Cosmology problems

Unless otherwise stated, express answers in standard form to 3.s.f.

AF. May 2015.

Distance, speed and time calculations on an astronomical scale

1. The Universe is about 13.8 billion years old. A *Physics hour* last 35 minutes. Express the age of the Universe in Physics hours, using standard form to 3.s.f.

2. The Universe is 13.8 billion years old, The Earth is 4.5 billion years old, and *homo sapiens* left Africa about 60,000 years ago. If the age of the Universe is *equivalent* to a calendar year of 365 days (ignore leap years), calculate:

   (i) The date of the Earth's birth

   (ii) The date (and time) of *homo sapiens* emergence from Africa

   (iii) The date (and time) of the foundation of Winchester College in 1384 AD

3. The known Universe has a *diameter* of 93 billion light years. The speed of light is $c = 2.998 \times 10^{8}\,\mathrm{ms^{-1}}$. The Universe has a total mass of about 10^{53} kg. By contrast, a *proton* has a mass of 1.673 x 10^{-27} kg.

   Calculate the following, expressing your answers in standard form to 3.s.f:

   (i) The radius of the Universe in metres

   (ii) The volume of the Universe in metres, assuming it is spherical

   (iii) The density of the Universe, expressed at the *number of protons per cubic metre*

4. The mass of *ordinary matter* in the Universe is about 10^{53} kg. This is thought to only constitute about 4.9% of the total.. (The remainder is 'dark matter' or 'dark energy' - the nature of these things is currently mysterious!).

   (a) Using the formula $E = mc^2$ where m is mass, E is energy and the speed of light is $c = 2.998 \times 10^{8}\,\mathrm{ms^{-1}}$, calculate the total amount of energy in the Universe. Express your answer in Joules, using standard form to 3.s.f.

   (b) The most powerful thermonuclear bomb detonated was the *Tsar Bomba*, which released 210,000 TJ of energy. [1TJ = 10^{12} J]. If all the energy in the Universe were released from a giant explosion ("The Big Bang"), calculate the numbers of Tsar Bomba explosions this is equivalent to.

5. One *Astronomical Unit* (AU) is the mean distance between the Earth and the Sun.

1AU = 1.496×10^{11} m

Using the speed of light $c = 2.998 \times 10^{8} \text{ms}^{-1}$, calculate

(i) The distance in metres that light travels in a minute (a 'light-minute')

(ii) The distance in metres that light travels in one year (a 'light-year')

(iii) The Earth-Sun distance in *light-minutes*

(iv) The distance (in AU) to the star *Sirius A*, which is 8.6 light-years away.

6. The NASA space probe *New Horizons* was launched on January 19 2006 with the mission of making a close encounter with the dwarf planet Pluto and its moons. It escaped Earth with a velocity of 16.26 kms^{-1} and gained a further 4kms^{-1} following a gravitational assist from Jupiter. Pluto varies in distance between 4.28 billion km and 7.5 billion km from Earth. Jupiter is at minimum 4.202 AU from Earth.

Use this information to estimate the time in years it will take New Horizons to reach Pluto.

7. The *Horsehead nebula* in the constellation of Orion is about 1500 light years from Earth. In angular terms, it is 8 arc minutes high.

(i) Use $c = 2.998 \times 10^{8} \text{ms}^{-1}$ and some basic trigonometry to work out the height of the nebula in metres. [1 arc minute is 1/60 of a degree]. Convert this distance to AU and also light years.

(ii) If a futuristic alien spacecraft can travel at 1% of the speed of light, how long would it take to cover the full length of the nebula? (Ignore any relativistic effects).

8. Calculate the orbital speed of the Earth about the Sun in kmh^{-1}. Assume a perfectly circular orbit of radius 1AU. [1AU = 1.496×10^{11}m]

9. The Milky Way Galaxy has up to 400 billion stars. It can be thought of as a disk of thickness 2000 light years and diameter 120,000 light years.

(i) Calculate the volume of the Milky Way in cubic metres.

(ii) Calculate the volume in AU^3 [1AU = 1.496×10^{11}m]

(iii) If each star has a solar system contained within a sphere of radius 50 AU, estimate the fraction of the milky way's volume which is 'deep interstellar space.'

(iv) It takes the Sun 240 million years to orbit the centre of the Milky Way. It is approximately 27,000 light years from the galactic centre. Calculate its average speed in kms^{-1} as it rotates, assuming a circular orbit.

10. The *Drake equation* is used to estimate how many advanced civilizations might evolve in a Galaxy of similar size as the Milky Way. The answers vary, but suggest about 40 million civilizations at any one time is possible. Assuming the planets upon which each civilization lives are evenly spread throughout the galactic disc, work out the time it would take to send a radio signal from one civilization to another. Based upon your answer, do you think it is likely that the Earth will make contact with aliens anytime soon?

[Milky Way is a disk of thickness 2000 light years and diameter 120,000 light years. The speed of light $c = 2.998 \times 10^{8} \text{ms}^{-1}$. HINT: divide up the volume of the galaxy into 40 million spheres].

The Solar System, gravity & orbits

For the following questions you will need the following data about the planets in the solar system

Earth mass: $M_\oplus = 5.97 \times 10^{24}\,\text{kg}$ Earth radius: $R_\oplus = 6.38 \times 10^{6}\,\text{m}$ 1 AU = 1.496×10^{11}m

Object	Mass in Earth masses	Distance from Sun in AU	Radius in Earth radii	Rotational period /days	Orbital period /years
Sun	332,837	-	109.123	-	-
Mercury	0.055	0.387	0.383	58.646	0.241
Venus	0.815	0.723	0.949	243.018	0.615
Earth	1	1	1	1	1
Mars	0.107	1.523	0.533	1.026	1.881
Jupiter	317.85	5.202	11.209	0.413	11.861
Saturn	95.159	9.576	9.449	0.444	29.628
Uranus	14.5	19.293	4.007	0.718	84.747
Neptune	17.204	30.246	3.883	0.671	166.344
Pluto	0.003	39.509	0.187	6.387	248.348

1. I am 36 years old. If I had spent my days on Mercury rather than on Earth, how many sunrises could I have witnessed?

2. Calculate the average *density* (in kgm^{-3}) of (i) Earth, (ii) Mars, (iii) Jupiter and (iv) Saturn. What does this tell you about the composition of these planets?

3. *Bode's Law* is an approximate relationship between the *planet number* n in the Solar System and the orbital radius from the Sun R. If R is measured in AU

$$10R = 4 + 3 \times 2^{n}$$

   Estimate (or if you can use logarithms, calculate) n for each planet. Is there anything wrong with the pattern?

4. Order the (main) Solar System objects in the table in ascending order of mass. Express your answers in Jupiter masses.

5. A space tourist spends a day (Sunrise to Sunrise) on each planet in the Solar System, before travelling to the next planet aboard a futuristic spacecraft which can travel at 0.1% of the speed of light, $c = 2.998 \times 10^{8}\,\text{ms}^{-1}$

   Ignoring relativistic effects and also differences in distances between the planets due to their various positions within their orbits, calculate how long the trip would last. Assume a start from Earth, with Mars being the next stop. After Pluto, a direct route to Mercury, then Venus before returning to Earth. Don't include ay days on Earth.

6. *Kepler's Third Law* states that *"the square of the orbital period is proportional to the cube of the distance from the star"*. Plot a suitable graph to indicate whether this is true for the solar system.

7. *Newton's Law of Universal Gravitation* states that the gravitational acceleration at the surface of a planet of mass M and radius R is given by:

$$g = \frac{GM}{R^2}, \quad G = 6.67 \times 10^{-11} \text{kg}^{-1}\text{m}^3\text{s}^{-2}$$

(i) Calculate g for each planet. On the surface of which planet will you (i) feel most heavy (ii) feel most 'weightless'.

(ii) A game of Winkies is played on Mercury and then in a space station orbiting close to the surface of Saturn. In both cases a ball is kicked upwards with a velocity of 10ms^{-1} from a height of one metre. How high will the ball rise in each case, and how long will it take for the ball to hit the ground?

8. Use *Newton's Law of Universal Gravitation* to work out the distance from the Sun where the gravitational pull from the Sun equals the pull from Jupiter. Ignore the effect of the other planets.

$$F = \frac{GMm}{R^2}, \quad G = 6.67 \times 10^{-11} \text{kg}^{-1}\text{m}^3\text{s}^{-2}$$

m, M are masses which feel a mutual gravitational force of F Newtons when they are separated by distance R.

9. The rotational kinetic energy of a sphere of mass M and radius R is given by the following equation

$$E = \tfrac{1}{2} I \omega^2$$

where 'moment of inertia' $I = \tfrac{2}{5} MR^2$ and 'angular velocity' $\omega = \dfrac{2\pi}{T}$. T is the rotation period.

(i) Work out the ratio of rotational kinetic energies for Earth : Saturn : Jupiter : Neptune

(ii) An exotic effect causes Jupiter's radius to contract by 50%. Assuming no energy change occurs in this process, calculate Jupiter's new rotation period.

10. Use *Kepler's Third Law* (see question 6) to predict the orbital period of dwarf planet *Haumea*, which has a maximum orbital radius of 43.2 AU about the Sun.

Observing the cosmos & calculating distances

1. A futuristic space telescope has an angular *resolution* of 10^{-3} arc seconds. Approximately, what would the telescope aperture diameter have to be if the radiation was

   (i) Visible ($\lambda = 10^{-7}\,\mathrm{m}$)

   (ii) X-Ray ($\lambda = 10^{-10}\,\mathrm{m}$)

   (iii) Infra-Red ($\lambda = 10^{-5}\,\mathrm{m}$)

2. *Voyager 2* passed Neptune on the 15th August 1989. Neptune is approximately 30.2 AU from Earth. Assuming an inverse square law, calculate the power (in Watts) received by a 70m diameter antenna which is part of the NASA Deep Space Network. Voyager transmits with a power of 22.4W, and the antenna gain at X-band (8-12GHz) is 48 dB i.e. a gain of $10^{48/10}$ in the direction of Earth.

   [1 AU = 1.496×10^{11}m].

3. *Barnard's Star* is the fourth closest star from the Sun. When observed from Earth over the course of half a year, it is observed to change angular positions by 1.0908 arc seconds.

   (i) Using 1AU = 1.496×10^{11}m, show that Barnard's Star is approximately 5.98 light years away. How many AU is this?

   (ii) Voyager 2 is now travelling at 57,888 kmh^{-1}. How many years would it take to reach Barnard's Star, starting from Earth?

4. *Barnard's Star* is currently *approaching* our Solar with a radial velocity v of 110.6 kms^{-1}.

   Changes in the energy state of Hydrogen atoms in Barnard's star cause the emission of ultra-violet photons with wavelength 121.6 nm.

   (i) Calculate the Doppler frequency shift Δf of this radiation using the formulae $c = f\lambda$ and

   $$\Delta f = \frac{v}{c} f$$

   (ii) Hence calculate the *redshift* z of this radiation $z = \dfrac{\lambda_{\text{observed}} - \lambda_{\text{emmitted}}}{\lambda_{\text{emmitted}}}$

5. The *Crab Nebula* is a remnant from a *Supernova*. It is essentially a gas cloud which is expanding radially at about 1500 kms^{-1}. In twenty years the angle between the centre of the nebula and the outer rim has changed by 3.16 arc seconds. Use this information to calculate the distance to the Crab Nebula. Express your answer in light years.

6. The *Andromeda* galaxy emits radiation at λ = 21.11cm corresponding to an energy change in Hydrogen atoms. This radiation is received on earth with a redshift of z = -3.668×10^{-4} (i.e. a *blueshift*).

   (i) Use this information to calculate how fast the Andromeda galaxy is approaching the Milky Way.

   (ii) The Andromeda Galaxy is about 2.5 million light years away from Earth. After how many years will it take for the Milky Way and the Andromeda galaxy to merge from the point of view of Earth? Do you think it likely there will be an Earth-bound observatory to record this event?