# DATA ANALYSIS & ERRORS

1/ (i) $t = \{7.52, 7.86, 7.15, 7.33, 7.44\}$

$$\bar{t} = \frac{1}{5}\sum_{i=1}^{5} t_i = \boxed{7.46} \text{ (s)}$$

$$\sigma_t = \sqrt{\frac{1}{4}\sum_{i=1}^{5}(t_i - \bar{t})^2} = \sqrt{\frac{1}{4} \times 0.277}$$

unbiased estimator of standard deviation.

$$= \boxed{0.26} \text{ (s)}$$

$$\therefore \boxed{t = 7.46 \pm 0.26} \text{ (s)}$$

(ii) $v = \frac{x}{t}$ $\therefore$ $\frac{9.8\text{ km}}{43/60\text{ hr}} < v < \frac{10.3\text{ km}}{39/60\text{ hr}}$

$$\Rightarrow \boxed{13.7\text{ km/h} < v < 15.8\text{ km/h}} \text{ (b)}$$

$$\therefore \frac{9.8 \times 10^3\text{ m}}{43 \times 60\text{ s}} < v < \frac{10.3 \times 10^3\text{ m}}{39 \times 60\text{ s}}$$

$$\Rightarrow \boxed{3.80\text{ m/s} < v < 4.40\text{ m/s}} \text{ (a)}$$

(iii) $E = \frac{1}{2}mv^2$ $\therefore$ $v = \sqrt{2E/m}$ CLASSICAL KE.

$$\sqrt{\frac{2 \times 5.0 \times 10^6 \times 1.602 \times 10^{-19}}{9.109 \times 10^{-31}}} \leq v < \sqrt{\frac{2 \times 9.8 \times 10^6 \times 1.602 \times 10^{-19}}{9.109 \times 10^{-31}}}$$

$$\boxed{4.42c \leq v < 6.19c}$$

So since $v \leq c$ the classical formula cannot be valid.

RELATIVISTIC KE: $(\gamma - 1)mc^2 = E$

$$\gamma = E/mc^2 + 1$$

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = E/mc^2 + 1$$

$$1 - \frac{v^2}{c^2} = \frac{1}{\left(1 + E/mc^2\right)^2}$$

$$\boxed{v = c\sqrt{1 - \frac{1}{\left(1 + E/mc^2\right)^2}}}$$

So $v < c\sqrt{1 - \frac{1}{\left(1 + \frac{9.8\times10^6 \times 1.602\times10^{-19}}{9.109\times10^{-31}\times(2.998\times10^8)^2}\right)^2}}$

$$\boxed{v < 0.999c}$$

$$v \geq c\sqrt{1 - \frac{1}{\left(1 + \frac{5.0\times10^6 \times 1.602\times10^{-19}}{9.109\times10^{-31}\times(2.998\times10^8)^2}\right)^2}}$$

$$\boxed{v \geq 0.996c}$$

(iv) $F = \frac{GMm}{r^2}$ $\therefore$ $G = \frac{Fr^2}{Mm}$

$[G] = \frac{[F]m^2}{kg^2}$ $[F] = kg\,ms^{-2}$ $\therefore$ $[G] = \frac{kg\,ms^{-2}\,m^2}{kg^2}$

$\therefore$ $\boxed{[G] = kg^{-1}m^3s^{-2}}$

(v) $c = k \rho^a p^b$

$[c] = m s^{-1}$

$[\rho] = kg\, m^{-3}$

$[p] = [\text{force}/\text{area}] = \dfrac{kg\, m s^{-2}}{m^2} = kg\, m^{-1} s^{-2}$ if $[k]$ are — (ie dimensionless)

$\therefore \quad m s^{-1} = kg^a m^{-3a} kg^b m^{-b} s^{-2b}$

comparing powers: kg: $0 = a + b \Rightarrow \boxed{a = -b}$

m: $1 = -3a - b \Rightarrow b = -3a - 1$

$\Rightarrow b = 3b - 1$

$\Rightarrow 1 = 2b$

$\Rightarrow \boxed{\frac{1}{2} = b}$

$\therefore \boxed{a = -\frac{1}{2}}$ check with s: $-1 = -2b$

$\boxed{\frac{1}{2} = b}$ ✓

So $\boxed{c = k\sqrt{\dfrac{p}{\rho}}}$ ie speed of sound c is $\propto \sqrt{\text{pressure}}$ and inversely proportional to $\sqrt{\text{density}}$ of a gas.

[$k = \sqrt{c_p/c_v}$ where $c_p$, $c_v$ are specific heat capacities at constant pressure and volume respectively].

(vi) $P = k r^2 v^3$

P power of a wind turbine

r Blade radius

v wind speed

$0.9R \leq r < 1.1R$ where R is the mean radius

$0.7u \leq v < 1.3u$ " u is the mean wind speed

So $k \times 0.9^2 \times 0.7^3 R^2 u^3 \leq P < k \times 1.1^2 \times 1.3^3 R^2 u^3$