# Cryptography 

Mathma Talk
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The study and practice of techniques for secure communication In other words, the reading and writing of SECRET CODES*


## Case studies

1. Substitution cipher
2. Frequency analysis \& code breaking
3. Vigenère \& polyalphabetic ciphers
4. Steganography
5. Enigma \& the story of Bletchley Park
6. Modern encryption \& internet security

## A brief history of cryptography

Cryptography, the study and practice of techniques for secure communication, is related to Cryptanalysis (from the Greek kryptós, "hidden", and analýein, "to loosen" or "to untie") which is the study of the systems used for cryptography.

Cryptography is therefore the study of codes, ciphers, algorithms (mathematical recipes) and computational machines


The ATBASH Cipher


## Ancient

1900BC - 850AD
Egyptian
Sumerian
Hebrew
Greek/Roman
Arabic

$\mathbf{2 0}^{\text {th }}$ Century
WWI,WWII
code breaking
Enigma
RSA
Birth of the internet


21 ${ }^{\text {st }}$ Century
Growth of the internet
Secure Wi-Fi
Quantum cryptography ${ }^{3}$


The ATBASH Cipher


1900 BC Non－standard hieroglyphs in the Old Kingdom of Egypt
（Hieroglyphs themselves translated via the Rossetta Stone by Champollion／Young 1820）

1500 BC
Mesopotamian recipe for
u，ū pottery glaze

600－500 BC
Hebrew Atbash cipher
500BC－450AD
Ancient Greek \＆Roman cryptography：
－Scytale transposition cipher
－Caeser Shift substitution cipher
－Secret messages tatooed on a slave＇s head that are hidden when the hair regrows i．e steganography

850AD Arabic／Muslim philosophy Al－Kindi＂Manuscript on Deciphering Cryptographic Messages＂



Blaise de Vigènere


Antoine Rossignol

1312-1361 Ibn al-Durayhim 1355-1418 Ahmad al-Qalqashandi Frequency analysis, substitution ciphers

## 1404-1472 Leon Battista Alberti

1523-1596 Blaise de Vigènere
Polyalphabetic cipher
1586 Thomas Phelippes deciphers letters from Mary Queen of Scots relating to the 'Babingdon plot' to assassinate Elizabeth I


1600-1682 Antoine Rossignol \& The Great Cipher (broken by Étienne Bazeries in 1900)

1860s Charles Babbage, Friedrich Kasiski break le chiffre indéchiffrable-"the indecipherable (polyalphabetic) cipher"


## 1917

Breaking the encoded Zimmermann Telegram brought the USA into WWI

1919 UK Government Communications Headquarters (GCHQ) founded

## 1940s

Enigma and Lorenz ciphers broken by the Allies at Bletchley Park during WWII

1945 Claude Shannon "A mathematical theory of cryptography"

1952 US National Security Agency (NSA) founded

1970s Data Encryption Standard (DES)
1976 Public Key Cryptography, RSA (Ronald Rivest, Adi Shamir, Leonard Adleman)

1970s-1990s Network infrastructure developed that became the Internet ${ }^{6}$

*Wired Equivalent Privacy (WEP) is a security algorithm for IEEE 802.11 wireless networks.

1989 Tim Berners-Lee invents the World Wide Web (WWW)

Mid 1990s
Exponential expansion of the Internet

2000s
Secure Socket Layer (SSL) WEP* Wi-Fi encryption

## ASecure hthos:"

2001 Advanced Encryption Standard (AES)

2011 Tempora (formerly) secret fibre-optic cable internet mass 'wiretap' by GCHQ, NSA etc

2014 First commercial quantum cryptography system

2013 Edward Snowdon discloses NSA classified documents

Internet Growth - Usage Phases - Tech Events


## Case study \#1: The substitution cipher

A cipher transforms the letters of a plaintext from one alphabet to another It is important the plaintext alphabet and cipher alphabet are shuffled versions of the same letters, otherwise decryption could not occur unambiguously

plaintext
ciphertext

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

 abcdefghijklmnopqrstuvwxyzBCDEFGHIJKLMNOPQRSTUVWXYZA bcdefghijklmnopqrstuvwxyza
plaintext alphabet
cipher alphabet

In this example a Caesar Shift is used
i.e. an 'circular' alphabetic shift by one letter
$\mathrm{a} \rightarrow \mathrm{b}$
b $\rightarrow$ c
$z \rightarrow a$ etc

## Case study \#1: The substitution cipher

Manual encryption of a plaintext using a substitution cipher is straightforward, but clearly tedious for messages of more than ten or so characters!
A machine is ideally suited for this repetitive task.


Display encryption or decryption time. 0.006 s for the previous example!

```
plaintext alphabet
AaBbCcDdEeFfGgHhIiJjKkLlMmNnOoPpQqRrSsTtUuVvWwXxYyZz
    !.,"?#@><;:()£$%^&*_-{}[]~1234567890\/| +=
cipher alphabet
ptkq+|Rc,B%h^F.oW]>{~_;&ZQD8s# [3\unIUX2gw":=d$6/
5@1Cx!OKMzS(PYN<0y9mVJvH-Ar4*L)abi7}lTGE£j?ef
```

Extending the alphabet and randomizing the order can make a cipher harder to break. However, repetitions of common words such as and can catalyse an attempt to find the cipher alphabet. In our case and $\rightarrow$ t8c. We could try the same for $\circ f$, by, to etc.

Also, the text structure (sentence length, paragraphs) also gives clues to what is written

| The Comedy of Errors by William Shakespeare | \| | 20B +\#QBc@ \#h ,II\#IX q@ d]\&\&]ta Uot_BX3BtIB |
| :---: | :---: |
|  | \| |
| ACT I | p+2 W |
| SCENE I. A hall in DUKE SOLINUS'S palace. | $\mathrm{U}+\mathrm{D}, \mathrm{W}$ ! pot\&\& ]8 Rw~, Us;WDwU'U $3 \mathrm{t} \& \mathrm{t} \mid \mathrm{B}$ ! |
| Enter DUKE SOLINUS, AEGEON, Gaoler, Officers, and other Attendants | \| | ,8gBl Rw~, Us;WDwUO p,^,sDO ^t\#\&BIO shh]|BIXO t8c | \# \#goBl pggB8ct8gX |
| AEGEON | \| $\mathrm{p}, \wedge, \mathrm{sD}$ |
| Proceed, Solinus, to procure my fall |  |
| And by the doom of death end woes and all. | p8c q@ goB c\#\#Q \#h cBtgo B8c \$\#BX t8c t\&\&! |

To remove structure，we can incorporate the line return character into our alphabet
i．e．char（10）or char（13）in many programming languages

|  | ASCll control |  |
| :---: | :---: | :---: |
| Characters |  |  |
| 00 | NULL | （Null character） |
| 01 | SOH | （Start of Header） |
| 02 | STX | （Start of Text） |
| 03 | ETX | （End of Text） |
| 04 | EOT | （End of Trans．） |
| 05 | ENQ | （Enquiry） |
| 06 | ACK | （Acknowledgement） |
| 07 | BEL | （Bell） |
| 08 | BS | （Backspace） |
| 09 | HT | （Horizontal Tab） |
| 10 | LF | （Line feed） |
| 11 | VT | （Vertical Tab） |
| 12 | FF | （Form feed） |
| 13 | CR | （Carriage return） |
| 14 | SO | （Shift Out） |
| 15 | SI | （Shift In） |
| 16 | DLE | （Data link escape） |
| 17 | DC1 | （Device control 1） |
| 18 | DC2 | （Device control 2） |
| 19 | DC3 | （Device control 3） |
| 20 | DC4 | （Device control 4） |
| 21 | NAK | （Negative acknowl．） |
| 22 | SYN | （Synchronous idle） |
| 23 | ETB | （End of trans．block） |
| 24 | CAN | （Cancel） |
| 25 | EM | （End of medium） |
| 26 | SUB | （Substitute） |
| 27 | ESC | （Escape） |
| 28 | FS | （File separator） |
| 29 | GS | （Group separator） |
| 30 | RS | （Record separator） |
| 31 | US | （Unit separator） |
| 127 | DEL | （Delete） |


| ASCII printable characters |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | space | 64 | （0） | 96 | － |
| 33 | ！ | 65 | A | 97 | a |
| 34 | ＂ | 66 | B | 98 | b |
| 35 | \＃ | 67 | C | 99 | c |
| 36 | \＄ | 68 | D | 100 | d |
| 37 | \％ | 69 | E | 101 | e |
| 38 | \＆ | 70 | F | 102 | f |
| 39 | ， | 71 | G | 103 | g |
| 40 | 1 | 72 | H | 104 | h |
| 41 | ） | 73 | I | 105 | i |
| 42 | ＊ | 74 | J | 106 | j |
| 43 | ＋ | 75 | K | 107 | k |
| 44 | ， | 76 | L | 108 | I |
| 45 | － | 77 | M | 109 | m |
| 46 | － | 78 | N | 110 | n |
| 47 | 1 | 79 | 0 | 111 | 0 |
| 48 | 0 | 80 | P | 112 | p |
| 49 | 1 | 81 | Q | 113 | q |
| 50 | 2 | 82 | R | 114 | r |
| 51 | 3 | 83 | S | 115 | s |
| 52 | 4 | 84 | T | 116 | t |
| 53 | 5 | 85 | U | 117 | u |
| 54 | 6 | 86 | V | 118 | v |
| 55 | 7 | 87 | W | 119 | w |
| 56 | 8 | 88 | X | 120 | x |
| 57 | 9 | 89 | Y | 121 | y |
| 58 | ： | 90 | Z | 122 | z |
| 59 | ； | 91 | ［ | 123 | \｛ |
| 60 | $<$ | 92 | 1 | 124 | I |
| 61 | $=$ | 93 | ］ | 125 | \} |
| 62 | $>$ | 94 | $\wedge$ | 126 | $\sim$ |
| 63 | ？ | 95 | － |  |  |


| Extended ASCII characters |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 128 | Ç | 160 | á | 192 | L | 224 | 0 |
| 129 | ü | 161 | i | 193 | $\perp$ | 225 | B |
| 130 | é | 162 | ó | 194 | T | 226 | Ô |
| 131 | â | 163 | ú | 195 | F | 227 | Ò |
| 132 | ä | 164 | ñ | 196 | － | 228 | 0 |
| 133 | à | 165 | Ñ | 197 | $t$ | 229 | 0 |
| 134 | $\dot{\text { a }}$ | 166 | a | 198 | ã | 230 | $\mu$ |
| 135 | ç | 167 | － | 199 | Ã | 231 | p |
| 136 | ê | 168 | c | 200 | L | 232 | p |
| 137 | ë | 169 | （2） | 201 | ［ | 233 | Ú |
| 138 | è | 170 | 7 | 202 | $\xrightarrow{[ }$ | 234 | Û |
| 139 | i | 171 | 1／2 | 203 | $\bar{T}$ | 235 | U̇ |
| 140 | î | 172 | $1 / 4$ | 204 | － | 236 | y |
| 141 | i | 173 | i | 205 | ． | 237 | $\dot{Y}$ |
| 142 | Ä | 174 | ＊ | 206 | $\pm$ | 238 |  |
| 143 | A | 175 | ＊ | 207 | － | 239 | ， |
| 144 | É | 176 | 殮 | 208 | $\delta$ | 240 | 三 |
| 145 | æ | 177 | ＂ | 209 | Đ | 241 | $\pm$ |
| 146 | AE | 178 | 틏 | 210 | $\hat{\mathbf{E}}$ | 242 |  |
| 147 | ô | 179 |  | 211 | $\ddot{\mathrm{E}}$ | 243 | 5／4 |
| 148 | 0 | 180 | － | 212 | È | 244 | TI |
| 149 | ò | 181 | A | 213 | 1 | 245 | § |
| 150 | û | 182 | Â | 214 | i | 246 | $\div$ |
| 151 | ù | 183 | À | 215 | İ | 247 |  |
| 152 | y | 184 | （c） | 216 | İ | 248 | $\bigcirc$ |
| 153 | 0 | 185 | 4 | 217 | 」 | 249 | － |
| 154 | Ü | 186 | \｜ | 218 |  | 250 | － |
| 155 | 6 | 187 | ］ | 219 |  | 251 | 1 |
| 156 | £ | 188 | ］ | 220 | － | 252 | 3 |
| 157 | $\emptyset$ | 189 | $\not \subset$ | 221 | ＋ | 253 | 2 |
| 158 | $\times$ | 190 | $¥$ | 222 | I | 254 | － |
| 159 | $f$ | 191 | 7 | 223 | － | 255 | nbsp |


| WWW． theASCIIcode <br> ．com．ar |  |
| :---: | :---: |
|  | most consulted |
| กี | énye，$n$ with tilde （alt＋164） |
| $\square$ | black square $\text { (alt }+254 \text { ) }$ |
| 2 | superscript two，square $\text { (alt }+253 \text { ) }$ |
| － | degree symbol （alt +248 ） |
| ＇ | apostrophe，single quote （alt +39 ） |
| $\mu$ | letter Mu，micro，micron （alt +230 ） |
| （C） | copyright symbol （alt＋184） |
| （B） | registered trademark （alt +169 ） |
| 3 | superscript three，cube $\text { (alt }+252 \text { ) }$ |
| á | a with acute accent （alt +160 ） |


| frequently－used |  |
| :---: | :---: |
| （spanish language） |  |
| ñ | alt +164 |
| $\tilde{N}$ | alt +165 |
| ＠ | alt +64 |
| i | alt +168 |
| $?$ | alt +63 |
| i | alt +173 |
| ！ | alt +33 |
| $:$ | alt +58 |
| $i$ | alt +47 |
| i | alt +92 |


| vowels acute accent （spanish language） |  | vowels with diaresis |  |
| :---: | :---: | :---: | :---: |
| á | alt +160 | ä | alt +132 |
| é | alt +130 | ë | alt +137 |
| i | alt +161 | I | alt +139 |
| ó | alt +162 | $\ddot{0}$ | alt +148 |
| ú | alt +163 | ü | alt +129 |
| Á | alt +181 | Ä | alt +142 |
| É | alt +144 | $\overrightarrow{\mathrm{E}}$ | alt +211 |
| i | alt +214 | İ | alt +216 |
| 0 | alt +224 | $0 ̈$ | alt +153 |
| Ú | alt +233 | Ü | alt +154 |


| mathematical <br> symbols |  |
| :---: | :---: |
| $1 / 2$ | alt +171 |
| $1 / 4$ | alt +172 |
| $3 / 4$ | alt +243 |
| $\mathbf{1}$ | alt +251 |
| $\mathbf{3}$ | alt +252 |
| $\mathbf{2}$ | alt +253 |
| $f$ | alt +159 |
| $\pm$ | alt +241 |
| $\mathbf{x}$ | alt +158 |
| $\div$ | alt +246 |


| commercial／trade symbols |  |
| :---: | :---: |
| \＄ | alt +36 |
| $£$ | alt +156 |
| $¥$ | alt +190 |
| $\not \subset$ | alt +189 |
| 口 | alt +207 |
| （2） | alt +169 |
| （c） | alt +184 |
| $a$ | alt +166 |
| － | alt +167 |
| $\bigcirc$ | alt +248 |


| quotes and |  |
| :---: | ---: |
| parenthesis |  |
| ＂ | alt +34 |
| b | alt +39 |
| （ | alt +40 |
| ） | alt +41 |
| ［ | alt +91 |
| ］ | alt +93 |
| \｛ | alt +123 |
| \} | alt +125 |
| s | alt +174 |
| ＂ | alt +175 |

## ACTI

SCENE I. A hall in DUKE SOLINUS'S palace.

AaBbCcDdEeFfGgHhlijjKkLIMmNnOoPpQqRrSsTtUuVvWwXxYyZz!.,"?\#@><;:()£\$\%^\&*_-\{\}[]~1234567890\/|+= y9\$X*]qu\}lok8IULMj>,R_sxViaNv6-F)d(rWOmQA"J/!<1\wBb@e\#g .hST+=H; $\mathrm{D}^{\sim}:\left\{7 \mathrm{t} 3\left[z 02 \mathrm{c} \& \mathrm{YECK}\right.\right.$ G\%54| $\mathrm{P}^{\wedge} \mathrm{nZf}$ ?p£

Enter DUKE SOLINUS, AEGEON, Gaoler, Officers, and other Attendants

## AEGEON

Proceed, Solinus, to procure my fall
And by the doom of death end woes and all.

## DUKE SOLINUS

Merchant of Syracuse, plead no more; I am not partial to infringe our laws: The enmity and discord which of late Sprung from the rancorous outrage of your duke To merchants, our well-dealing countrymen, Who wanting guilders to redeem their lives Have seal'd his rigorous statutes with their bloods, Excludes all pity from our threatening looks.
For, since the mortal and intestine jars 'Twixt thy seditious countrymen and us, It hath in solemn synods been decreed Both by the Syracusians and ourselves, To admit no traffic to our adverse towns Nay, more, If any born at Ephesus be seen
At any Syracusian marts and fairs;
Again: if any Syracusian born Come to the bay of Ephesus, he dies, His goods confiscate to the duke's dispose, Unless a thousand marks be levied, To quit the penalty and to ransom him.
Thy substance, valued at the highest rate, Cannot amount unto a hundred marks; Therefore by law thou art condemned to die.
ciphertext alphabet
mLI?*6iluB?6k?\}rr6rO?XB?!jxxj9i?WL9_IOFI9rl?£p£py* m?M£pW*\}a\}?M\#?y?L9xx?jN?qAR\}?WvsMaAW'W?F9x9 ]l\#£p£p\}NQIr?qAR\}?WvsMaAWg?y\}8\}vag?896xIrg?vkkj]I rOg?9Nu?6QLIr?yQQINu9NQO?fpy\}8\}va?£p-
r6]llug?W6xjN"Og?Q6?Fr6]"rl?iB?k9xxfpyNu?XB?QLI?u 66i?6k?ul9QL?INu?<6IO?9Nu?9xx\#£p£pqAR\}?WvsMaA W?£pVIr]L9NQ?6k?WBr9]"Olg?FxI9u?N6?i6rI=£pM?9i? N6Q?F9rQj9x?Q6?jNkrjNII?6"r?x9<OH£pmLI?INijQB?9N u?ujO]6ru?<Lj]L?6k?x9Ql£pWFr"NI?kr6i?QLI?r9N]6r6"O ?6"Qr9II?6k?B6"r?u"_Ifpm6?ilr]L9NQOg?6"r?<lxxzul9xj NI?]6"NQrBilNgfp!L6?<9NQjNI?I"jxulrO?Q6?rlulli?QLljr ?xj/IO£pU9/I?OI9x'u?LjO?rjI6r6"O?OQ9Q"QIO?<jQL?QLI jr?Xx66uOgfp\}<br>]x"ulO?9xx?FjQB?kr6i?6"r?QLrI9QINjNI? x66_O\#fpo6rg?OjNJI?QLI?i6rQ9x?9Nu?jNQIOQjNI?,9rO £p'm<j\Q?QLB?OlujQj6"O?]6"NQrBilN?9Nu?"Og£pMQ? L9QL?jN?O6xliN?OBN6uO?XIIN?ul]rllufp\$6QL?XB?QLI? WBr9]"Oj9NO?9Nu?6"rOlx/IOgfpm6?9uijQ?N6?Qr9kkj] ?Q6?6"r?9u/IrOI?Q6<NO?a9Bg?i6rlgfpMk?9NB?X6rN? 9Q?\}FLIO"O?XI?OIIN£pyQ?9NB?WBr9]"Oj9N?i9rQO?9N u?k9jrO=£pyl9jNH?jk?9NB?WBr9]"Oj9N?X6rNfp*6il?Q 6?QLI?X9B?6k?\}FLIO"Og?LI?ujIOg£pUjO?!66uO?]6NkjO] 9QI?Q6?QLI?u"_I'O?ujOF6Olg£pANxIOO?9?QL6"O9Nu?i 9r_O?XI?x//jlug£pm6?d"jQ?QLI?FIN9xQB?9Nu?Q6?r9N O6i?Lji\#£pmLB?O"XOQ9N]Ig?/9x"lu?9Q?QLI?LjILIOQ?r9 Qlg£p*9NN6Q?9i6"NQ?"NQ6?9?L"Nurlu?i9r_O=£pmLIrl k6rl?XB?x9<?QL6"?9rQ?]6NuliNlu?Q6?ujl

```
fid = fopen( filename, 'r' ); %Open file filename (read only)
%Store filename text in a row vector A of characters, then close file
A = fscanf(fid, '%c'); fclose(fid);
e.g A = 'The Comedy of Errors .....'
%Open file for writing
fid = fopen( strrep( filename,'.txt',['-',cipher_mode,'.txt'] ), 'w' );
%Step through cipher_key, replacing instances of the
%characters with their plaintext or enciphered equivalents
B = A; dim = size(cipher_key);
if strcmp(cipher_mode,'encrypt')==1
    %Encrypt file contents
    for n=1:dim(1)
            indices = strfind( A, cipher_key{n,1} );
            B(indices) = cipher_key{n,2};
    end
```

e.g.
plaintext.txt
would become
plaintext-
encrypt.txt

```
else
    %Decrypt file contents
    for n=1:dim(1)
        indices = strfind( A, cipher_key{n,2} );
        B(indices) = cipher_key{n,1};
```

MATLAB code for cipher.m

```
end
end
\%Write encrypted character array \(B\) to a appended, then close file fwrite(fid, B ); fclose(fid);

Case study \#2: \(\quad\) Frequency analysis \& code breaking


Bar chart describing fractions of alphabetic characters in a typical extended piece of written English

Different languages have different character usage statistics i.e. different 'signatures'


Frequency analysis for A Comedy of Errors
 char(13)


\section*{Case study \#3: Vigenère \& polyalphabetic ciphers}

How do we construct a code that is more resistant to frequency analysis? One answer is to use more than one cipher alphabet.

A polyalphabetic cipher cycles between \(N\) different alphabets in a fixed sequence. The first character of the plaintext is encoded with alphabet 1 , the second character with alphabet 2 etc.

\section*{Example:}
plaintext alphabet abcdefghijklmnopqrstuvwxyz cipher alphabet 1 cdefghijklmnopqrstuvwxyzab cipher alphabet 2 mnopqrstuvwxyzabcdefghijkl cipher alphabet 3 wxyzabcdefghijklmnopqrstuv

\section*{plaintext:}
```

a polyalphabetic cipher is harder to break So although
abcdefghijklmnopqrstuvqxyz
ciphertext:
c mqyvcymjnyggfe zkcege kf jnofro gl oognh
xdpagsdjvgmyjp.bmsepvhsskvb

```

These examples are Caesar Shifts of the plaintext alphabet

So although we have oo in the ciphertext, this means two different letters in the plaintext

\section*{Case study \#3: Vigenère \& polyalphabetic ciphers}



Blaise de Vigènere 1523-1596
Diplomat, Cryptographer, Alchemist(!)

\section*{The Vigènere}

Square
i.e. all A-Z

Caesar Shift
cipher alphabets

\section*{Case study \#3: Vigenère \& polyalphabetic ciphers}
\begin{tabular}{|c|c|}
\hline & ABCDEFGHIJKLMNOPQ \\
\hline & A B CDEFGHIJKLMNOPQRSTUVWXYZ \\
\hline & B CDEFGHI JKLMNOPQRSTUVWXYZA \\
\hline & CDEFGHIJKLMNOPQRSTUVWXYZ \\
\hline & DEFGHISKLMNOPQRSTUVWXYZABC \\
\hline & EFGHI J K L M O P Q R S T UVWXY Z A B \\
\hline & FGHI I K L M O P Q R ST UVWXYZABC \\
\hline & GHIJKLMNOPQRSTUVWXYZABCD \\
\hline &  \\
\hline & I J K L M N OPQRST UVWXY Z \\
\hline & IKLMNOPQRSTUVWXYZABCDEFG \\
\hline & KLMNOPQRSTUVWXYZABCDEFGH \\
\hline & L MNOPQRSTUVWXYZABCDEFGHI IK \\
\hline & M NOPQRSTUVWXYZABCDEFGHI J \\
\hline & NOPQRSTUVWXYZABCDEFGHISK \\
\hline & OPQRSTUVWXYZABCDEFGHIJK \\
\hline & PQRSTUVWXYZABCDEFGHI J K L M No \\
\hline & QRSTUVWXYZABCDEFGH I J K L M N \\
\hline & R S T UVWXYZABCDEFGHIJKLMNO \\
\hline & S TUVWXYZABCDEFGHIJKLMNOP \\
\hline & TUVWXYZABCDEFGHIJKLMNOPQ \\
\hline & UVWXYZA B C D E F G H I I K L M NOPQRST \\
\hline & VWXYZABCDEFGHI J K L M N OPQR S T U \\
\hline & WX Y Z A B CDEFGH I J K L M O P Q R S T U V \\
\hline & XY Z A B CDEFGHI JKLMNOPQRSTUVW \\
\hline & Y Z A B CDEFGHI JKLMNOPQRSTUVWX \\
\hline & ZABCDEFGHI IKLMNOPQRSTUVWX \\
\hline
\end{tabular}

The Vigènere Square can be used to generate cipher keys. The cycle of alphabets correspond to the repetition of a codeword.

If the codeword is POLY, alphabets are
\#1: PQRSTUVWXYZABCDEFGHIJKLMNO
\#2: OPQRSTUVWXYZABCDEFGHIJKLMN
\#3: LMNOPQRSTUVWXYZABCDEFGHIJK
\#4: YZABCDEFGHIJKLMNOPQRSTUVWX

\section*{plaintext:}

MR VIGENERE WAS A VERY CLEVER CHAP
POLYPOLYPOLYPOLYPOLYPOLYPOLYPOLYPO
ABCDEFGHIJKLMNOPQRSTUVWXYZ
LYPOLYPOLYPOLYPOLYPOLYPOLY

Repetition of codeword POLY defines the alphabet used to encrypt the plaintext characters
ciphertext:
BF TXUPLTFP LOD P GCGM AASGCG NEPD

\section*{Case study \#3: Vigenère \& polyalphabetic ciphers}

The Vigenère cipher was thought to be le chiffre indéchiffrable"the indecipherable cipher"

However, an attack was devised in the 1860s by Charles Babbage and Friedrich Kasiski.
It is based upon finding repeated strings of characters with the ciphertext. This can be used to work out the repeat length of a polyalphabetic cipher.
(For the previous example of codeword POLY, the repeat length is 4)
Once this is known, characters can be separated out, with each set corresponding to a unique alphabet.

Frequency analysis can then be used to work out each of the cipher alphabets.


Charles Babbage
1791-1871

Inventor of the first programmable (mechanical) computer

Case study \#4: Steganography - the practice of concealing messages or information within other non-secret text or data

If one receives an obviously encrypted message, one's interest is immediately piqued...... It is perhaps preferable to hide a secret message in normal communications. The secret bit is not expected, and hence is overlooked.

In ancient times, a message might be tattooed on the head of a slave. The slave could be sent to the recipient after a hair regrowing period, who would know to shave the slave upon arrival.

Other physical examples of steganography might be the use of invisible ink (which is revealed using UV light or application of a chemical), microdots, or simply in a letter
"Today her interest soared, in Spain a seedy European corporate rogue entered Toledo carrying Olga's distinctive emerald ..."

\section*{This iS a secreT cOde}

However, this is all rather time consuming! The digital age provides us with an excellent method for hiding messages: Photographs

Case study \#4: Modern steganography - concealing messages in digital photographs


What is the difference between these images?
The one on the left also contains the entire works of Shakespeare.

Case study \#4: Modern steganography - concealing messages in digital photographs


Digital (colour) images consist of three matrices of numbers. These correspond to intensities of the colours Red, Green and Blue

A typical (JPEG) image from a 12 MegaPixel digital camera might have three \(4000 \times 3000\) matrices of integer numbers within the range 0 ... 255

Blue matrix


Green matrix


Red matrix


Resultant Image


\section*{Case study \#4: Modern steganography - concealing messages in digital photographs}

The trick of 'digital steganography' is to modify the R,G,B pixel values by a small amount corresponding to the ASCII codes of characters which form a message. The difference between an encrypted image and a 'genuine' one might be imperceptible to the human eye, but a computer can use the numerical difference to encode or extract a message.

The Project Gutenberg eBook of the Complete Works of Shakespeare contains \(5,589,886\) characters (which includes line returns and spaces).

Each character has an ASCII code between 0 and 255.
- To encode an image, use the three integers which make up the ASCII code of a character. For example a is 097 , therefore the numbers are \(\mathbf{0 , 9 , 7}\)
- After making sure the maximum colour intensity is \(\mathbf{2 4 6}\) (any \(R, G, B\) values above 246 are set at 246), I will add my three ASCII numbers, respectively, to the \(R, G, B\) values
- This means I can store one character of my plaintext in every pixel in the image.

Given a 6 million pixel image is fairly low-resolution these days, storing the Complete Works of Shakespeare is quite easy.

\section*{ASCII control characters}
\begin{tabular}{|c|c|c|}
\hline 00 & NULL & （Null character） \\
\hline 01 & SOH & （Start of Header） \\
\hline 02 & STX & （Start of Text） \\
\hline 03 & ETX & （End of Text） \\
\hline 04 & EOT & （End of Trans．） \\
\hline 05 & ENQ & （Enquiry） \\
\hline 06 & ACK & （Acknowledgement） \\
\hline 07 & BEL & （Bell） \\
\hline 08 & BS & （Backspace） \\
\hline 09 & HT & （Horizontal Tab） \\
\hline 10 & LF & （Line feed） \\
\hline 11 & VT & （Vertical Tab） \\
\hline 12 & FF & （Form feed） \\
\hline 13 & CR & （Carriage return） \\
\hline 14 & SO & （Shift Out） \\
\hline 15 & SI & （Shift In） \\
\hline 16 & DLE & （Data link escape） \\
\hline 17 & DC1 & （Device control 1） \\
\hline 18 & DC2 & （Device control 2） \\
\hline 19 & DC3 & （Device control 3） \\
\hline 20 & DC4 & （Device control 4） \\
\hline 21 & NAK & （Negative acknowl．） \\
\hline 22 & SYN & （Synchronous idle） \\
\hline 23 & ETB & （End of trans．block） \\
\hline 24 & CAN & （Cancel） \\
\hline 25 & EM & （End of medium） \\
\hline 26 & SUB & （Substitute） \\
\hline 27 & ESC & （Escape） \\
\hline 28 & FS & （File separator） \\
\hline 29 & GS & （Group separator） \\
\hline 30 & RS & （Record separator） \\
\hline 31 & US & （Unit separator） \\
\hline 127 & DEL & （Delete） \\
\hline
\end{tabular}

ASCII printable characters
\begin{tabular}{|c|c|c|c|c|c|}
\hline 32 & space & 64 & ＠ & 96 & － \\
\hline 33 & ！ & 65 & A & 97 & a \\
\hline 34 & ＂ & 66 & B & 98 & b \\
\hline 35 & \＃ & 67 & C & 99 & c \\
\hline 36 & \＄ & 68 & D & 100 & d \\
\hline 37 & \％ & 69 & E & 101 & e \\
\hline 38 & \＆ & 70 & F & 102 & f \\
\hline 39 & ， & 71 & G & 103 & g \\
\hline 40 & 1 & 72 & H & 104 & h \\
\hline 41 & ） & 73 & I & 105 & i \\
\hline 42 & ＊ & 74 & J & 106 & j \\
\hline 43 & ＋ & 75 & K & 107 & k \\
\hline 44 & ， & 76 & L & 108 & I \\
\hline 45 & － & 77 & M & 109 & m \\
\hline 46 & － & 78 & N & 110 & n \\
\hline 47 & 1 & 79 & 0 & 111 & 0 \\
\hline 48 & 0 & 80 & P & 112 & p \\
\hline 49 & 1 & 81 & Q & 113 & q \\
\hline 50 & 2 & 82 & R & 114 & r \\
\hline 51 & 3 & 83 & S & 115 & s \\
\hline 52 & 4 & 84 & T & 116 & t \\
\hline 53 & 5 & 85 & U & 117 & u \\
\hline 54 & 6 & 86 & V & 118 & v \\
\hline 55 & 7 & 87 & W & 119 & w \\
\hline 56 & 8 & 88 & X & 120 & x \\
\hline 57 & 9 & 89 & Y & 121 & y \\
\hline 58 & ： & 90 & Z & 122 & z \\
\hline 59 & ； & 91 & ［ & 123 & \｛ \\
\hline 60 & ＜ & 92 & 1 & 124 & I \\
\hline 61 & \(=\) & 93 & ］ & 125 & \} \\
\hline 62 & ＞ & 94 & \(\wedge\) & 126 & \(\sim\) \\
\hline 63 & ？ & 95 & － & & \\
\hline
\end{tabular}

Extended ASCII characters
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline 128 & Ç & 160 & á & 192 & L & 224 & Ó \\
\hline 129 & ü & 161 & i & 193 & \(\perp\) & 225 & B \\
\hline 130 & é & 162 & ó & 194 & T & 226 & Ô \\
\hline 131 & â & 163 & ú & 195 & \(F\) & 227 & Ò \\
\hline 132 & ä & 164 & กี & 196 & － & 228 & O \\
\hline 133 & à & 165 & Ñ & 197 & ＋ & 229 & O \\
\hline 134 & à & 166 & ， & 198 & ã & 230 & \(\mu\) \\
\hline 135 & ç & 167 & － & 199 & Ã & 231 & p \\
\hline 136 & ê & 168 & c & 200 & L & 232 & p \\
\hline 137 & ë & 169 & （2） & 201 & ［ & 233 & Ú \\
\hline 138 & è & 170 & 7 & 202 & \(\underline{L}\) & 234 & Û \\
\hline 139 & i & 171 & 1／2 & 203 & T & 235 & Ù \\
\hline 140 & i & 172 & 1／4 & 204 & － & 236 & y \\
\hline 141 & i & 173 & i & 205 & \(=\) & 237 & \(\dot{Y}\) \\
\hline 142 & Ä & 174 & ＊ & 206 & \(\ddagger\) & 238 & \\
\hline 143 & A & 175 & \＃ & 207 & 口 & 239 & \\
\hline 144 & É & 176 & 罂 & 208 & \(\delta\) & 240 & 三 \\
\hline 145 & æ & 177 & 츷 & 209 & Đ & 241 & \(\pm\) \\
\hline 146 & FE & 178 & 틏 & 210 & \(\hat{\mathbf{E}}\) & 242 & \\
\hline 147 & ô & 179 & & 211 & Ë & 243 & 3／4 \\
\hline 148 & ö & 180 & － & 212 & È & 244 & II \\
\hline 149 & ò & 181 & A & 213 & 1 & 245 & § \\
\hline 150 & û & 182 & Â & 214 & i & 246 & \(\div\) \\
\hline 151 & ù & 183 & À & 215 & î & 247 & \\
\hline 152 & \(\ddot{\text { y }}\) & 184 & © & 216 & İ & 248 & ： \\
\hline 153 & Ö & 185 & \(\xlongequal{1}\) & 217 & 」 & 249 & \\
\hline 154 & Ü & 186 & 1 & 218 & & 250 & － \\
\hline 155 & ■ & 187 & ］ & 219 & & 251 & 1 \\
\hline 156 & £ & 188 & ］ & 220 & － & 252 & s \\
\hline 157 & \(\emptyset\) & 189 & ¢ & 221 & I & 253 & 2 \\
\hline 158 & \(\times\) & 190 & \(¥\) & 222 & 1 & 254 & ■ \\
\hline 159 & \(f\) & 191 & 7 & 223 & ■ & 255 & nbsp \\
\hline
\end{tabular}
www． theASCIIcode ．com．ar
most consulted
ñ énye， n with tilde （alt +164 ）
black square （alt +254 ）
superscript two，square （alt +253 ）
－degree symbol （alt +248 ） （alt +39 ） （alt +230 ）
copyright symbol （alt +184 ）
registered trademark （alt＋169）
superscript three，cube （alt＋252）
a
frequently－used （spanish language）
\begin{tabular}{cc} 
ñ & alt +164 \\
N & alt +165 \\
＠ & alt +64 \\
\(i\) & alt +168 \\
\(?\) & alt +63 \\
\(i\) & alt +173 \\
\(\vdots\) & alt +33 \\
\(\vdots\) & alt +58 \\
\(i\) & alt +47 \\
1 & alt +92 \\
\hline
\end{tabular}

It +92
vowels acute accent （spanish language）
\begin{tabular}{|ll|}
\hline á & alt +160 \\
é & alt +130 \\
i & alt +161 \\
ó & alt +162 \\
ú & alt +163 \\
Á & alt +181 \\
É & alt +144 \\
\(\mathbf{i}\) & alt +214 \\
O & alt +224 \\
Ú & alt +233 \\
\hline
\end{tabular}
vowels with diaresis
\begin{tabular}{ll}
\(\ddot{a}\) & alt +132 \\
\(\ddot{\mathrm{e}}\) & alt +137 \\
\(\overrightarrow{\mathbf{i}}\) & alt +139 \\
\(\ddot{\mathbf{o}}\) & alt +148 \\
\(\ddot{\mathbf{u}}\) & alt +129 \\
\(\ddot{\mathbf{A}}\) & alt +142 \\
\(\ddot{\mathbf{E}}\) & alt +211 \\
\(\ddot{\mathbf{I}}\) & alt +216 \\
\(\ddot{\mathbf{O}}\) & alt +153 \\
\(\ddot{\mathbf{U}}\) & alt +154 \\
\hline
\end{tabular}
mathematical symbols
\begin{tabular}{|cl|}
\hline \(\mathbf{1} 2\) & alt +171 \\
\(1 / 4\) & alt +172 \\
\(\mathbf{3} / 4\) & alt +243 \\
\(\mathbf{1}\) & alt +251 \\
\(\mathbf{3}\) & alt +252 \\
\(\mathbf{z}\) & alt +253 \\
\(\mathbf{f}\) & alt +159 \\
\(\mathbf{\pm}\) & alt +241 \\
\(\mathbf{x}\) & alt +158 \\
\(\div\) & alt +246 \\
\hline
\end{tabular}

\section*{commercial／trade} symbols
\begin{tabular}{ll}
\(\$\) & alt +36 \\
\(\mathbf{£}\) & alt +156 \\
\(¥\) & alt +190 \\
\(\not \subset\) & alt +189 \\
a & alt +207 \\
＠ & alt +169 \\
（C） & alt +184 \\
a & alt +166 \\
\(\circ\) & alt +167 \\
\(\circ\) & alt +248 \\
\hline
\end{tabular}
quotes and parenthesis
alt +34
alt +39
alt +40
alt +41
alt +91
alt +93
alt +123
alt +125
alt +174
alt +175

\section*{Case study \#4: Modern steganography - concealing messages in digital photographs}


Original image
e.g. \(3000 \times 2000\) pixels

R: 155, G: 23, B: 190

Secret message e.g. 5,589,886 characters for the Complete Works ....

Each character converted to a three digit ASCII code. a is 097, A is 065
\[
+0
\]

Step through the first 5,589,886 pixels of the 6,000,000 pixel image and add the ASCII code numbers to the R,G,B pixel values

Save the encoded image. It should look very similar!


R: 155, G: 29, B: 195

\section*{encoded original}

Compare the encoded image with an original. From the differences in \(\mathrm{R}, \mathrm{G}, \mathrm{B}\) values work out the ASCII codes of the characters of the hidden message

\(64 \times 82\) pixel original image (5248 pixels)
\(64 \times 82\) pixel encrypted image, with the first 5248 characters of \(A\) Comedy of Errors

ACT I
SCENE I. A hall in DUKE SOLINUS'S palace.
Enter DUKE SOLINUS, AEGEON, Gaoler, Officers, and other Attendants
AEGEON
Proceed, Solinus, to procure my fall And by the doom of death end woes and all.

DUKE SOLINUS
Merchant of Syracuse, plead no more;
I am not partial to infringe our laws:
The enmity and discord which of late
Sprung from the rancorous outrage of your duke To merchants, our well-dealing countrymen, Who wanting guilders to redeem their lives Have seal'd his rigorous statutes with their bloods, Excludes all pity from our threatening looks. For, since the mortal and intestine jars 'Twixt thy seditious countrymen and us, It hath in solemn synods been decreed Both by the Syracusians and ourselves, To admit no traffic to our adverse towns Nay, more, If any born at Ephesus be seen
At any Syracusian marts and fairs;
Again: if any Syracusian born
Come to the bay of Ephesus, he dies,
His goods confiscate to the duke's dispose,
Unless a thousand marks be levied,
To quit the penalty and to ransom him.
Thy substance, valued at the highest rate,
Cannot amount unto a hundred marks; Therefore by law thou art condemned to die.

\section*{AEGEON}

Yet this my comfort: when your words are done, My woes end likewise with the evening sun.

\section*{DUKE SOLINUS}

Well, Syracusian, say in brief the cause Why thou departed'st from thy native home And for what cause thou camest to Ephesus.

AEGEON
A heavier task could not have been imposed Than I to speak my griefs unspeakable: Yet, that the world may witness that my end Was wrought by nature, not by vile offence, I'll utter what my sorrows give me leave.
In Syracusa was I born, and wed
Unto a woman, happy but for me, And by me, had not our hap been bad. With her I lived in joy; our wealth increased By prosperous voyages I often made To Epidamnum; till my factor's death And the great care of goods at random left Drew me from kind embracements of my spouse: From whom my absence was not six months old Before herself, almost at fainting under The pleasing punishment that women bear, Had made provision for her following me And soon and safe arrived where I was. There had she not been long, but she became A joyful mother of two goodly sons; And, which was strange, the one so like the other, As could not be distinguish'd but by names.
That very hour, and in the self-same inn,
A meaner woman was delivered
Of such a burden, male twins, both alike:
Those,--for their parents were exceeding poor,--
I bought and brought up to attend my sons. My wife, not meanly proud of two such boys, Made daily motions for our home return: Unwilling I agreed. Alas! too soon, We came aboard.
A league from Epidamnum had we sail'd, Before the always wind-obeying deep Gave any tragic instance of our harm: But longer did we not retain much hope; For what obscured light the heavens did grant Did but convey unto our fearful minds A doubtful warrant of immediate death; Which though myself would gladly have embraced, Yet the incessant weepings of my wife,


Weeping before for what she saw must come, And piteous plainings of the pretty babes, That mourn'd for fashion, ignorant what to fear, Forced me to seek delays for them and me. And this it was, for other means was none: The sailors sought for safety by our boat, And left the ship, then sinking-ripe, to us: My wife, more careful for the latter-born, Had fasten'd him unto a small spare mast, Such as seafaring men provide for storms; To him one of the other twins was bound, Whilst I had been like heedful of the other: The children thus disposed, my wife and I, Fixing our eyes on whom our care was fix'd, Fasten'd ourselves at either end the mast; And floating straight, obedient to the stream, Was carried towards Corinth, as we thought. At length the sun, gazing upon the earth, Dispersed those vapours that offended us; And by the benefit of his wished light, The seas wax'd calm, and we discovered Two ships from far making amain to us, Of Corinth that, of Epidaurus this:
But ere they came,--O, let me say no more! Gather the sequel by that went before.

\section*{DUKE SOLINUS}

Nay, forward, old man; do not break off so; For we may pity, though not pardon thee.

\section*{AEGEON}

O, had the gods done so, I had not now Worthily term'd them merciless to us! For, ere the ships could meet by twice five leagues, We were encounterd by a mighty rock; Which being violently borne upon, Our helpful ship was splitted in the midst; So that, in this unjust divorce of us, Fortune had left to both of us alike What to delight in, what to sorrow for. Her part, poor soul! seeming as burdened With lesser weight but not with lesser woe, Was carried with more speed before the wind; And in our sight they three were taken up By fishermen of Corinth, as we thought. At length, another ship had seized on us; And, knowing whom it was their hap to save, Gave healthful welcome to their shipwreck'd guests; And would have reft the fishers of their prey, Had not their bark been very slow of sail; And therefore homeward did they bend their course. Thus have you heard me sever'd from my bli


The Project Gutenberg eBook of The King James Bible (1,100,823 characters) stored in The Last Supper by Leonardo da Vinci


The Project Gutenberg eBook of War \& Peace (3,291,645 characters) stored in a painting of Leo Tolstoy


The Project Gutenberg eBook of Arthur Conan-Doyle's The Adventures of Sherlock Holmes (594,930 characters) stored in a photograph of the actor Benedict Cumberbatch

Using MATLAB running on a Windows 7 (64bit) operating system ( 3.2 GHz Intel i5, 8GB RAM) War \& Peace took 4.4888 seconds to be encrypted. The Adventures of Sherlock Holmes took 2.4903s)


Bletchley Park housed the British codebreaking operation during WWII

A major challenge was posed by the Enigma and Lorenz encryption devices

The machines

created at Bletchley to help break the codes are the ancestors of modern computers
\(1.59 \times 10^{20}\) combinations for 3-rotor Enigma


\section*{Case study \#5: Enigma \& the story of Bletchley Park}

Around ten thousand people worked at Bletchley Park and its associated outstations.
It is estimated that the codebreaking work may have shortened the war by two years, saving countless lives.

The success of the D-Day landings was in no small part due to a 'misinformation campaign' which misled the Axis about the intended target. This relied on not only breaking the Enigma code, but ensuring that the Axis were not aware that it had been broken.


\section*{Case study \#5: Enigma \& the story of Bletchley Park}
'Bombes'


\section*{Case study \#5: Enigma \& the story of Bletchley Park}

The Bombe machine was developed by Alan Turing and Gordon Welchman to speed up the breaking of Enigma.

The name was inspired by the 'bomba', an earlier machine used by the Polish Cypher Bureau.


> Alan Turing 1912-1954

Bombes were used to identify Engima rotor settings that led to contradictions in the decryption of an intercepted plaintext. With these settings removed, a more manageable number could be investigated.

\section*{Case study \#6: Modern encryption \& internet security}

Alice and Bob can communicate securely if they both have the same cipher key. e.g a polyalphabetic cipher, based upon random alphabets with a large repeat sequence, will be computationally hard to break. In fact, a 'one time pad', where a plaintext is encoded with a random alphabet the same length as the plaintext, is impossible to break.

However, how do you securely communicate the cipher key? (e.g. the randomized alphabets in the polyalphabetic cipher)


ALICE

Communication channel (e.g. fibre optic cable, wireless link etc)


EVE


BOB

\section*{Case study \#6: Modern encryption \& internet security}

A solution, via the Diffie-Hellman key exchange protocol (1976) is to use an algorithm which generates the same key, but from different secret parts which are not communicated on their own.


\section*{Case study \#6: Modern encryption \& internet security}

Ron Rivest, Adi Shamir, and Leonard Adleman proposed the RSA algorithm for performing the Diffie-Hellman key exchange in 1977

\section*{Alice}


Secret colours
\(=\)


A paint-mixing analogy of the Diffie-Hellman key exchange


\section*{Clifford Cocks} discovered the RSA algorithm three years earlier
.... but he worked for GCHQ so he had to keep it secret till 1997!


Alice chooses two secret prime numbers \(p, q\) e.g. \(p=17, q=11\)

Alice's Public Key

e.g. \(N=17 \times 11=187\) \(e=7\)

Alice computes \(M\) using
\[
M=C^{d}(\bmod N)
\]
\[
\text { s.t. } e d=1(\bmod (p-1)(q-1))
\]
e.g. \(7 \times 23=1(\bmod 16 \times 10)\)
\[
\begin{aligned}
& \therefore d=23 \\
& \therefore 11^{23}(\bmod 187)=88
\end{aligned}
\]

This works because exponentials in modular arithmetic are one-way functions i.e. it is very hard to find \(M\) from \(C\) without \(p, q\)
(Adapted from The Code Book by Simon Singh pp379)

Bigger primes mean much more security as \(N\) will be harder to factorize

Bob encodes a message into a number \(M\) e.g. the decimal ASCII code for the letter X is
\[
M=88
\]

Enciphered message to Alice is: \(C=M^{e}(\bmod N)\)
e.g. \(88^{7}(\bmod 187)=11\)

Only Alice can decrypt Bob's message because only she knows \(p, q\) whereas Bob only needs to know \(N\)

RSA uses modular arithmetic \(x \bmod y=x-n y\) where \(n\) is an integer e.g \(32 \bmod 5=2\) since \(2=32-6 \times 5\)

Here you can tell whether Eve has been listening!

The Quantum future of cryptography


> Alice's lab



DEAD\&ALIVE
SCHRODINGERSCAT


Erwin Schrödinger (1887-1961)

If you intercept a photon, you will force its polarization to be that of the detector. In Quantum Mechanics your act of measurement collapses the wavefunction.

Alice sends Bob a message based upon photons of different polarizations. Alice \& Bob communicate to agree which photons were intercepted with the correct detector, but not what the polarizations were. This sequence forms the basis of a cipher key.

It is very hard for Eve to intercept this, as if she guesses Alice's polarization wrong she will change what Bob receives. This means Alice \& Bob can detect whether Eve has been listening!


\section*{Entangled} photons


Detector A

\(Y_{B}\)


Detector B

We shall assign probabilities for each detector's eigenstate to be based upon the statistics of the classical limit ie. billions and billions of photons! In this case we expect Malus' Law to hold ie. the square of the projection of the polarization yields transmitted power.


\section*{Classical scenario}
\[
P(\text { match })=P\left(X_{A}, X_{B}\right)+P\left(Y_{A}, Y_{B}\right)
\]
\[
\begin{aligned}
& P(\text { match })=\cos ^{2} \theta \cos ^{2} \phi+\sin ^{2} \theta \sin ^{2} \phi \\
& P(\text { mismatch })=1-\cos ^{2} \theta \cos ^{2} \phi-\sin ^{2} \theta \sin ^{2} \phi
\end{aligned}
\]
\[
\begin{array}{|ll}
P\left(X_{A}\right)=\cos ^{2} \theta, & P\left(Y_{A}\right)=\sin ^{2} \theta \\
P\left(X_{B}\right)=\cos ^{2} \phi, & P\left(Y_{B}\right)=\sin ^{2} \phi
\end{array}
\]


Alternatively, if we measure using detector \(A\) first, then QM says that the polarization will now be the measured eigenstate of detector \(A\). This will change the statistics of measurement of \(B\). This if course requires 'spooky action at a distance' between A and \(\mathrm{B} . .\).


\section*{Quantum scenario}
\[
\begin{aligned}
& P(\text { match })=P\left(X_{A}, X_{B}\right)+P\left(Y_{A}, Y_{B}\right) \\
& P(\text { match })=\cos ^{2} \theta \cos ^{2}(\phi-\theta)+\sin ^{2} \theta \cos ^{2}(\phi-\theta) \\
& P(\text { match })=\left(\cos ^{2} \theta+\sin ^{2} \theta\right) \cos ^{2}(\phi-\theta) \\
& P(\text { match })=\cos ^{2}(\phi-\theta) \\
& P(\text { mismatch })=1-\cos ^{2}(\phi-\theta) \\
& P(\text { mismatch })=\sin ^{2}(\phi-\theta)
\end{aligned}
\]

Note we get the same match and mismatch probabilities if we measure \(B\) first. However, what happens if \(A\) and \(B\) detections are simultaneous?

Example: \(\quad \theta=-30^{\circ}, \phi=30^{\circ}\)
Classical \(\quad P(\) mismatch \()=1-\cos ^{2} \theta \cos ^{2} \phi-\sin ^{2} \theta \sin ^{2} \phi\)

\[
\begin{aligned}
& P(\text { mismatch })=1-\left(\frac{\sqrt{3}}{2}\right)^{2}\left(\frac{\sqrt{3}}{2}\right)^{2}-\left(-\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2} \\
& P(\text { mismatch })=1-\frac{9}{16}-\frac{1}{16}=\frac{16-10}{16}=\frac{3}{8}
\end{aligned}
\]

The difference between the probabilities is significant, and therefore readily measurable. For this scenario the QM prediction is that the fraction of mismatches between the detector strings \(X X X Y Y X X Y X Y Y X \ldots\) is double the classical prediction.


Detector B

\section*{Ethical issues: Personal privacy vs national security?}


Quis custodiet ipsos custodes? "Who will guard the guards themselves?" Juvenal, Satire VI, lines 347-8

\section*{Ethical issues: Current UK privacy law}

\section*{Article 8 of the UK Human Rights Act 1998 (from the European Convention of Human Rights)}

Right to respect for private and family life:


Ian Hislop. Private Eye Editor
1. Everyone has the right to respect for his private and family life, his home and his correspondence.
2. There shall be no interference by a public authority with the exercise of this right except such as is in accordance with the law and is necessary in a democratic society in the interests of national security, public safety or the economic wellbeing of the country, for the prevention of disorder or crime, for the protection of health or morals, or for the protection of the rights and freedoms of others.
+ UK Data Protection Act 1998
(Has National Security exemptions)
Need a warrant granted by the Home Secretary for a wiretap etc. But monitoring of internet usage within the workplace is legal as a Lawful Business Practice.


Sara Cox (Radio 1 DJ)
"I concluded that however awful it may be, it's better to have a press which can expose MPs' private lives because it means we have a free press... It means we can expose corruption. Max Mosley has argued for the further advancement of the law whereas the editor of the Daily Mail newspaper Paul Dacre has accused Mr Justice Eady, the judge in the Mosley case, of bringing in a privacy law by the back door."

Mark Oaten
Liberal Democrat Home Affairs Spokesman 2003-2006 MP for Winchester 1997-2010
Resigned from Liberal front bench due to a sex scandal published in the News of the World in 2006


\section*{National}

\section*{Article 8 - Privacy}
- Mother and son stripped searched on prison visit for drugs
- Breach of Prison Rules, humiliated and distressed
- Son mentally impaired and developed PTSD
- Trespass against both persons had been committed.

Son's "trespass" amounted to battery
- Awarded \(£ 2,600\) and \(£ 4,500\) respectively
- Appealed trespass charges
- No common law tort of invasion of privacy
- Needed legislation not common law
- Used ECHR and HRA 1998 to fill gaps


References \& further reading


CODE' BOOK
THE SECRET HISTORY OF CODES AND CODE-BREAKING SIMON SINGH

www.eclecticon.info

\section*{Questions}


Prof. Paweł Horodecki (PG), Rafał Demkowicz-Dobrzański, PhD (FUW) and Michał Karpiński, PhD student (FUW). (Source: Grzegorz Krzyżewski, NLTK, University of Warsaw)```

