}$$

(About 7x speed of the Earth about the Sun)

10/ Divide the volume of the Milky Way into 40×10^6 spheres of radius R . R is the separation of 'Advanced civilizations', and we assume there are 40 million of them.

From Q9: $40 \times 10^6 \times \frac{4}{3}\pi R^3 = 2000 \times \pi \times 60,000^2$

(R is in light years)

$$\Rightarrow R = \sqrt[3]{\frac{2000 \times 60,000^2 \times \pi \times 3}{40 \times 10^6 \times 4\pi}}$$

$$= \boxed{51.3 \text{ light years}}$$

But physical contact is very unlikely!

(7) Since radio transmissions from Earth into space have occurred since the 1930s ... Perhaps our signals have already been intercepted! \rightarrow **SETI**

The Solar System, gravity & orbits

1/ Mercury day = 58.646 Earth days.

$$36 \text{ years} \approx 36 \times 365 \text{ days} = \frac{36 \times 365}{58.646} \text{ Mercury days}$$

$$= 224 \text{ Mercury days}$$

So \times 224 sunrises could have been witnessed on Mercury.

2/ $\bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3}$ (i) $\bar{\rho}_{\text{Earth}} = \frac{5.97 \times 10^{24}}{\frac{4}{3}\pi (6.38 \times 10^6)^3}$

$$= \boxed{5488 \text{ kg m}^{-3}}$$

(ii) $\bar{\rho}_{\text{Mars}} = \frac{0.67}{0.533^3} \times \bar{\rho}_{\text{Earth}} = \boxed{3878 \text{ kg m}^{-3}}$

(iii) $\bar{\rho}_{\text{Jupiter}} = \frac{317.85}{11.209^3} \times \bar{\rho}_{\text{Earth}} = \boxed{1239 \text{ kg m}^{-3}}$

(iv) $\bar{\rho}_{\text{Saturn}} = \frac{95.159}{9.449^3} \bar{\rho}_{\text{Earth}} = \boxed{619 \text{ kg m}^{-3}}$

$$\rho_{\text{Iron}} = 7870 \text{ kg m}^{-3} \quad \rho_{\text{water}} \approx 1000 \text{ kg m}^{-3}$$

So Earth, Mars are rocky planets, whereas Jupiter and Saturn are made of lighter materials. In fact they are 'gas giants' - although both Saturn and Jupiter are thought to have rocky cores

Note Saturn could float on water!

3/ Bode's Law

$$OR = 4 + 3 \times 2^n$$

R = orbital radius in AU

n = 'planet number'

$$\text{So } 2^n = \frac{OR - 4}{3}$$

$$n \log 2 = \log \left(\frac{OR - 4}{3} \right)$$

$$n = \frac{\log \left(\frac{OR - 4}{3} \right)}{\log 2}$$

	(R)	(n)	
Mercury	0.387	0.11	* ← OR < 4
Venus	0.723	0.11	(0)
Earth	1	1	(1)
Mars	1.523	1.90	(2)
Jupiter	5.202	4.00	(4)
Saturn	9.576	4.93	(5)
Uranus	19.203	5.98	(6)
Neptune	30.246	6.64	*
Pluto	39.509	7.03	(7) → Venus

So Bode's law is quite a good fit for Earth, Mars, Jupiter, Saturn, Uranus, Pluto - but not good for Mercury, Neptune.

It is perhaps coincidental, but nonetheless intriguing. It would be even more interesting to see if it works for planets in other solar systems.

What about $n=3$? Bode's law correctly anticipates the dwarf planet Ceres in the asteroid belt, which lies between the orbit of Mars & Jupiter.

4/	Sun	1047
	Jupiter	1
	Saturn	0.30
	Neptune	0.054
	Uranus	0.045
	Earth	0.00315
	Venus	0.00256
	Mars	0.000337
	Mercury	0.000173
	Pluto	9.44×10^{-6}

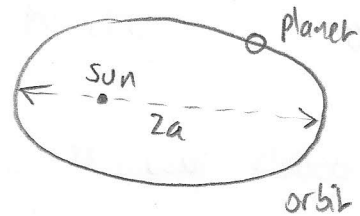
Mass in Jupiter masses

5/ See spreadsheet. ≈ 818.5 days ($\approx 2\frac{1}{4}$ years)

This is at a speed of $\frac{0.1}{100} \times 2.998 \times 10^8 \text{ ms}^{-1}$
 $= 299,800 \text{ ms}^{-1}$

[New Horizons travels at $\approx 20 \text{ kms}^{-1} = 20,000 \text{ ms}^{-1}$
 At this speed the trip would take 21.7 years]

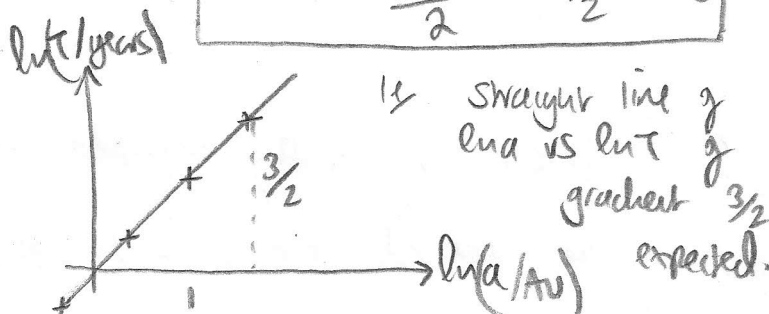
6/ Kepler II: $T^2 \propto a^3$
 \uparrow period $\quad \uparrow$ semi-major axis



So $T^2 = k a^3$

$2 \ln T = \ln k + 3 \ln a$

$\ln T = \frac{\ln k}{2} + \frac{3}{2} \ln a$



[Note if T is in years and a is AU
 $\Rightarrow k$ is very close to unity
 So $\ln k \approx 0$]

6) From spreadsheet, agreement is high (1.4999) from line of best fit.

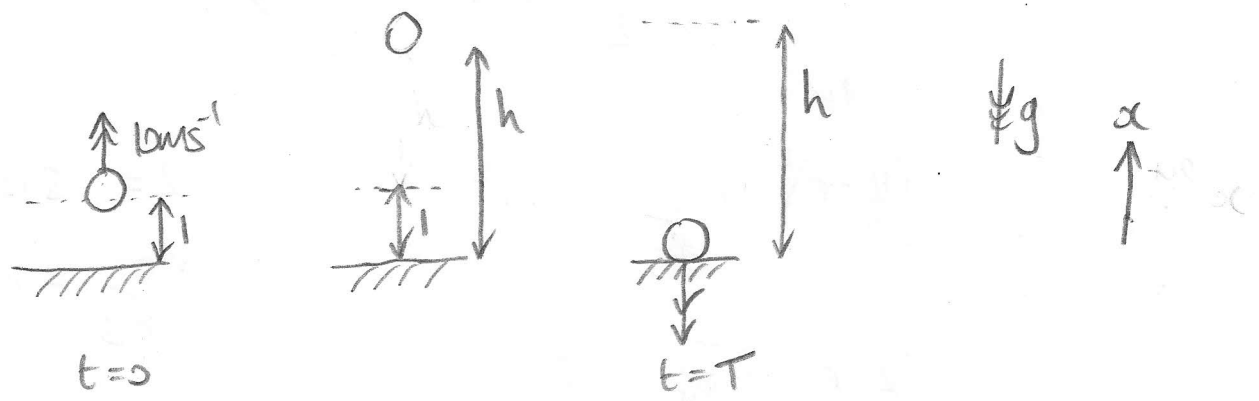
7/

g (relative to Earth)

$g_{\oplus} = 9.81 \text{ ms}^{-2}$

(i)	Mercury	0.37
	Venus	0.9
	Earth	1
	Mars	0.38
	Jupiter	2.53 ← Most heavy
	Saturn	1.07
	Uranus	0.90
	Neptune	1.14
	Pluto	0.09 ← Most weightless
	[Sun	27.95]

(ii)



Constant acceleration motion

$$x = 10t - \frac{1}{2}gt^2 + 1$$

when $x=0$

$$\Rightarrow t = \frac{-10 - \sqrt{100 - 4(-\frac{1}{2}g)(1)}}{-g}$$

conservation of energy

$$\frac{1}{2}10^2 + g \times 1 = gh$$

$$\Rightarrow h = \frac{100}{2g} + 1$$

*

$$\text{So } t = \frac{10}{g} + \frac{1}{g} \sqrt{100 + 2g}$$

$$h_{\text{Mercury}} = \frac{100}{2 \times 0.37 \times 9.81} + 1 = \boxed{14.8 \text{ m}}$$

$$t_{\text{Mercury}} = \frac{10 + \sqrt{100 + 2 \times 0.37 \times 9.81}}{9.81 \times 0.37} = \boxed{5.16 \text{ s}}$$

$$h_{\text{Saturn}} = \frac{100}{2 \times 1.07 \times 9.81} + 1 = \boxed{5.76 \text{ m}}$$

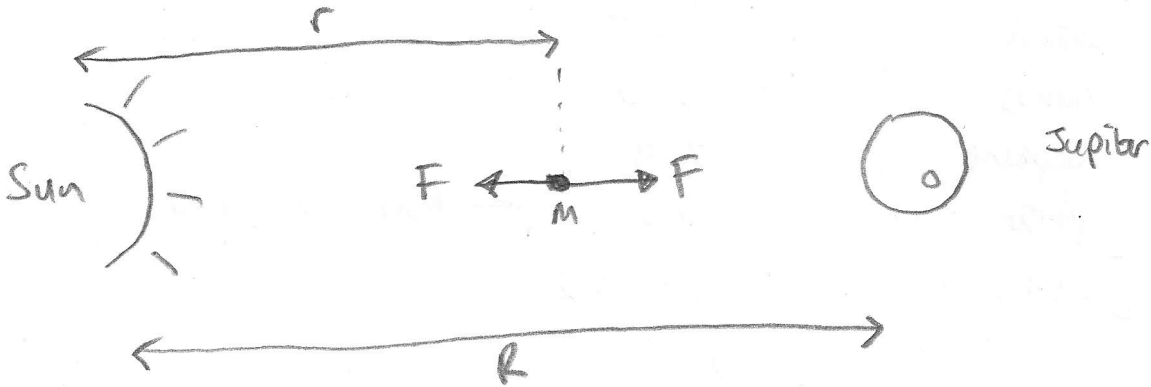
$$t_{\text{Saturn}} = \frac{10 + \sqrt{100 + 2 \times 1.07 \times 9.81}}{9.81 \times 1.07} = \boxed{2.00 \text{ s}}$$

①

$\{ \text{Earth is } 2.13 \text{ s} \}$

$$\frac{GM_{\odot}}{r^2} = \frac{GM_S}{(R-r)^2}$$

if mass m feels no net gravitational force from the Sun or Jupiter.



$$(R-r)^2 \frac{M_{\odot}}{M_S} = r^2$$

$$R = 5.202 \text{ AU}$$

$$\frac{M_{\odot}}{M_S} = 1046.91$$

$$(R-r) \sqrt{\frac{M_{\odot}}{M_S}} = r$$

$$R \sqrt{\frac{M_{\odot}}{M_S}} = r (1 + \sqrt{\frac{M_{\odot}}{M_S}})$$

$$r = \frac{R \sqrt{\frac{M_{\odot}}{M_S}}}{1 + \sqrt{\frac{M_{\odot}}{M_S}}}$$

$$\therefore r = \frac{5.202 \times \sqrt{1046.91}}{1 + \sqrt{1046.91}} \approx \boxed{5.05 \text{ AU}}$$

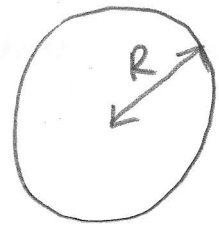
9/

$$E = \frac{1}{2} I \omega^2$$

$$I_{\text{sphere}} = \frac{2}{5} MR^2$$

$$T = \frac{2\pi}{\omega}$$

$$\text{So } \omega^2 = \left(\frac{2\pi}{T}\right)^2$$



$$\therefore E = 2\pi^2 \times \frac{2}{5} MR^2 \frac{1}{T^2}$$

$$E = \frac{4\pi^2}{5} \frac{MR^2}{T^2}$$

	M	R	T	E
(i) Earth	1	1	1	1
Saturn	95.159	9.449	0.444	43,098
Jupiter	317.85	11.209	0.413	234,129
Neptune	17,204	3.883	0.671	576

So rotational kinetic energy ratios are

1 ; 43098 ; 234129 ; 576
 ↑ ↑ ↑ ↑
 Earth Saturn Jupiter Neptune

$$(ii) E = \frac{4\pi^2}{5} \frac{MR^2}{T^2}$$

$E = \text{constant}$ but $R \rightarrow R/2$ for Jupiter

\therefore let new period be τ

$$\frac{R^2}{T^2} = \frac{(R/2)^2}{\tau^2} \Rightarrow \tau^2 = \frac{T^2}{4}$$

$$\Rightarrow \tau = \frac{T}{2}$$

\therefore period would also halve.

(13)

If it has 50% of its mass instead, and
we take $M = \frac{4}{3}\pi R^3 \rho$ [ρ is mean density]

$$\therefore R = \left(\frac{3M}{4\pi\rho} \right)^{1/3}$$

$$\therefore E = \frac{4\pi^2}{5} M \frac{1}{T^2} \times \left(\frac{3}{4\pi\rho} \right)^{2/3} \times M^{1/3}$$

$$\text{So } E \propto \frac{M^{4/3}}{T^2}$$

If $E = \text{constant}$ then $T^2 \propto M^{4/3}$
 $\Rightarrow \boxed{T \propto M^{2/3}}$

So if $M \rightarrow M/2$

$$T \rightarrow \frac{T}{2^{2/3}}$$

$\therefore \boxed{T \rightarrow 0.63T}$ i.e. a reduction by 37%

10/ Haumea

$$a = 43.2 \text{ AU}$$

$$T^2 \approx a^3$$

if T in years and
 a in AU

$$\therefore T \approx 43.2^{3/2} = \boxed{283.9 \text{ years}}$$

