DATA ANALYSIS & ERRORS

1/ (i) $t = \{7.52, 7.86, 7.15, 7.33, 7.44\}$

$$\bar{t} = \frac{1}{5}\sum_{i=1}^{5} t_i = \boxed{7.46} \text{ (s)}$$

$$\sigma_t = \sqrt{\frac{1}{4}\sum_{i=1}^{5}(t_i-\bar{t})^2} = \sqrt{\frac{1}{4}\times 0.277}$$

unbiased estimator of standard deviation.

$$= \boxed{0.26} \text{ (s)}$$

$$\therefore \boxed{t = 7.46 \pm 0.26} \text{ (s)}$$

(ii) $v = \frac{x}{t}$ $\therefore$ $\frac{9.8\,km}{43/60\,hr} < v < \frac{10.3\,km}{39/60\,hr}$

$$\Rightarrow \boxed{13.7\,km/h < v < 15.8\,km/h} \text{ (b)}$$

$$\therefore \frac{9.8\times 10^3\,m}{43\times 60\,s} < v < \frac{10.3\times 10^3\,m}{39\times 60\,s}$$

$$\Rightarrow \boxed{3.80\,m/s < v < 4.40\,m/s} \text{ (a)}$$

(iii) $E = \frac{1}{2}mv^2$ $\therefore$ $v = \sqrt{2E/m}$ CLASSICAL KE.

$$\sqrt{\frac{2\times 5.0\times 10^6\times 1.602\times 10^{-19}}{9.109\times 10^{-31}}} \leq v < \sqrt{\frac{2\times 9.8\times 10^6\times 1.602\times 10^{-19}}{9.109\times 10^{-31}}}$$

$$\boxed{4.42c \leq v < 6.19c}$$

So since $v \leq c$ the classical formula cannot be valid.

RELATIVISTIC KE: $(\gamma-1)mc^2 = E$

$$\gamma = E/mc^2 + 1$$

$$\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = E/mc^2 + 1$$

$$1-\frac{v^2}{c^2} = \frac{1}{\left(1+E/mc^2\right)^2}$$

$$\boxed{v = c\sqrt{1-\frac{1}{\left(1+E/mc^2\right)^2}}}$$

So $$v < c\sqrt{1-\frac{1}{\left(1+\frac{9.8\times10^6\times1.602\times10^{-19}}{9.109\times10^{-31}\times(2.998\times10^8)^2}\right)^2}}$$

$$\boxed{v < 0.999c}$$

$$v \geq c\sqrt{1-\frac{1}{\left(1+\frac{5.0\times10^6\times1.602\times10^{-19}}{9.109\times10^{-31}\times(2.998\times10^8)^2}\right)^2}}$$

$$\boxed{v \geq 0.996c}$$

(iv) $F = \frac{GMm}{r^2}$ $\therefore$ $G = \frac{Fr^2}{Mm}$

$[G] = \frac{[F]m^2}{kg^2}$ $[F] = kg\,m\,s^{-2}$ $\therefore$ $[G] = \frac{kg\,m\,s^{-2}\,m^2}{kg^2}$

$\therefore$ $\boxed{[G] = kg^{-1}m^3s^{-2}}$

(v) $c = k\rho^a p^b$

$[c] = ms^{-1}$

$[\rho] = kg\,m^{-3}$

$[p] = [\text{force}/\text{area}] = \frac{kg\,ms^{-2}}{m^2} = kg\,m^{-1}s^{-2}$ if $[k]$ are — (ie dimensionless)

$\therefore\ ms^{-1} = kg^a m^{-3a}\, kg^b m^{-b} s^{-2b}$

comparing powers: kg: $0 = a + b \Rightarrow \boxed{a = -b}$

m: $1 = -3a - b \Rightarrow b = -3a - 1$

$\Rightarrow b = 3b - 1$

$\Rightarrow 1 = 2b$

$\Rightarrow \boxed{\tfrac{1}{2} = b}$

$\therefore \boxed{a = -\tfrac{1}{2}}$ check with s: $-1 = -2b$

$\boxed{\tfrac{1}{2} = b}$ ✓

So $\boxed{c = k\sqrt{\frac{p}{\rho}}}$ ie speed of sound c is $\propto \sqrt{\text{pressure}}$ and inversely proportional to $\sqrt{\text{density}}$ of a gas.

[$k = \sqrt{c_p/c_v}$ where c_p, c_v are specific heat capacities at constant pressure and volume respectively].

(vi) $P = kr^2v^3$

P power of a wind turbine
r Blade radius
v wind speed

$0.9R \leq r < 1.1R$ where R is the mean radius

$0.7u \leq v < 1.3u$ " u is the mean wind speed

So $k \times 0.9^2 \times 0.7^3 R^2u^3 \leq P < k \times 1.1^2 \times 1.3^3 R^2u^3$

$\therefore$ if $\bar{P} = k R^2 u^3$

$$\boxed{0.278 \leq P/\bar{P} < 2.66}$$

ie power has a % error of $\boxed{\begin{matrix} +166\% \\ -72\% \end{matrix}}$

ie <u>asymmetric</u> (if using upper and lower bounds).

if errors are Gaussian $\frac{\sigma_r}{r} \simeq 0.1$ $\quad \frac{\sigma_v}{v} \simeq 0.3$

$$\boxed{\sigma_p^2 = \left(\frac{\partial P}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial P}{\partial v}\right)^2 \sigma_v^2}$$ "Law of errors"

$P = kr^2v^3$

$$\therefore \sigma_p^2 = (2krv^3)^2 \sigma_r^2 + (3kr^2v^2)^2 \sigma_v^2$$

$$\left(\frac{\sigma_p}{P}\right)^2 = \frac{(2krv^3)^2 \sigma_r^2}{(kr^2v^3)^2} + \frac{(3kr^2v^2)^2 \sigma_v^2}{(kr^2v^3)^2}$$

$$\left(\frac{\sigma_p}{P}\right)^2 = 4\left(\frac{\sigma_r}{r}\right)^2 + 9\left(\frac{\sigma_v}{v}\right)^2$$

Generalize:
if $f = x^a y^b z^c \ldots$
$$\left(\frac{\sigma_f}{f}\right)^2 = a^2\left(\frac{\sigma_x}{x}\right)^2 + b^2\left(\frac{\sigma_y}{y}\right)^2 + \ldots$$

$$\therefore \frac{\sigma_p}{P} = \sqrt{4 \times 0.1^2 + 9 \times 0.3^2}$$

$$= 0.92$$

So expect, assuming normally distributed errors,
% in P to be $\boxed{\pm 92\%}$

(vii) $P = \frac{V^2}{R}$ for lightbulb power $\therefore$ $\boxed{R = \frac{V^2}{P}}$

$V = \{110.1, 110.4, 109.8, 109.9\}$ (Volts)

$\boxed{\bar{V} = 110.1}$ Volts ($\bar{V} = 110.05$ in Calc. memory A)

ie $\frac{1}{4}\sum_{i=1}^{4} V_i$ $\qquad \sigma_V = \sqrt{\frac{1}{3}\sum_{i=1}^{4}(V_i - \bar{V})^2}$

$\sigma_V = \sqrt{\frac{1}{3} \times 0.21} = \boxed{0.3 \text{ Volts}}$ (0.265... in memory B)

$P = \{60.2, 59.4, 60.5, 59.8\}$ (W)

$\boxed{\bar{P} = 60.0\,W}$ (59.975 in Cal. memory C)

$\sigma_P = \sqrt{\frac{1}{3} \times 0.6875} = \boxed{0.5\,W}$ (0.479... in Calc memory D)

Inequality method:

$$\frac{(110.1 - 0.3)^2}{60.0 + 0.5} < R < \frac{(110.1 + 0.3)^2}{60.0 - 0.5}$$

But use Calc memories instead

ie $\frac{(A-B)^2}{(C+D)} < R < \frac{(A+B)^2}{C-D}$

$\Rightarrow \boxed{199.4\,\Omega < R < 204.5\,\Omega}$

$\boxed{\therefore R = 201.9^{+2.6}_{-2.5}\ \Omega}$

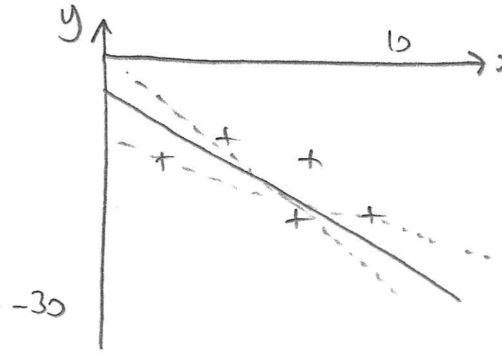
Normally distributed errors:

$$\frac{\sigma_R}{\bar{R}} = \sqrt{4\left(\frac{\sigma_V}{\bar{V}}\right)^2 + \left(\frac{\sigma_P}{\bar{P}}\right)^2}$$

$$= \sqrt{4\left(\frac{B}{A}\right)^2 + \left(\frac{D}{C}\right)^2} = 9.32 \times 10^{-3} \Rightarrow \boxed{\sigma_R = 1.9\,\Omega}$$

$\left[\bar{R} = \frac{A^2}{C}\right]$ so $\boxed{R = (201.9 \pm 1.9)\,\Omega}$

(viii) See Excel sheet.

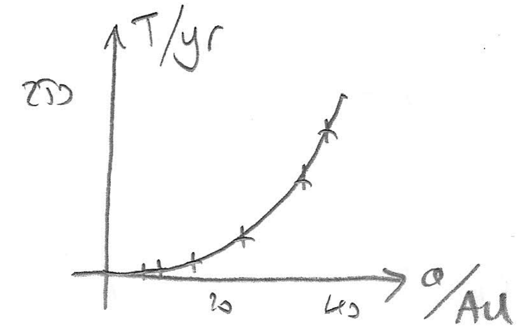


$$y = (-2.5 \pm 0.3)x + (-1.3 \pm 2.3)$$

Line of best fit.

$r = -0.933$ (product moment correlation coefficient)

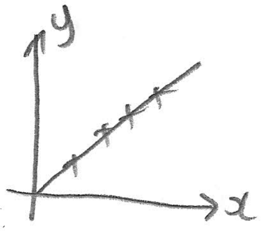
2/ See Excel sheet.



$$T/\text{yr} \approx \left(a/\text{AU}\right)^{3/2}$$

Kepler III.

$$\underbrace{\log_{10}\left(\frac{T}{\text{yr}}\right)}_{y} = \frac{3}{2}\underbrace{\log_{10}\left(\frac{a}{\text{AU}}\right)}_{x}$$



$y = (1.503 \pm 0.019)x$ ← Excel sheet

$r = 1.000$ is very strong correlation.

$\boxed{\text{Bode's law:}}$

$$10\left(\frac{a}{\text{AU}}\right) \approx 4 + 3\times 2^n \Rightarrow n = \frac{\log_{10}\left[\frac{10a-4}{3}\right]}{\log_{10} 2}$$

	a/AU	n
Venus	0.7	0
Earth	1.0	1.0
Mars	1.5	1.9
Asteroids	2.1	2.5
Jupiter	5.2	4.0
Saturn	9.8	5.0
Uranus	19.3	6.0
Neptune	30.2	6.6
Pluto	39.5	7.0

So a tantalising pattern! Asteroids perhaps fill the $n=3$ position between Mars and Jupiter.

3/

a) $\frac{e}{M_e} = \frac{2V}{B^2 r^2}$ charge to Mass ratio of the electron.

$V = (170 \pm 5)$ Volts $\quad B = (8.0 \pm 0.2) \times 10^{-4}\,T$

$r = (5.50 \pm 0.30) \times 10^{-2}\,m$

$$\frac{2 \times (170-5)}{[(8.0+0.2)\times 10^{-4}]^2 [(5.5+0.3)\times 10^{-2}]^2} < e/M_e < \frac{2 \times (170+5)}{[(8.0-0.2)\times 10^{-4}]^2 [(5.5-0.3)\times 10^{-2}]^2}$$

$$\boxed{1.46 \times 10^{11}\ C/kg < \frac{e}{M_e} < 2.13 \times 10^{11}\ C/kg}$$

i/ $\frac{e}{M_e} = \left(1.76 {}^{+0.37}_{-0.30}\right) \times 10^{11}\ C/kg$

b) Now assume normally distributed errors:

let $e/M_e = \bar{k} \pm \sigma_k$ $\qquad \bar{k} = \frac{2(170)}{(8.0\times 10^{-4})^2 (5.50 \times 10^{-2})^2}$

$$\boxed{\bar{k} = 1.76 \times 10^{11}\ C/kg}$$

$$\left(\frac{\sigma_k}{k}\right)^2 = \left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{2\sigma_B}{B}\right)^2 + \left(\frac{2\sigma_r}{r}\right)^2$$

$$\therefore \sigma_k = 1.76 \times 10^{11}\ C/kg \times \sqrt{\left(\frac{5}{170}\right)^2 + \left(\frac{2\times 0.2}{8.0}\right)^2 + \left(\frac{2\times 0.3}{5.5}\right)^2}$$

$$= 1.76 \times 10^{11}\ \frac{C}{kg} \times 0.124$$

$$= \boxed{0.22 \times 10^{11}\ C/kg}$$

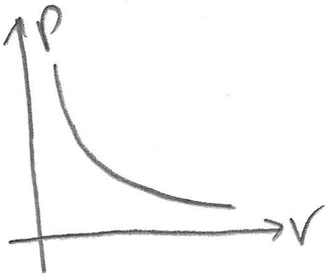
So $\boxed{\frac{e}{M_e} = (1.76 \pm 0.22) \times 10^{11}\ C/kg}$ { 12.5% error }

(c) $\dfrac{e}{M_e} = \dfrac{1.602\times10^{-19}\,C}{9.109\times10^{-31}\,kg} = \boxed{1.76\times10^{11}\ C/kg}$

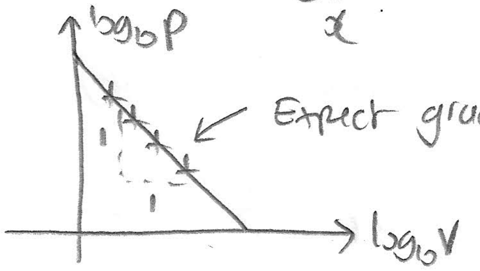
which is exactly what $\bar{k}$ is. ie/so the 'actual' e/M_e is within our error range. So no discernable <u>systematic</u> error.

4/ See Excel sheet. $PV = nRT$ ideal gas equation

Boyle's law: T, n constant $\therefore$ $PV = \text{constant}$



$\therefore \underbrace{\log_{10}P}_{y} = \log_{10}(nRT) - \underbrace{\log_{10}V}_{x}$



Expect gradient -1.

Excel: $\boxed{y = (-1.02\pm0.01)x + (0.504\pm0.003)}$

Intercept (C) is $\log_{10}(nRT)$

$R = 8.314\ J/mol/k$

$T = 294k$

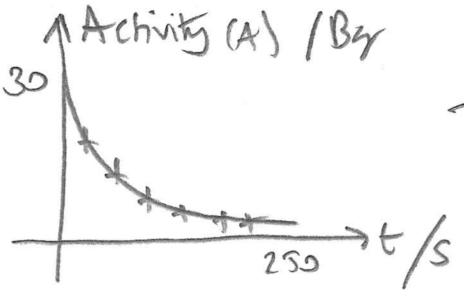
So $n = \dfrac{10^{C}}{RT}$

\# moles

$$\frac{10^{0.501}}{8.314\times294} < n < \frac{10^{0.507}}{8.314\times294}$$

$\boxed{1.30\times10^{-3} < n < 1.31\times10^{-3}}$ moles

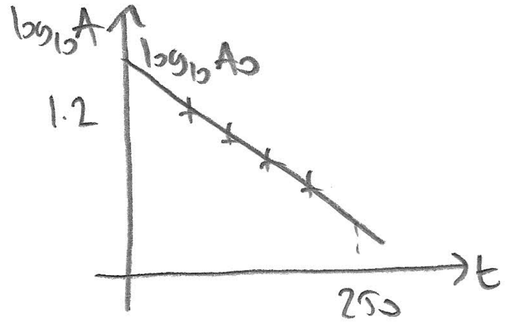
5/ Pa-234 decay



$T =$ half-life

$$A = A_0 \times 2^{-t/T}$$

$$\overbrace{\log_{10} A}^{y} = \overbrace{\log_{10} A_0}^{c} - \overbrace{\frac{t}{T}\log_{10} 2}^{mx}$$



$y = \log_{10}(A/\mathrm{Bq})$

$x = t/s$

Excel: $\boxed{y = (-3.931 \pm 0.208)\times 10^{-3}\, x + (1.487 \pm 0.028)}$

so $10^{1.459} < A_0 < 10^{1.515}$

$$\boxed{28.8\ \mathrm{Bq} < A_0 < 32.7\ \mathrm{Bq}}$$

Best-fit gradient $\quad m = -\dfrac{\log_{10} 2}{T} \quad \therefore \quad T = -\dfrac{\log_{10} 2}{m}$

$$\therefore \quad \frac{\log_{10} 2}{4.139\times 10^{-3}} < T/s < \frac{\log_{10} 2}{3.723\times 10^{-3}}$$

$$\boxed{72.7 < T/s < 80.9}$$

So half life is $\simeq \boxed{T = (76.8 \pm 4.1)\,\mathrm{s}}$

Next steps... ← which is systematically higher than official value of 70.2 s, but not hugely. $\left(\frac{6.6}{70.2} = 9.4\%\ \text{error}\right)$.

→ Has background been correctly subtracted? More samples needed...