Cosmology problems

1/ One physics hour (ph) = 35×60 s So $\boxed{1\,s = \frac{1\,ph}{35 \times 60}}$

Age of Universe $(T) = 13.8 \times 10^9$ yr

$1\,yr = 365 \times 24 \times 3600 \quad (\approx \pi \times 10^7)$ s

$\therefore T = 13.8 \times 10^9 \times 365 \times 24 \times 3600 \times \frac{1}{35 \times 60}$ ph

$\boxed{T = 2.07 \times 10^{14} \text{ ph}}$

2/

Real time	Cosmic calendar
Big Bang	Midnight on January 1st
Now	Midnight on January 1st

+ 1 year

(i) Earth birth $\frac{4.5 \times 10^9}{13.8 \times 10^9} \times 365 = 119$ days

So $\boxed{\text{April 29th}}$

(ii) Homo Sapiens emerge from Africa $\frac{60{,}000}{13.8 \times 10^9} \times 365 = 1.59 \times 10^{-3}$ days

= 137 seconds before end of year

= $\boxed{\text{2 mins 17 seconds " " " "}}$

		Cumulative
Jan	31	31
Feb	28	59
Mar	31	90
Apr	30	120
May	31	151
Jun	30	181
Jul	31	212
Aug	31	243
Sept	30	273
Oct	31	304
Nov	30	334
Dec	31	365
	365	

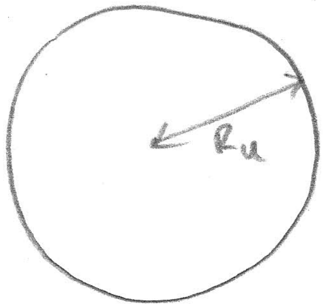
(iii) $2015 - 1384 = 631$ years

(Age of Winchester College)

$\frac{631}{13.8 \times 10^9} \times 365 = 1.67 \times 10^{-5}$ days

= $\boxed{\text{1.44 seconds}}$ before the end of the year.

3/



"Universe"

$2R_u = 93 \times 10^9$ light years

1 light year $= 2.998 \times 10^8 \times 365 \times 24 \times 3600$

$= \boxed{9.45 \times 10^{15} \text{ m}}$

$\therefore$ (i) $R_u = \dfrac{93 \times 10^9}{2} \times 9.45 \times 10^{15}$

$= \boxed{4.40 \times 10^{26} \text{ m}}$

(ii) $V_u = \frac{4}{3}\pi R_u^3$

$= \frac{4}{3}\pi (4.40 \times 10^{26})^3 = \boxed{3.56 \times 10^{80} \text{ m}^3}$

(iii) Total mass of universe $\approx 10^{53}$ kg

$\therefore$ if proton mass is $M_p = 1.673 \times 10^{-27}$ kg

protons in Universe $\approx \dfrac{10^{53}}{1.673 \times 10^{-27}}$

$\therefore$ # protons / cubic metre $\simeq \dfrac{10^{53}}{1.673 \times 10^{-27} \times 3.56 \times 10^{80}}$

$\simeq \boxed{0.17}$ protons per cubic metre

[According to WMAP*, actual density is $\approx$ $\boxed{\text{0.25 protons per cubic metre}}$. BUT mean 'energy density' $\Rightarrow$ mass density of $\simeq \boxed{6 \text{ protons} / \text{m}^3}$ ($E = mc^2$) $\therefore$ 95% of mass/energy in the universe is 'dark matter' or 'dark energy' .... or our models of gravity on this scale are wrong!]

* Accurate measurements of the Cosmic Microwave Background Radiation

4/ $M_{ordinary} \simeq 10^{53}$ kg

a) $M_{Tot} \times 0.049 = M_{ordinary}$

$$\therefore M_{TOT} \simeq \frac{10^{53}}{0.049} = \boxed{2.04 \times 10^{54}} \text{ kg}$$

$$\therefore E_{Tot} = M_{Tot} c^2 = 2.04 \times 10^{54} \times (2.998 \times 10^8)^2$$

$$\boxed{E_{TOT} = 1.83 \times 10^{71} \text{ J}}$$

b) If Big Bang 'released' 1.83×10^{71} J

Tsar Bomba energy $E_{TB} = 210{,}000 \times 10^{12}$ J

$\therefore$ Big Bang is equivalent to $\dfrac{1.83 \times 10^{71}}{210{,}000 \times 10^{12}}$

$$= \boxed{8.73 \times 10^{53}} \text{ Tsar Bombas!}$$

Note 1 kg of matter has energy $= 1 \text{ kg} \times (2.998 \times 10^8)^2$

$$= 8.99 \times 10^{16} \text{ J}$$

This is $\dfrac{8.99 \times 10^{16}}{210{,}000 \times 10^{12}} = \boxed{0.43}$ Tsar Bombas

So in $\boxed{\text{every } 2\tfrac{1}{3} \text{ kg}}$ of matter, we have sufficient energy to equal the most powerful explosion detonated by humans. If only this energy could be easily released!

5/ $\boxed{1\text{ AU} = 1.496 \times 10^{11}\text{ m}}$ $\boxed{c = 2.998 \times 10^{8}\text{ ms}^{-1}}$

(i) 1 light-minute $= 2.998 \times 10^{8} \times 60$

$= \boxed{1.80 \times 10^{10}\text{ m}}$

(ii) 1 light-year $= 2.998 \times 10^{8} \times 365 \times 24 \times 3600$

$= \boxed{9.45 \times 10^{15}\text{ m}}$

(iii) $1\text{ AU} = \dfrac{1.496 \times 10^{11}}{1.80 \times 10^{10}}$ light minutes

$= \boxed{8.3\text{ light minutes}}$

(iv) Sirius A is 8.6 light years away

8.6 light years $= 8.6 \times 9.45 \times 10^{15}\text{ m}$

$= \dfrac{8.6 \times 9.45 \times 10^{15}}{1.496 \times 10^{11}}$ AU

$= \boxed{5.43 \times 10^{5}\text{ AU}}$

i.e. $\approx$ 543,000 $\times$ distance of Earth to Sun

6/ $\boxed{\text{New Horizons}}$

Spacecraft	speed	distance/m	time/yrs
Earth – Jupiter	16.26 km/s	$4.202 \times \underbrace{1.496 \times 10^{11}}_{1\text{ AU}}$	t_1
Jupiter – Pluto (Assume closest approach)	20.26 km/s	x	t_2

Furthest $x = 7.5 \times 10^{9} \times 10^{3} - 4.202 \times 1.496 \times 10^{11} = 6.87 \times 10^{12}\text{ m}$

Closest $x = 4.28 \times 10^{9} \times 10^{3} - 4.202 \times 1.496 \times 10^{11} = 3.65 \times 10^{12}\text{ m}$

$t_1 = \dfrac{4.202 \times 1.496 \times 10^{11}}{16.26 \times 10^{3}} \times \dfrac{1}{365 \times 24 \times 3600} = \boxed{1.23\text{ years}}$ to reach Jupiter.

$$\min t_2 = \frac{3.65 \times 10^{12}}{20.26 \times 10^3} \times \frac{1}{365 \times 24 \times 3600} = \boxed{5.71 \text{ years}}$$

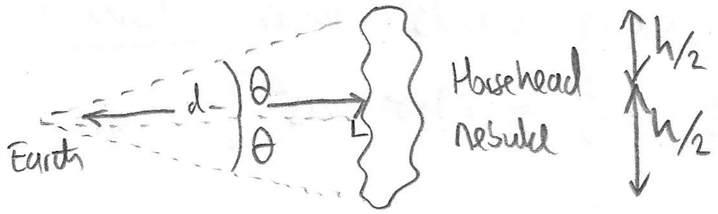
$$\max t_2 = \frac{6.87 \times 10^{12}}{20.26 \times 10^3} \times \frac{1}{365 \times 24 \times 3600} = \boxed{10.75 \text{ years}}$$

So total time to reach Pluto will be between 6.94 and 11.98 years.

[In 2015 it hasn't reached pluto yet! so perhaps a rendezvous anytime 2016 – 2018]

→ NASA mission website predicts July 14, 2015 as the encounter date. It was launched on Jan 19, 2005

7/ (i)



$d = 1500$ light years

$2\theta = \frac{8}{60}$ degrees   let height be h

$$\therefore \tan\theta = \frac{h/2}{d}$$

$$\therefore h = 2d\tan\theta$$

$$= 2 \times 1500 \times \underbrace{2.998 \times 10^8 \times 365 \times 24 \times 3600}_{1 \text{ light-year in m}} \times \tan\left(\frac{8}{120}\right)$$

$$= \boxed{3.30 \times 10^{16} \text{ m}}$$

$$= \frac{3.30 \times 10^{16}}{1.496 \times 10^{11}} \text{ AU} = \boxed{220,604 \text{ AU}}$$

$$= 2 \times 1500 \times \tan\left(\frac{8}{120}\right) \text{ light years}$$

$$= \boxed{3.49 \text{ light years}}$$

(ii) If a spacecraft travels at $0.01c$
it will take 349 years to traverse the Nebula, since it takes light 3.49 years to cover the same distance.

8/ The Earth's orbit is $\approx$ circular about the Sun. In one year it covers $2\pi \times 1.496 \times 10^{11}$ m

$\therefore$ orbital speed (in km / hour) is

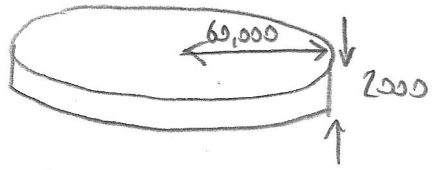
$$v = \frac{2\pi \times 1.496 \times 10^{11} \times \frac{1}{1000} \text{ km}}{365 \times 24 \text{ hours}} = \boxed{107{,}302 \text{ km/h}}$$

ie every second the Earth travels about $\boxed{30 \text{km}}$

$$\left[107{,}302 \text{ km/h} = \frac{107{,}302}{3600} \text{ km/s} = 29.8 \text{ kms}^{-1} \right]$$

9/ $\boxed{\text{Milky Way}}$ (distances in light years)

Contains $\approx 400 \times 10^{9}$ stars



(i) Volume $V \approx 2000 \times \pi \times 60{,}000^2$ cubic light years

$= 2.26 \times 10^{13}$ (light-year)^3

$= 2.26 \times 10^{13} \times (2.998 \times 10^{8} \times 365 \times 24 \times 3600)^3$

$= \boxed{1.91 \times 10^{61} \text{ m}^3}$

(ii) $1 \text{ AU} = 1.496 \times 10^{11}$ m $\therefore$ $1 \text{ m}^3 = \left(\frac{1}{1.496 \times 10^{11}}\right)^3 \text{ AU}^3$

$$\therefore V = \frac{1.91 \times 10^{61}}{(1.496 \times 10^{11})^3} = \boxed{5.70 \times 10^{27} \text{ AU}^3}$$

(iii) let each star of the 400×10^9 in the Milky Way correspond to a volume $\frac{4}{3}\pi \times 50^3$ AU^3

$\therefore$ fraction of empty space is

$$1 - \frac{\frac{4}{3}\pi \times 50^3 \times 400 \times 10^9}{5.70 \times 10^{27}} = \boxed{1 - 3.67 \times 10^{-11}}$$

ie $\boxed{99.9999999963\ \%}$

(iv) orbital speed of the Sun about the galactic centre is

$$v = \frac{2\pi \times 27{,}000 \times 2.998 \times 10^8 \times 365 \times 24 \times 3600 \times \frac{1}{1000}\ \text{km}}{240 \times 10^6 \times 365 \times 24 \times 3600\ \text{s}}$$

$$= \boxed{212\ \text{kms}^{-1}}$$

(About 7x speed of the Earth about the Sun)

10/ Divide the volume of the Milky Way into 40×10^6 spheres of radius R. R is the ~~x~~ separations of 'Advanced civilizations', and we assume there are 40 million of them.

From Q9: $$40 \times 10^6 \times \frac{4}{3}\pi R^3 = 2000 \times \pi \times 60{,}000^2$$

(R is in light years)

$$\Rightarrow \quad R = \sqrt[3]{\frac{2000 \times 60{,}000^2 \times \pi \times 3}{40 \times 10^6 \times 4\pi}}$$

$$= \boxed{51.3\ \text{light years}}$$

But physical contact is very unlikely!

Since radio transmissions from Earth into space have occured since the 1930s .... Perhaps our signals have already been intercepted! $\rightarrow$ $\boxed{\text{SETI}}$

The Solar System, gravity & orbits

1/ Mercury day $= 58.646$ Earth days.

36 years $\approx 36 \times 365$ days $= \dfrac{36 \times 365}{58.646}$ Mercury days

$= 224$ Mercury days

So $\approx \boxed{224}$ sunrises could have been witnessed on Mercury.

2/ $\bar{\rho} = \dfrac{M}{\frac{4}{3}\pi R^3}$ (i) $\bar{\rho}_{Earth} = \dfrac{5.97 \times 10^{24}}{\frac{4}{3}\pi \left(6.38 \times 10^{6}\right)^3}$

$= \boxed{5488 \text{ kg m}^{-3}}$

(ii) $\bar{\rho}_{Mars} = \dfrac{0.107}{0.533^3} \times \bar{\rho}_{Earth} = \boxed{3878 \text{ kg m}^{-3}}$

(iii) $\bar{\rho}_{Jupiter} = \dfrac{317.85}{11.209^3} \times \bar{\rho}_{Earth} = \boxed{1239 \text{ kg m}^{-3}}$

(iv) $\bar{\rho}_{Saturn} = \dfrac{95.159}{9.449^3} \bar{\rho}_{Earth} = \boxed{619 \text{ kg m}^{-3}}$

$\rho_{Iron} = 7870 \text{ kg m}^{-3}$ $\rho_{water} \simeq 1000 \text{ kg m}^{-3}$

So Earth, Mars are rocky planets, whereas Jupiter and Saturn are made of lighter materials. In fact they are 'gas giants' – although both Saturn and Jupiter are thought to have rocky cores

Note Saturn could float on water!

3/ Bode's law $10R = 4 + 3\times 2^n$

R = orbital radius in AU

n = 'planet number'

So $2^n = \dfrac{10R - 4}{3}$

$\therefore \; n \log 2 = \log\left(\dfrac{10R-4}{3}\right)$

$$\boxed{n = \dfrac{\log\left(\frac{10R-4}{3}\right)}{\log 2}}$$

	R	n	
Mercury	0.387	— ← $10R < 4$	*
Venus	0.723	0.11	0
Earth	1	1	1
Mars	1.523	1.90	2
Jupiter	5.202	4.00	4
Saturn	9.576	4.93	5
Uranus	19.293	5.98	6
Neptune	30.246	6.64	*
Pluto	39.509	7.03	7

So Bode's law is quite a good fit for Venus, Earth, Mars, Jupiter, Saturn, Uranus, pluto - but not good for Mercury, Neptune.

It is perhaps coincidental, but nonetheless intriguing. It would be even more interesting to see if it works for planets in other solar systems.

What about $\boxed{n = 3}$? Bode's law correctly anticipates the dwarf planet Ceres in the asteroid belt, which lies between the orbits of Mars & Jupiter.

4/

Sun	1047
Jupiter	1
Saturn	0.30
Neptune	0.054
Uranus	0.045
Earth	0.00315
Venus	0.00256
Mars	0.000337
Mercury	0.000173
Pluto	9.44×10^{-6}

mass in Jupiter masses

5/ See spreadsheet. $\simeq 818.5$ days ($\simeq 2\frac{1}{4}$ years)

This is at a speed of $\frac{0.1}{100} \times 2.998 \times 10^8 \text{ ms}^{-1}$

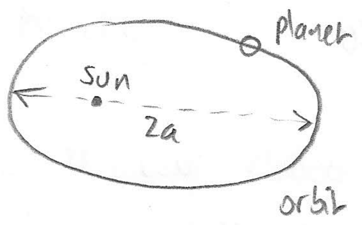
$= 299{,}800 \text{ ms}^{-1}$

[New Horizons travels at $\simeq 20 \text{ kms}^{-1} = 20{,}000 \text{ ms}^{-1}$

At this speed the trip would take 21.7 years]

6/ Kepler III: $T^2 \propto a^3$

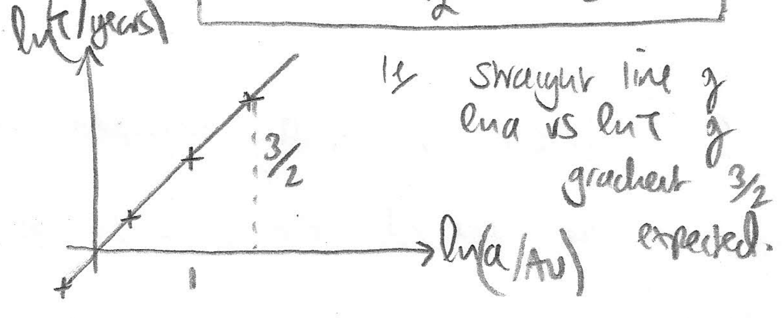
T ↑ period, a ↑ semi-major axis



So $T^2 = k a^3$

$2 \ln T = \ln k + 3 \ln a$

$$\boxed{\ln T = \frac{\ln k}{2} + \frac{3}{2} \ln a}$$



I.e. straight line of $\ln a$ vs $\ln T$ of gradient $3/2$ expected.

[Note if T is in years and a in AU $\Rightarrow$ k is very close to unity So $\ln k \approx 0$]

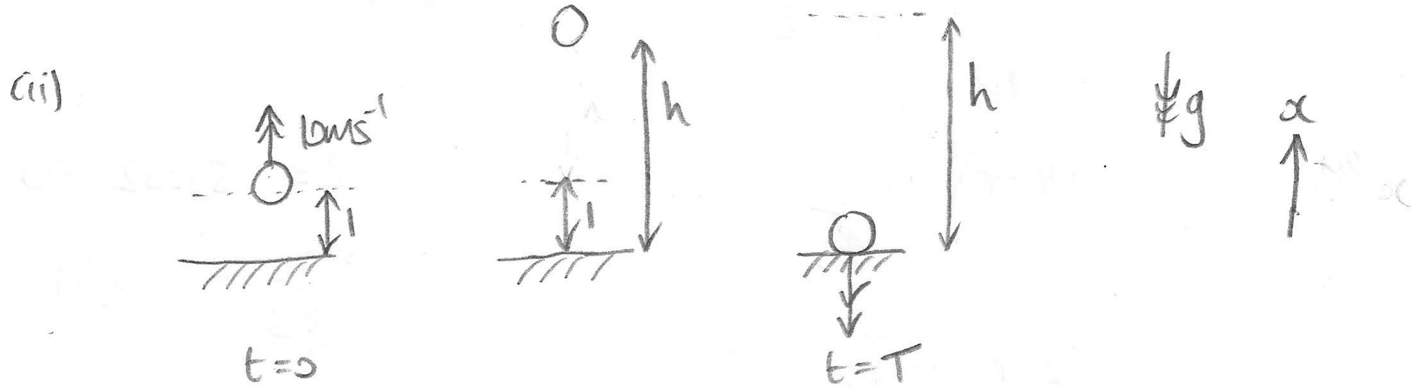
From spreadsheet, agreement is high (1.4999) from line of best fit.

7/

(i)

	g (relative to Earth)	$g_{\oplus} = 9.81\,ms^{-2}$
Mercury	0.37	
Venus	0.9	
Earth	1	
Mars	0.38	
Jupiter	2.53	← Most heavy
Saturn	1.07	
Uranus	0.90	
Neptune	1.14	
pluto	0.09	← Most weightless
[Sun	27.95]	

(ii)



Constant acceleration motion

$$x = 10t - \tfrac{1}{2}gt^2 + 1 \quad \Rightarrow$$

when $x = 0$

$$t = \frac{-10 - \sqrt{100 - 4(-\frac{1}{2}g)(1)}}{-g} \quad *$$

conservation of energy

$$\tfrac{1}{2}10^2 + g \times 1 = gh \quad \Rightarrow \quad \boxed{h = \frac{100}{2g} + 1}$$

*

$$\text{So } t = \frac{10}{g} + \frac{1}{g}\sqrt{100 + 2g}$$

$$t_{Mercury} = \frac{10 + \sqrt{100 + 2 \times 0.37 \times 9.81}}{9.81 \times 0.37} = \boxed{5.61\,s}$$

$$t_{Saturn} = \frac{10 + \sqrt{100 + 2 \times 1.07 \times 9.81}}{9.81 \times 1.07} = \boxed{2.00\,s}$$

{Earth is 2.13s}

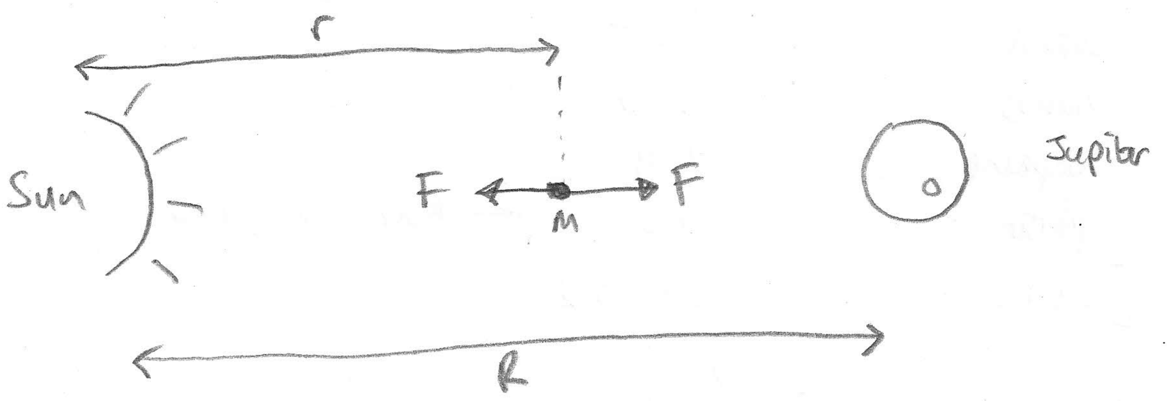
$$\therefore h_{Mercury} = \frac{100}{2 \times 0.37 \times 9.81} + 1 = \boxed{14.8\,m}$$

$$h_{Saturn} = \frac{100}{2 \times 1.07 \times 9.81} + 1 = \boxed{5.76\,m}$$

8/

$$\frac{GM_\odot}{r^2} = \frac{GM_J}{(R-r)^2}$$

if mass M feels no net gravitational force from the Sun or Jupiter.



$$\therefore (R-r)^2 \frac{M_\odot}{M_J} = r^2$$

$R = 5.1202$ AU

$\frac{M_\odot}{M_J} = 1046.91$

$$(R-r)\sqrt{M_\odot/M_J} = r$$

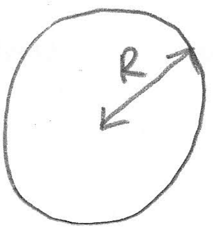
$$R\sqrt{\frac{M_\odot}{M_J}} = r\left(1+\sqrt{M_\odot/M_J}\right)$$

$$\therefore \boxed{r = \frac{R\sqrt{M_\odot/M_J}}{1+\sqrt{M_\odot/M_J}}}$$

$$\therefore r = \frac{5.1202 \times \sqrt{1046.91}}{1+\sqrt{1046.91}} \simeq \boxed{5.05 \text{ AU}}$$

9/ $E = \frac{1}{2} I \omega^2$ $\qquad I_{sphere} = \frac{2}{5} M R^2$

$T = 2\pi / \omega$



Mass M

So $\omega^2 = \left(\frac{2\pi}{T}\right)^2$

$\therefore E = 2\pi^2 \times \frac{2}{5} M R^2 / T^2$

$$\boxed{E = \frac{4\pi^2}{5} \frac{MR^2}{T^2}}$$

(i)

	M	R	T	E
Earth	1	1	1	1
Saturn	95.159	9.449	0.444	43,098
Jupiter	317.85	11.209	0.413	234,129
Neptune	17.204	3.883	0.671	576

So rotational kinetic energy ratios are

1 : 43098 : 234129 : 576

Earth Saturn Jupiter Neptune

(ii) $E = \frac{4\pi^2}{5} MR^2 / T^2$

E = constant but $R \to R/2$ for Jupiter

$\therefore$ let new period be τ

$$\frac{R^2}{T^2} = \frac{(R/2)^2}{\tau^2} \Rightarrow \tau^2 = T^2/4$$

$$\Rightarrow \boxed{\tau = T/2}$$

i.e. period would also halve.

if it lost 50% of its mass instead, and we take $M = \frac{4}{3}\pi R^3 \rho$ [ρ is mean radius]

$$\therefore R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

$$\therefore E = \frac{4\pi^2}{5} M/T^2 \times \left(\frac{3}{4\pi\rho}\right)^{2/3} \times M^{2/3}$$

So $E \propto \frac{M^{4/3}}{T^2}$

If $E =$ constant then $T^2 \propto M^{4/3}$.

$$\Rightarrow \boxed{T \propto M^{2/3}}$$

So if $M \to M/2$

$$T \to T/2^{2/3}$$

ie $\boxed{T \to 0.63T}$ ie a reduction by 37%

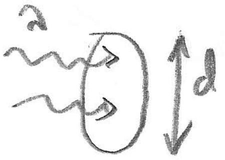
10/ Haumea $a = 43.2$ AU

$T^2 \simeq a^3$ if T in years and a in AU

$$\therefore T \simeq 43.2^{3/2} = \boxed{283.9 \text{ years}}$$

Observing the Cosmos & Calculating distances

1/ Resolution (in an angular sense) of an optical device $= \Delta\theta \sim \dfrac{\lambda}{d}$ ← radians



let $\Delta\theta = 10^{-3}$ arc seconds

$$\Delta\theta = 10^{-3} \times \frac{1}{3600} \text{ degrees}$$

$$\Delta\theta = \frac{\pi}{180 \times 3600} \times 10^{-3} \text{ radians}$$

$$\boxed{d \sim \lambda / \Delta\theta}$$

(i) $\therefore \; d_{vis} \sim \dfrac{10^{-7}}{\pi \times 10^{-3}} \times 180 \times 3600$

$$\sim \boxed{20.63}$$

(ii) $d_X \sim \dfrac{10^{-10}}{\pi \times 10^{-3}} \times 180 \times 3600$

$$\sim \boxed{0.021\,m}$$

(iii) $d_{IR} \sim \dfrac{10^{-5}}{\pi \times 10^{-3}} \times 180 \times 3600$

~ 2063 m (!) ie an array of telescopes is required

Note Hubble has a resolution of about 0.05 arc seconds

$$\therefore \; d \sim \frac{10^{-7}}{\pi \times 0.05} \times 180 \times 3600 = 0.413 \text{ m}.$$

[The optics is a little more complicated than this requirement indicates — primary mirror is 2.4 m diameter, secondary mirror is 0.3 m].

→ So perhaps our sizes above are $\simeq \frac{1}{10}$ of what they should be.

2/ $P_{Rec} \approx \dfrac{\pi (70/2)^2}{4\pi (30.2 \times 1.496 \times 10^{11})^2} \times 22.4 \times 10^{4.8}$

← collecting area

↑ radiated area

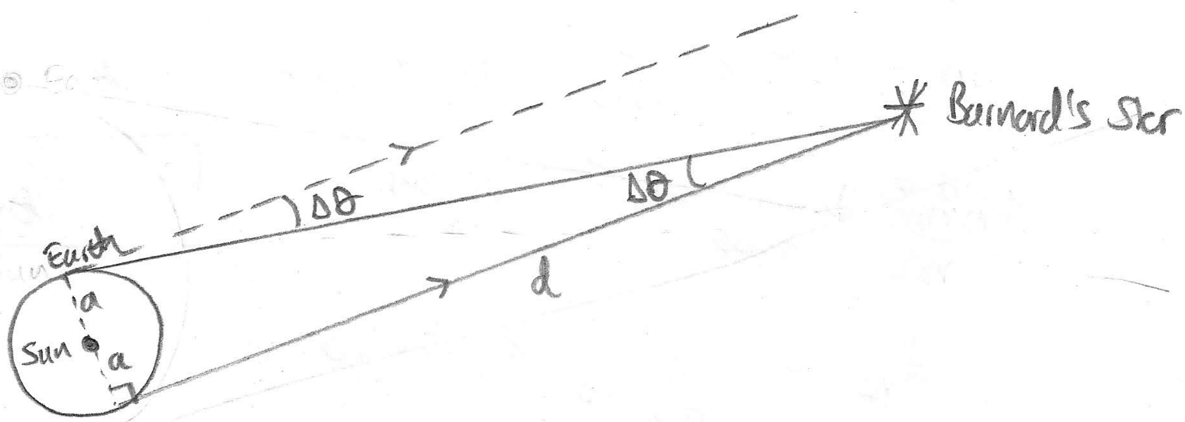
transmit power

gain

$= \boxed{2.12 \times 10^{-17}\ W}$

3/

(i)



Parallax $\Delta\theta = \dfrac{1.0908}{3600}$ degrees $= \dfrac{1.0908 \times \pi}{3600 \times 180}$ radians

$2a = d \tan \Delta\theta$ Since $\Delta\theta \ll 1$ radian

$2a \simeq d\Delta\theta$ $\therefore$ $\boxed{d \simeq \dfrac{2a}{\Delta\theta}}$

[a = 1 AU]

So $d \simeq \dfrac{2 \times 1.496 \times 10^{11}}{1.0908 \times \pi} \times 3600 \times 180$

$= 5.66 \times 10^{16}$ m

$= \boxed{378{,}000 \text{ AU}}$

$= \boxed{5.98}$ light years

(ii) If Voyager 2 travels at 57,888 km/h

It will take $\dfrac{5.66 \times 10^{16}}{57{,}888 \times 10^{3}}$ hours $=$ $\boxed{\text{111 thousand years}}$

4/ Barnard's Star approaches our Solar System at 110.6 kms^{-1}

(i) Doppler shift of UV Hydrogen emission is

$$\Delta f = \underbrace{\frac{110.6 \times 10^3}{2.998 \times 10^8}}_{v/c} \times \underbrace{\frac{2.998 \times 10^8}{121.6 \times 10^{-9}}}_{f = c/\lambda}$$

$\boxed{\Delta f = \frac{v}{c} f}$

$\boxed{c = f\lambda}$

$$= \boxed{9.095 \times 10^{11} \text{ Hz}}$$

Now $f = \frac{2.998 \times 10^8}{121.6 \times 10^{-9}} = 2.47 \times 10^{15}$ Hz

So $\Delta f / f = \frac{v}{c} \simeq 3.69 \times 10^{-4}$

ie $\boxed{0.037 \%}$ shift.

(ii) $$\lambda_{obs} = \frac{c}{f + \Delta f}$$

$$= \frac{c}{f + \frac{v}{c} f} = \frac{c}{f} \frac{1}{1 + v/c} = \lambda_{emitted}$$

$$\therefore z = \frac{\frac{1}{1+v/c} - 1}{1} = \frac{1}{1+v/c} - 1 = \frac{-v/c}{1 + v/c}$$

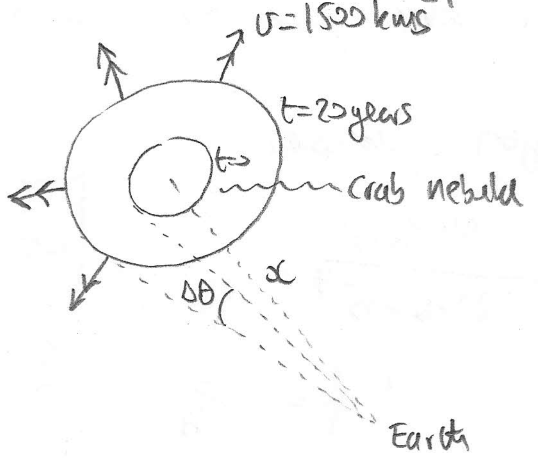
$$= \boxed{\frac{-v}{c+v}}$$

This assumes non-relativistic effects.

$$\therefore z = -\frac{110.6 \times 10^3}{2.998 \times 10^8 + 110.6 \times 10^3}$$

$$= \boxed{-3.67 \times 10^{-4}}$$

5/



$$\boxed{x \approx \frac{v \Delta t}{\Delta \theta}}$$

$$\Delta\theta = \frac{3.16}{3600} \times \frac{\pi}{180} \quad \text{radians}$$

$$\Delta t = 20 \times 365 \times 24 \times 3600 \ \text{s}$$

$$\therefore \ x \approx \frac{1500 \times 10^3 \times 20 \times 365 \times 24 \times 3600}{\frac{3.16 \times \pi}{3600 \times 180}}$$

$$= 6.18 \times 10^{19} \ \text{m}$$

$$= \frac{6.18 \times 10^{19}}{2.998 \times 10^8 \times 365 \times 24 \times 3600} \quad \text{light years}$$

$$= \boxed{6{,}530} \ \text{light years}$$

6/

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

$$\Delta f = \frac{v}{c} f_{em}$$ ← v is approaching

$$f_{obs} = f_{em} + \Delta f$$

$$f_{obs} = f_{em}(1 + v/c)$$

$$c = f\lambda \ \therefore \ \lambda = \frac{c}{f} \quad \therefore \ \lambda_{obs} = \frac{c}{f_{em}(1 + v/c)}$$

$$\frac{c}{f_{em}} = \lambda_{em} \quad \text{so} \quad \boxed{\lambda_{obs} = \frac{\lambda_{em}}{1 + v/c}}$$

$$\therefore \ z = \frac{1}{1 + v/c} - 1 = \boxed{\frac{-v}{c+v}}$$

$\therefore \quad (c+v)z = -v$

For Andromeda

$z = -3.1668 \times 10^{-4}$

(i)

$v(z+1) = -cz$

$$\boxed{v = \frac{-cz}{z+1}}$$

$$\therefore \quad v = -\frac{2.998 \times 10^{8} \times -3.1668 \times 10^{-4}}{1 + (-3.1668 \times 10^{-4})} \ \mathrm{ms^{-1}}$$

$$= \boxed{110 \ \mathrm{kms^{-1}}}$$

(ii) 2.5×10^{6} light years $= 2.5 \times 10^{6} \times 2.998 \times 10^{8} \times 365 \times 24 \times 3600$

$= 2.364 \times 10^{22}$ m

$\therefore$ Andromeda will collide with the Milky Way 'fully' (ie from Earth's perspective) after

$$\frac{2.364 \times 10^{22}}{110 \times 10^{3} \times 365 \times 24 \times 3600} = \boxed{6.8 \text{ billion years}}$$

In about 5.4 billion years our Sun will become a Red Giant, this will eventually shed about half its mass into a planetary nebula leaving a white dwarf star.

Earth will not survive this! it will be engulfed by the Swelling Sun.

HF. May 2015.