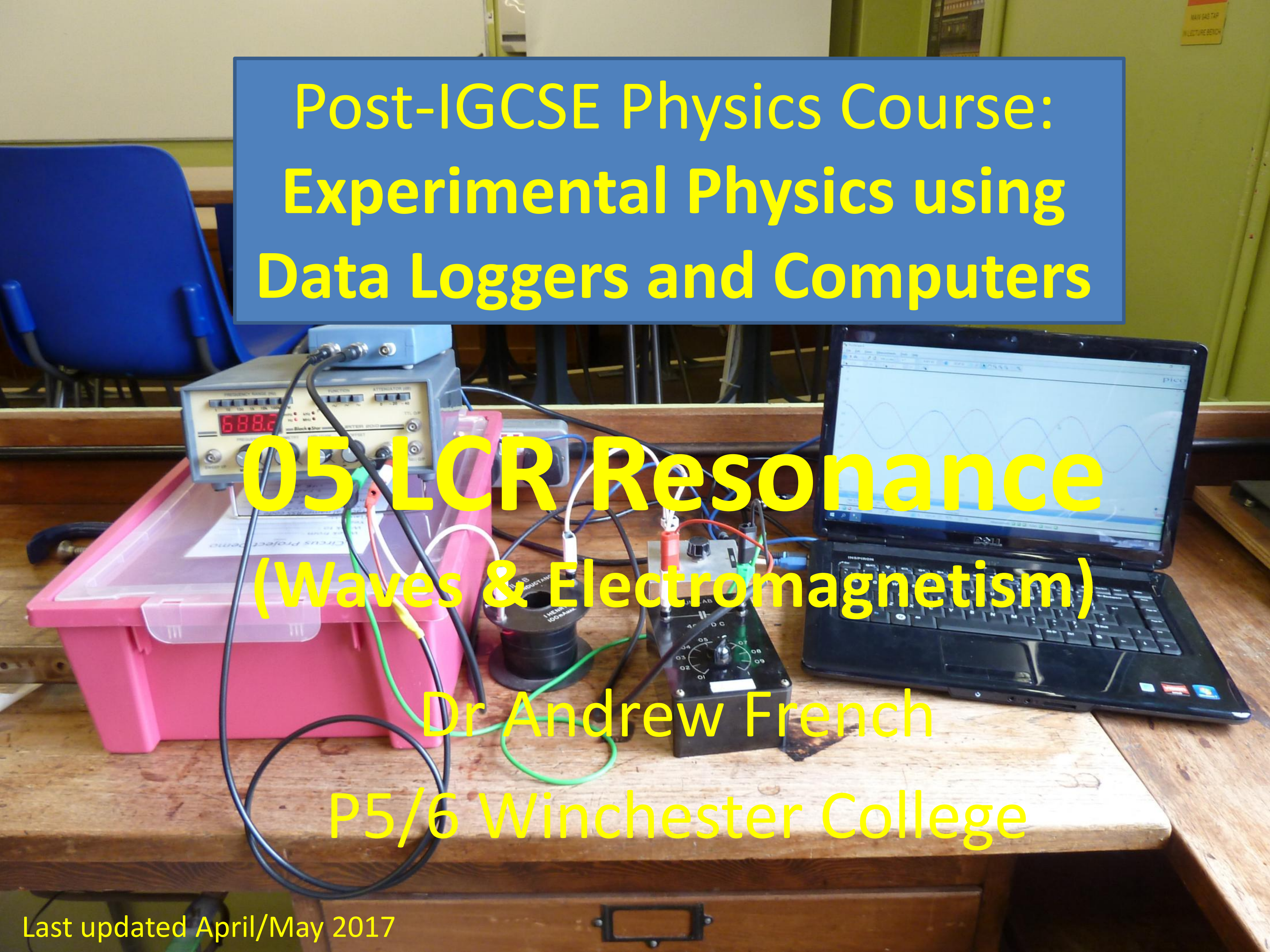

Post-IGCSE Physics Course: **Experimental Physics using Data Loggers and Computers**

05 LCR Resonance

(Waves & Electromagnetism)

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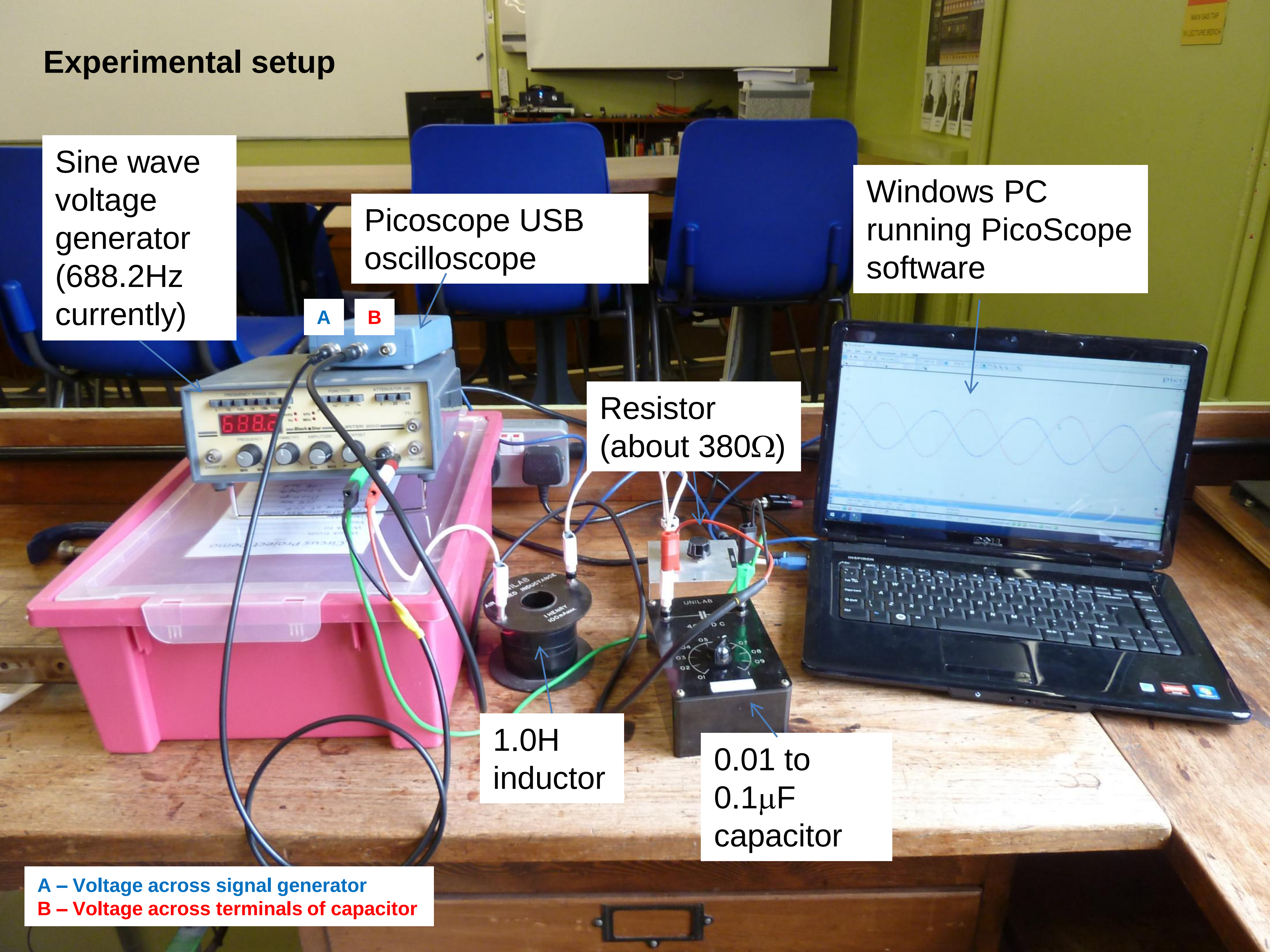
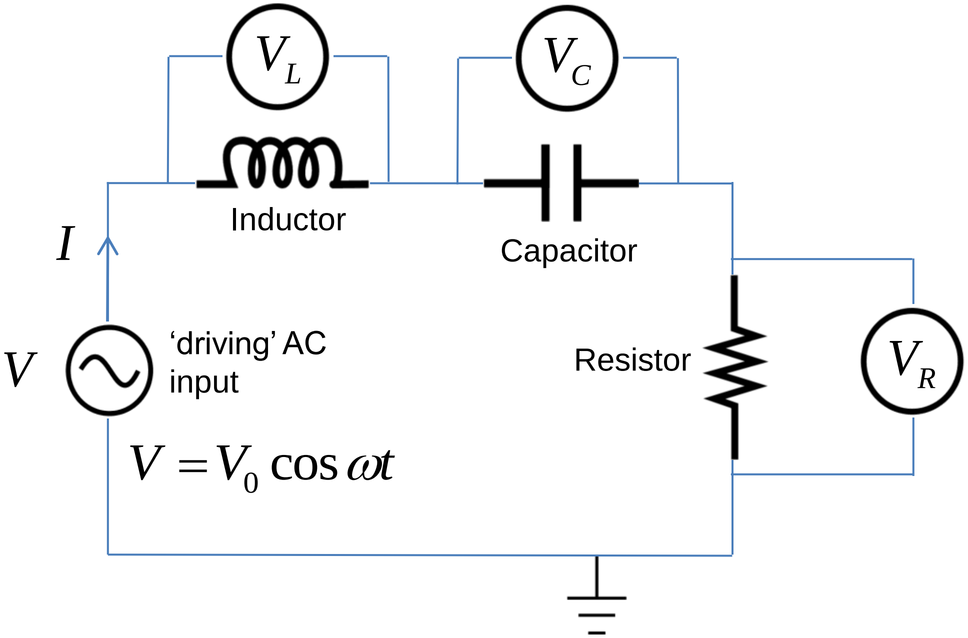
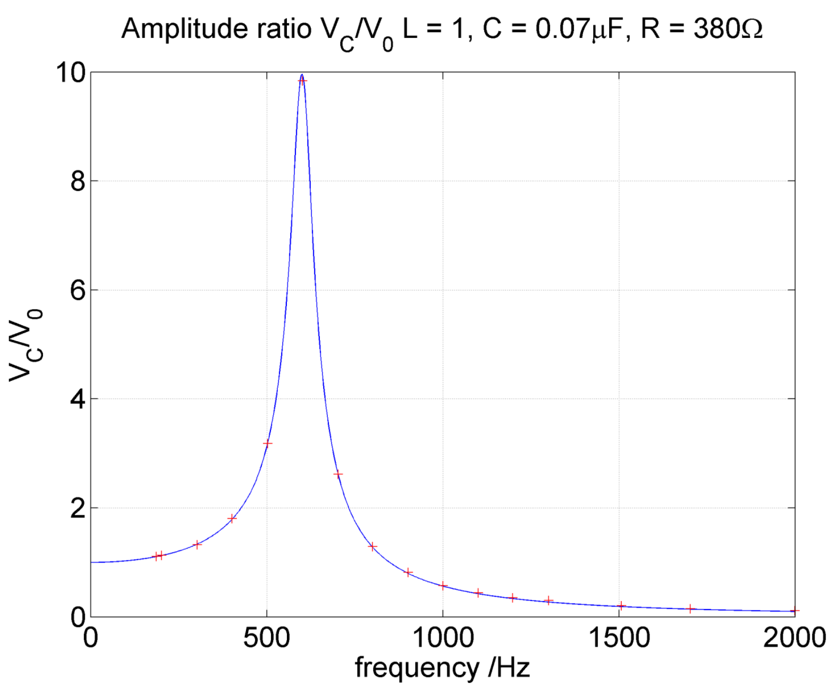
Last updated April/May 2017

Experimental setup





An **LCR circuit** is a single current loop which consists of an **Inductor** (essentially a coil of wire), a **Capacitor** (plates to store charge separated by an insulator) and a fixed **resistor**.



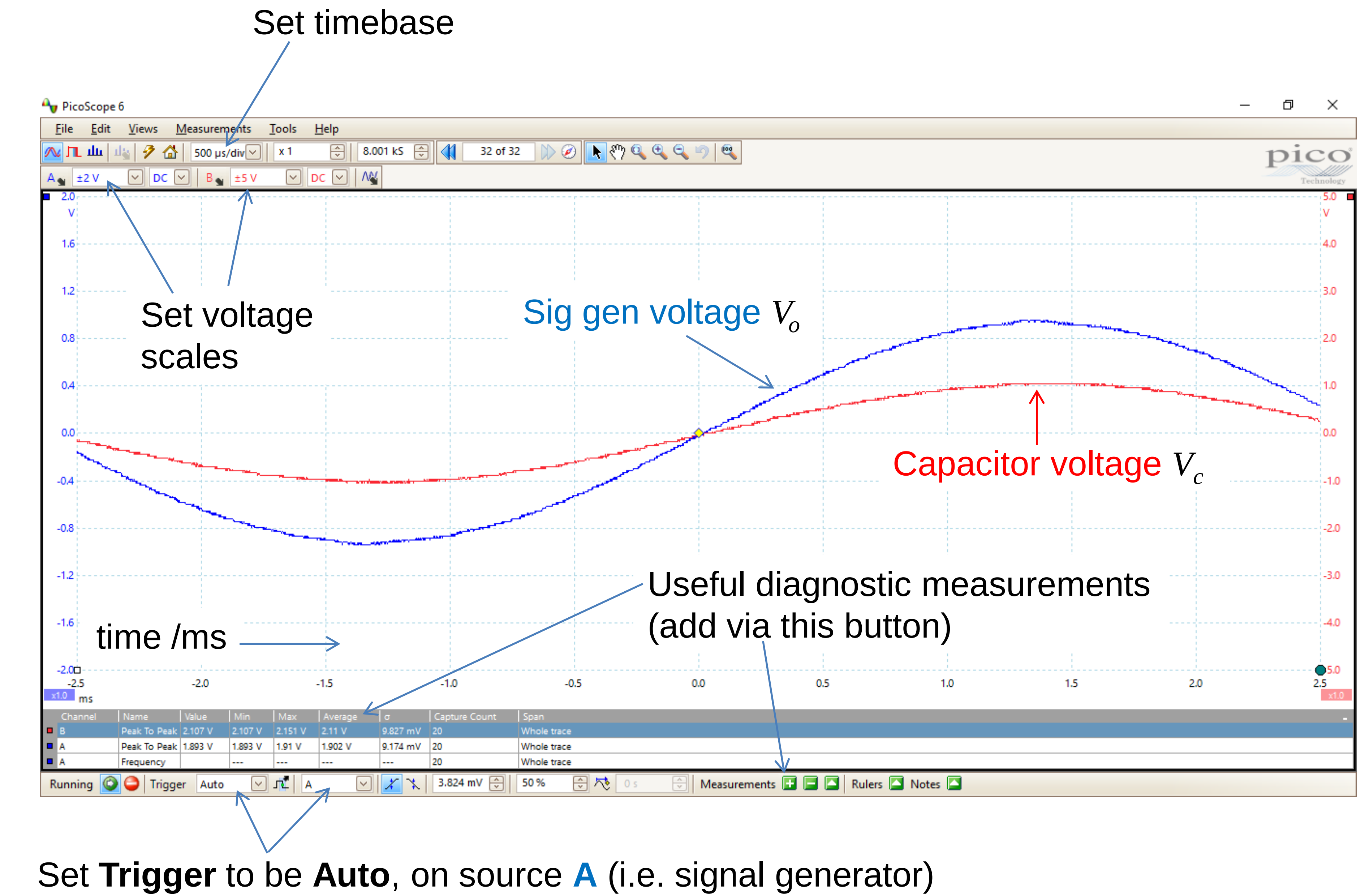
If an AC source drives current through the circuit, **resonant effects** can be observed.

Around the resonant frequency

$$f_{\max} = \tfrac{1}{2\pi}\sqrt{\frac{1}{LC}}\sqrt{1-\frac{(RC)^2}{LC}} \approx \tfrac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

the voltage response across the capacitor will have a *greater amplitude* than the driving voltage.

The *phase* of V_c will also vary with driving frequency.



Screenshot from PicoScope

Experimental procedure

Once **Picoscope** has been set up and both traces visible clearly, sweep the frequency over an appropriate range and then take screenshots (**ctrl+alt+print screen**) and paste into **IrfanView**. Save a PNG bitmap image file with a filename which records the current frequency. e.g. 123p4 Hz.png means ‘100.1 Hz’.

A screenshot every 100Hz might be a sensible first move for L = 1.0H and C = 0.07μF, since

$$f_{\max} \approx \tfrac{1}{2\pi}\sqrt{\frac{1}{LC}} \approx 601.5\text{Hz}$$

.... Then more screenshots around the resonance peak might be a good idea (i.e. every 20Hz instead of 100Hz)

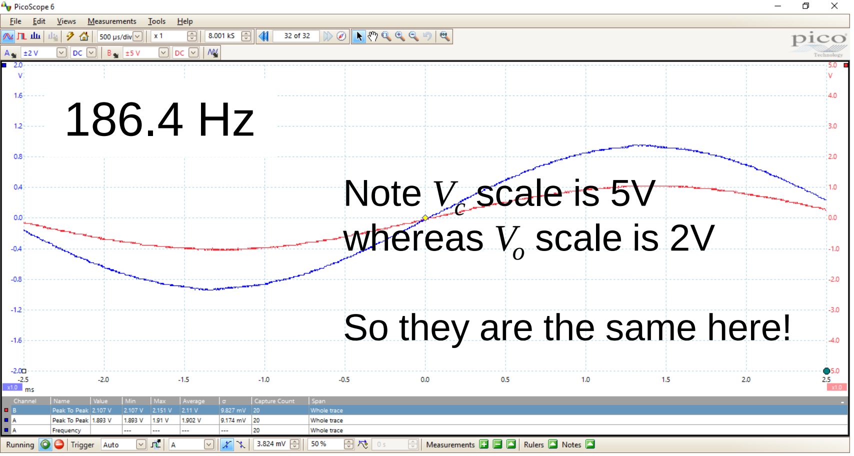
The resonance frequency is fairly weakly dependent upon resistance, but the peak amplitude *will* change significantly with the value chosen.

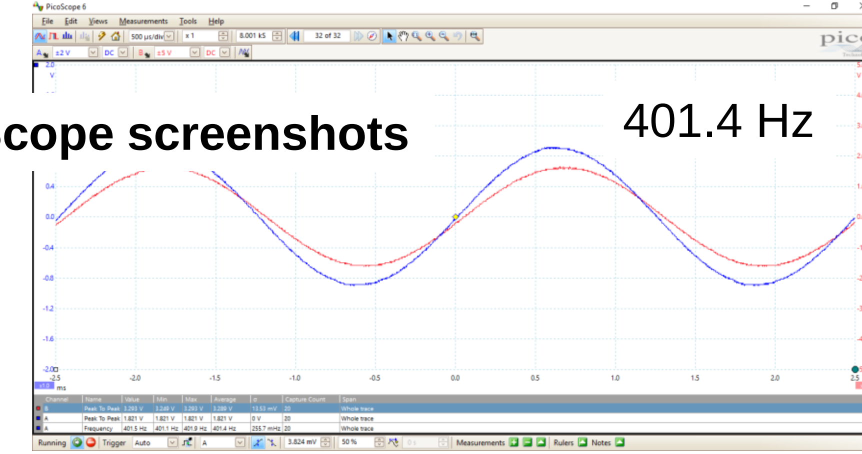
The goal of the experiment is to plot V_c amplitude and phase vs frequency for a variety of capacitances, so setting the resistor so that the resonance is ‘sharp enough to observe, but perhaps not too sharp to measure’ is the key idea.
Turn the resistor knob and observe the effect near resonance.

We need to measure resistance R to fit a model curve.

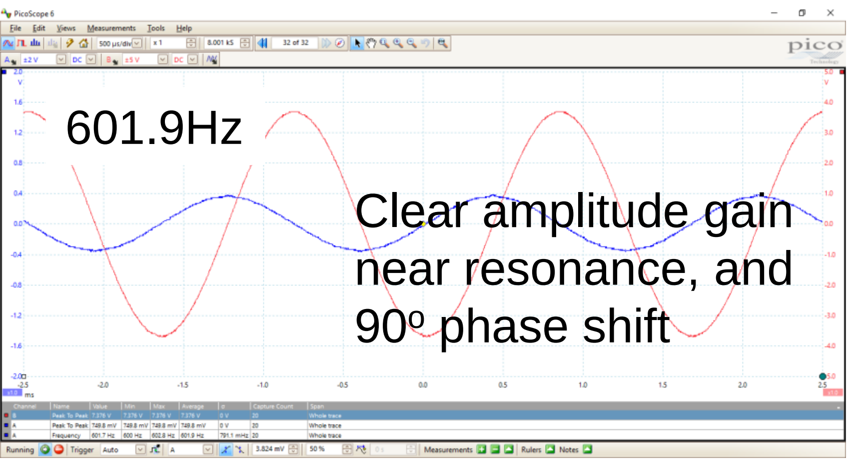
Use a multimeter to achieve this.

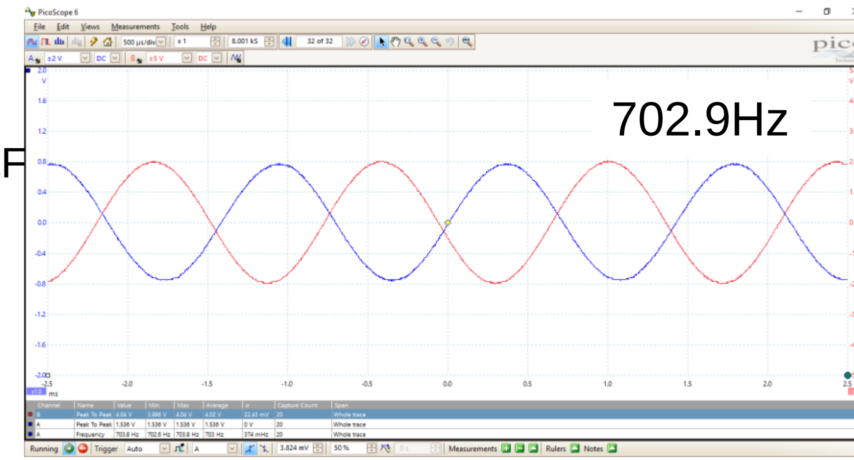
PicoScope screenshots

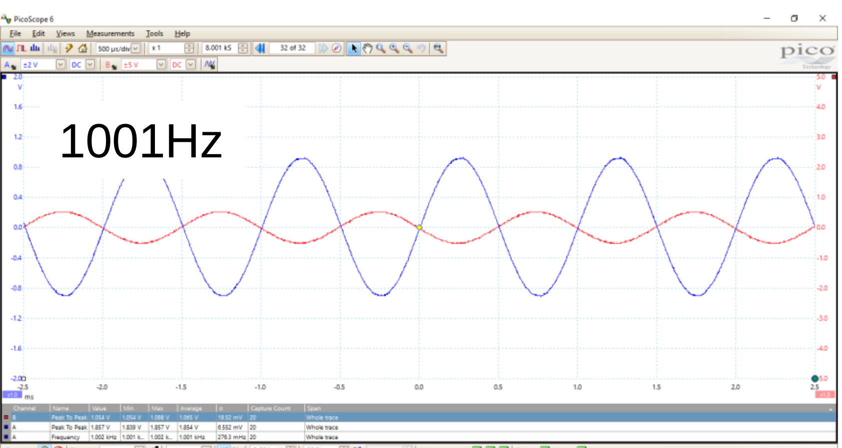


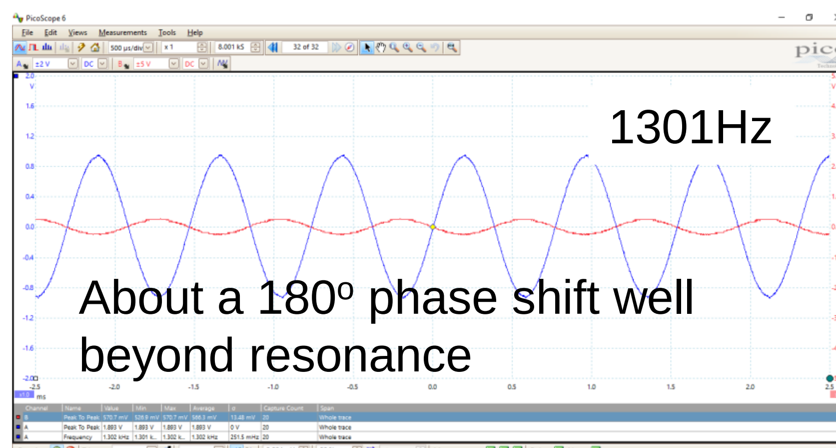


L = 1.0H
C = 0.07μF



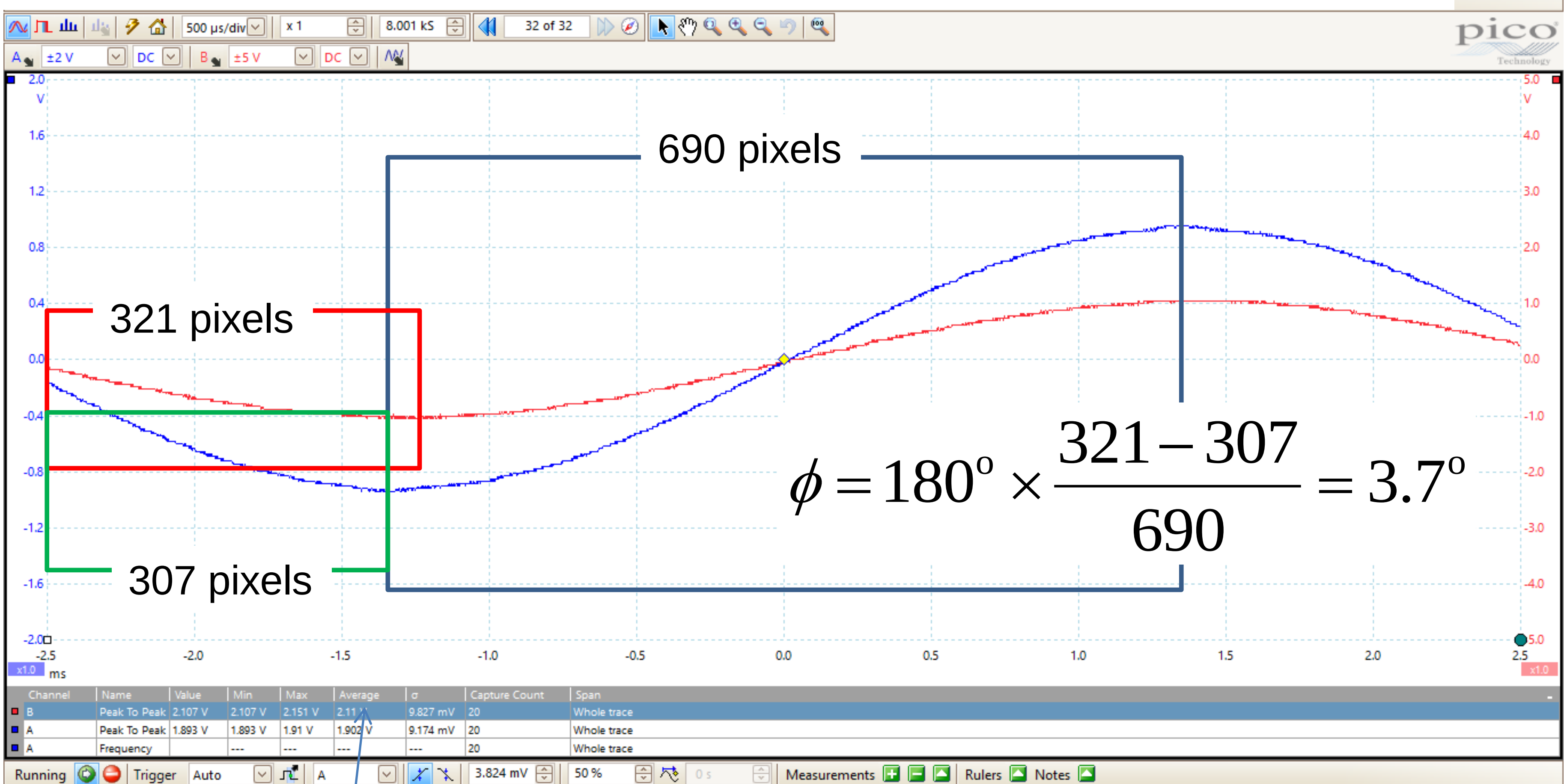






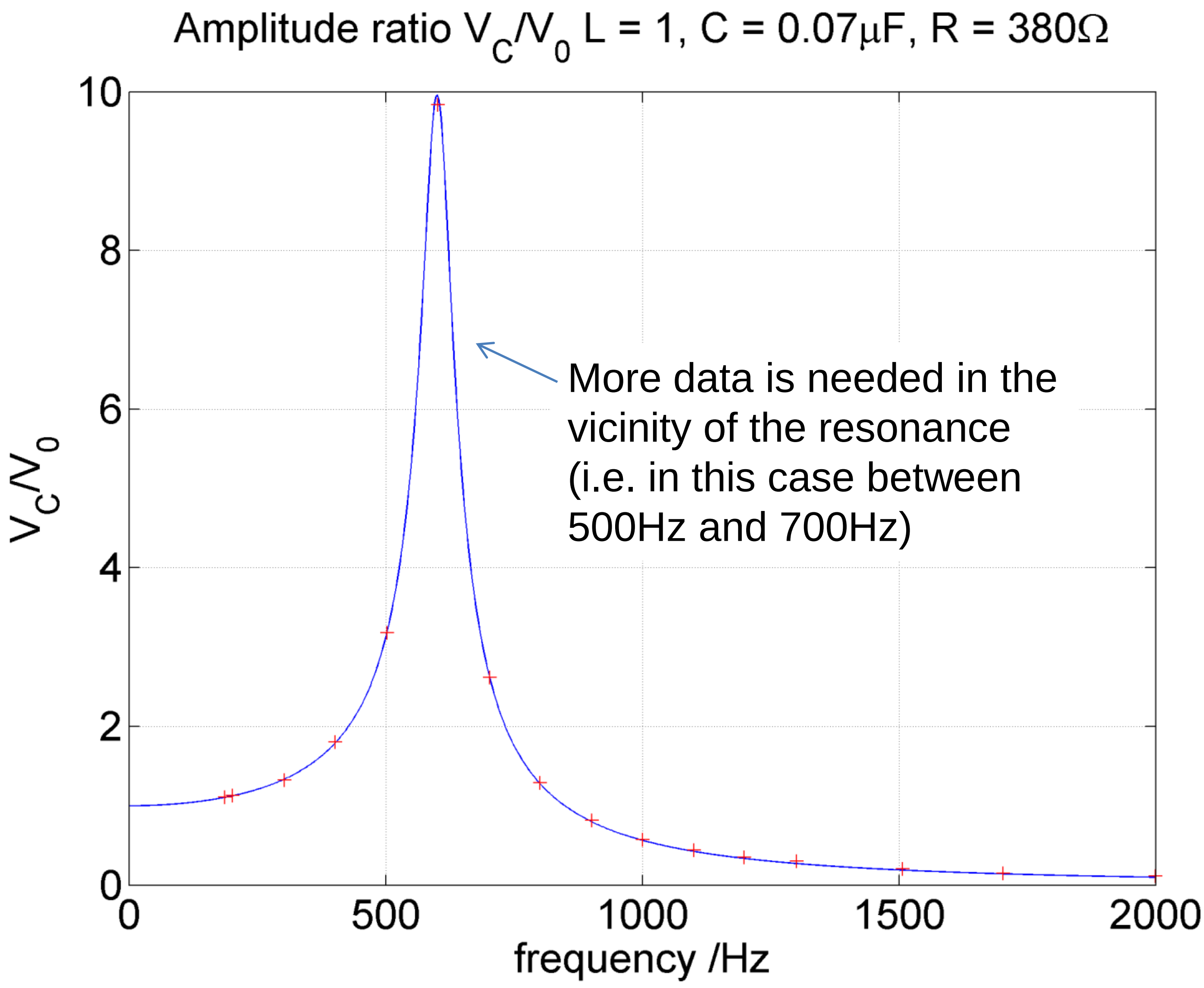
For **phase measurements**, drag a box in IrfanView between trough and peak of the V_0 trace. Record number of pixels for 180º of phase (look in the title bar).

Then work out pixel difference between corresponding peaks or troughs of V_c and V_o traces. Divide both numbers by each other and multiply by 180º

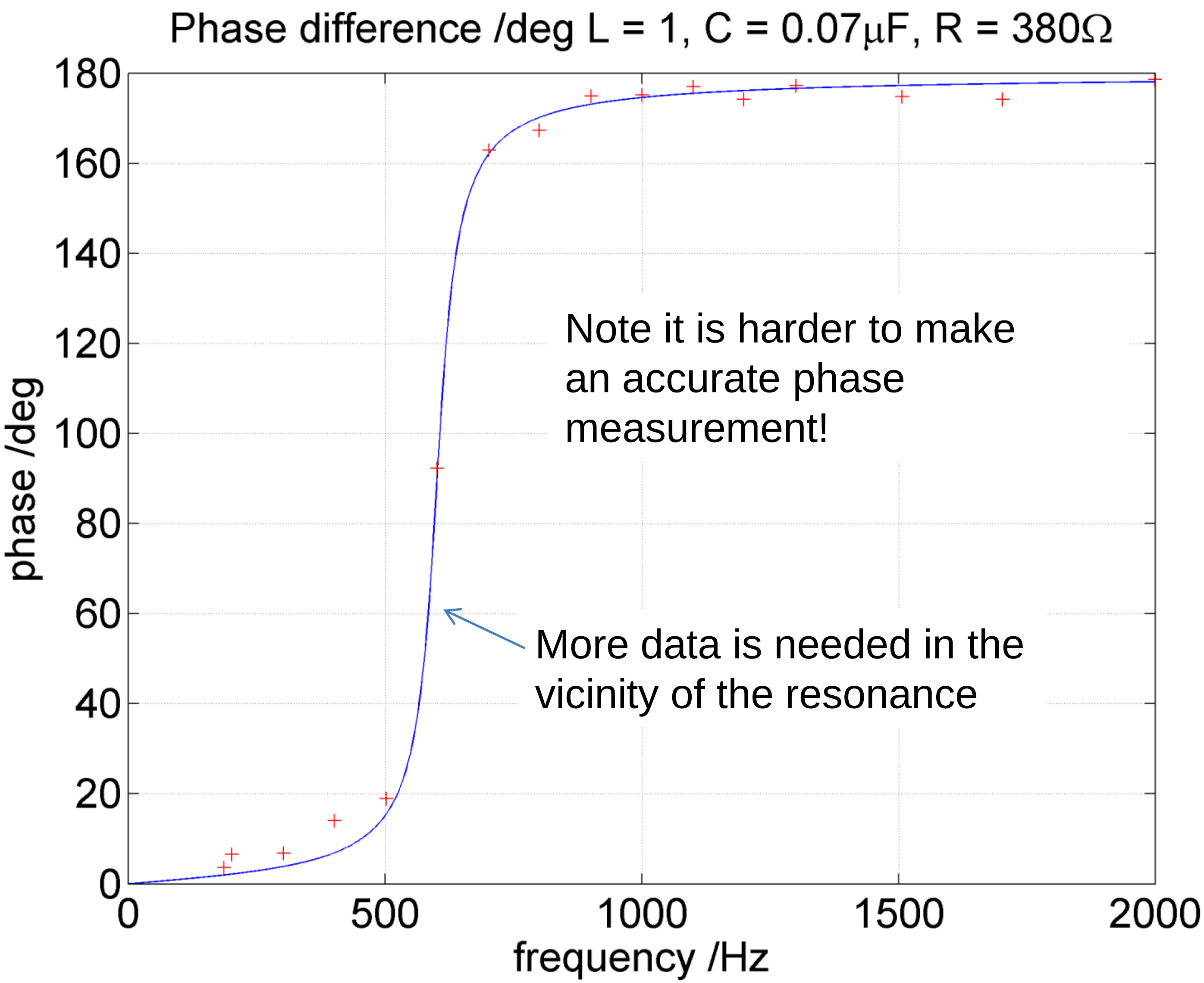


For **amplitude measurements**, simply halve the ‘Peak-to Peak’ values for each trace

Data from **Excel** sheet overlaid upon a model curve. Computations and graphics production done via a MATLAB program **`lcr.m`**



Data from Excel sheet overlaid upon a model curve. Computations and graphics production done via a MATLAB program **lcr.m**



```
%lcr
% Function which loads V0 and VC amplitude and phase information vs
% frequency for the LCR experiment. Experimental data is overlaid upon
% theoretical predictions.
%
% LAST UPDATED by Andy French May 2017

function lcr

%Define capacitance /F
C = 0.07*1e-6;

%Define inductance /H
L = 1.0;

%Define resistance /Ohms
R = 380;

%Max frequency for model
fmax = 2000;

%Number of data points for model
N = 1000;

%Fontsize for graphs
fsize = 18;

%

%Load Excel data file
[filename, pathname, filterindex] = ...
    uigetfile({'*.xlsx';'*.xls'}, 'Choose Excel file');
if filename==0
    return
else
   [num,txt,raw] = xlsread([pathname,filename]);
end

%Assign columns of data

%Frequency in Hz
fdata = num(:,1);

%V0 amplitude /volts
V0 = num(:,2);

%VC amplitude /volts
VC = num(:,3);

%Phase difference between VC and V0 /degrees
phase_deg = num(:,4);

%%
```

MATLAB code **lcr.m**

```
%LCR model
f = linspace(0,fmax,N);
w = 2*pi*f;
ZC = 1./(1i*w*C);
ZL = 1i*w*L;
ZR = R;
VC_by_V0 = ZC./(ZC+ZL+ZR);
```

i.e. *complex impedance* potential divider idea

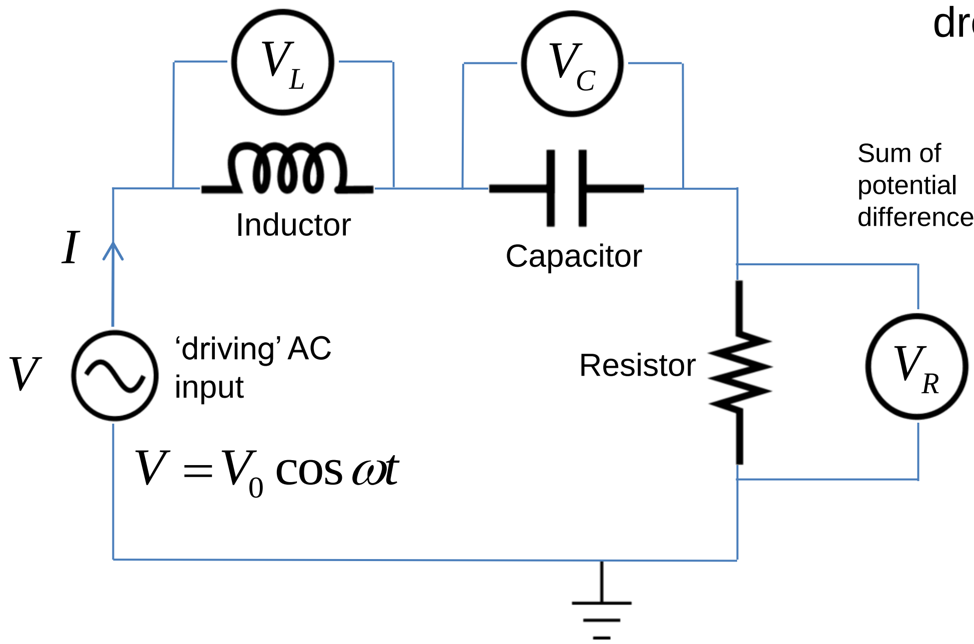
```
%Plot model amplitude and overlay with experimental data
plot( f, abs(VC_by_V0),'b',fdata,VC./V0,'r+' );
xlabel('frequency /Hz','fontsize',fsize);
ylabel('V_C/V_0','fontsize',fsize);
title(['Amplitude ratio V_C/V_0 L = ',num2str(L,2),', C = ',...
    num2str(C*1e6,2),'\mu','F, R = ',num2str(R,4),'\Omega'],...
    'fontsize',fsize)
grid on
box on
set(gca,'fontsize',fsize)
print(gcf,'amplitude ratio.png','-dpng','-r300');
clf;

%Plot model phase and overlay with experimental data
plot( f, -(180/pi)*angle(VC_by_V0),'b',fdata,phase_deg,'r+' );
xlabel('frequency /Hz','fontsize',fsize);
ylabel('phase /deg','fontsize',fsize);
title(['Phase difference /deg L = ',num2str(L,2),', C = ',...
    num2str(C*1e6,2),'\mu','F, R = ',num2str(R,4),'\Omega'],...
    'fontsize',fsize)
grid on
box on
set(gca,'fontsize',fsize)
print(gcf,'phase.png','-dpng','-r300');
close(gcf);

%End of code
```

Steady state solution to the LCR circuit

Let current *I* flow through the circuit. The net EMF $V - V_L$ must equal the sum of the potential drops across each electrical component.



$$V = V_0 \cos \omega t$$

Sum of potential differences →

$$V_R + V_C = V - V_L$$

← EMF – 'back EMF' due to induction in the coil

$$V_R = IR$$

$$V_C = \frac{1}{C}\int I dt \quad \leftarrow \quad Q = CV$$

$$I = \frac{dQ}{dt} \therefore CV = \int I dt$$

$$V_L = L\frac{dI}{dt}$$

$$IR + \frac{1}{C}\int I dt = V_0 \cos \omega t - L\frac{dI}{dt}$$

$$R\frac{dI}{dt} + \frac{I}{C} + L\frac{d^2 I}{dt^2} = -\omega V_0 \sin \omega t$$

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC} I = \frac{-\omega V_0}{L} \sin \omega t$$

$$\gamma = \frac{R}{2L} \;, \quad \omega_0^2 = \frac{1}{LC} \;, \quad A_0 = \frac{-\omega V_0}{L}$$

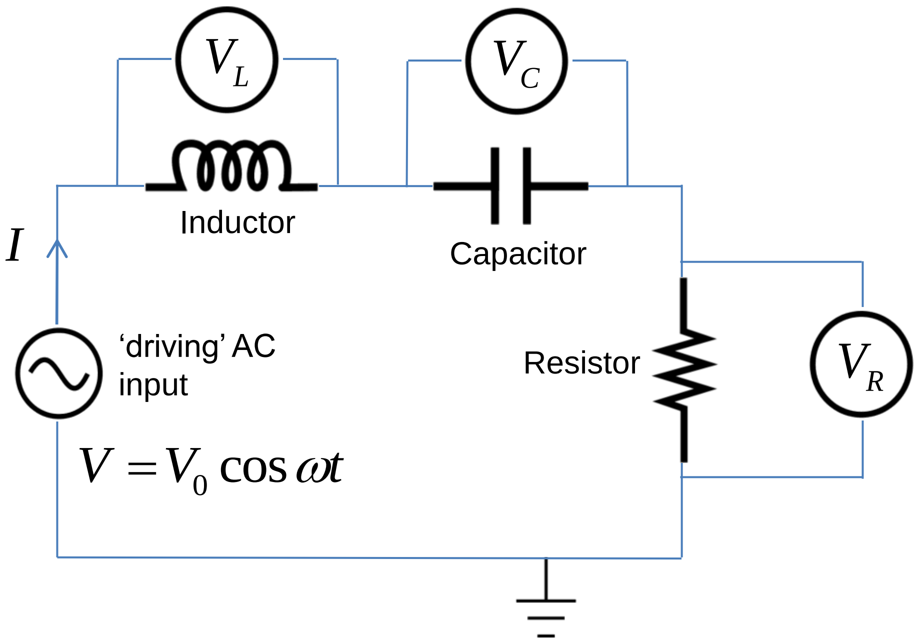
Steady state solution to SHM equation

$$\frac{d^2 x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = A_0 \sin \omega t$$

$$x = \frac{A_0}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\gamma^2\omega^2}} \sin\left(\omega t - \phi\right)$$

$$\phi = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

Steady state solution to the LCR circuit .... cont



$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{LC} = \frac{-\omega V_0}{L}\sin\omega t$$

$$\gamma = \frac{R}{2L}\ ,\quad \omega_0^2 = \frac{1}{LC}\ ,\quad A_0 = \frac{-\omega V_0}{L}$$

$$I = \frac{-\omega V_0 / L}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{RC}{LC}\right)^2 \omega^2}}\sin(\omega t - \phi)$$

$$\phi = \tan^{-1}\left(\frac{RC\omega}{1 - LC\omega^2}\right)$$

Steady state solution to SHM equation

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = A_0 \sin\omega t$$

$$x = \frac{A_0}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\gamma^2\omega^2}}\sin(\omega t - \phi)$$

$$\phi = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

I_{max} when $\omega = \sqrt{\omega_0^2 - 2\gamma^2}$

$$f_{max} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{RC}{LC}\right)^2}$$

$$f_{max} = f_0\sqrt{1 - \frac{(RC)^2}{LC}}$$

$$f_{max} = f_0\sqrt{1 - 4\pi^2 (f_0\tau)^2}$$

$$\omega = 2\pi f$$

$$f_0 = \tfrac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$\tau = RC$$

$$LC = \frac{1}{4\pi^2 f_0^2}$$

$$4\pi^2 (f_0\tau)^2 < 1$$

$$f_0\tau < \tfrac{1}{2\pi}$$

Using dimensionless variables ...

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{LC} = \frac{-\omega V_0}{L}\sin\omega t$$

$$\gamma = \frac{R}{2L}\ ,\quad \omega_0^2 = \frac{1}{LC}\ ,\quad A_0 = \frac{-\omega V_0}{L}$$

$$\frac{d^2 x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = A_0 \sin\omega t$$

$$x_0 = \frac{A_0}{\omega_0^2} \qquad z = \frac{\omega}{\omega_0} \qquad k = \frac{2\gamma}{\omega_0}$$

$$x = \frac{x_0}{\sqrt{\left(1-z^2\right)^2 + k^2 z^2}}\sin\left(\omega t - \phi\right)$$

$$\phi = \tan^{-1}\left(\frac{kz}{1-z^2}\right)$$

$$f_0 = \tfrac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$z = \frac{f}{f_0}$$

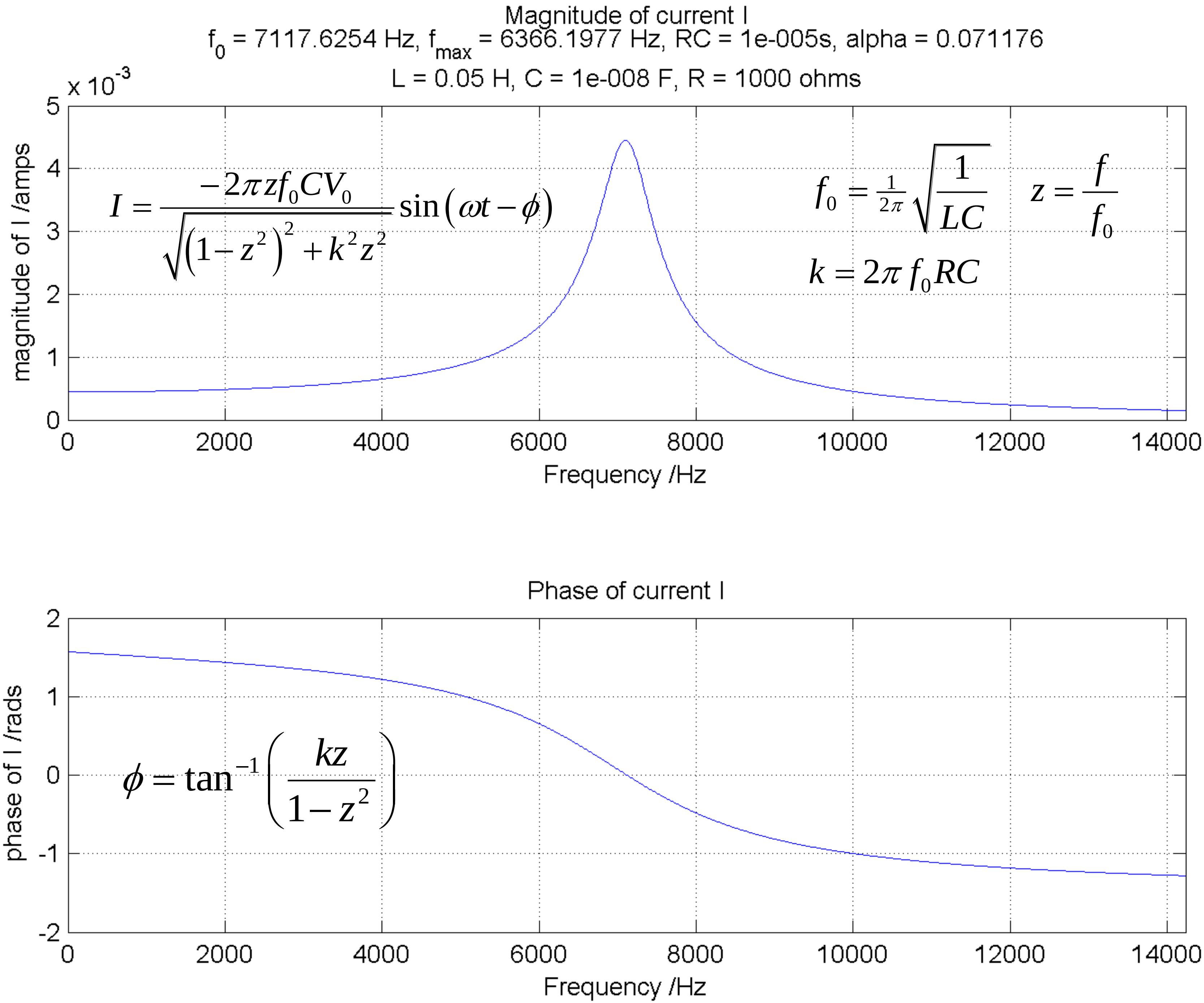
$$I_0 = \frac{-\omega V_0}{L\omega_0^2} = -\frac{2\pi f V_0 LC}{L} = -2\pi f C V_0 \qquad = -2\pi z f_0 C V_0$$

$$k = \frac{R}{L\omega_0} = \frac{R\sqrt{LC}}{L} = R\sqrt{\frac{C}{L}} = RC\sqrt{\frac{1}{LC}} = 2\pi f_0 RC$$

$$I = \frac{-2\pi z f_0 C V_0}{\sqrt{\left(1-z^2\right)^2 + k^2 z^2}}\sin\left(\omega t - \phi\right)$$

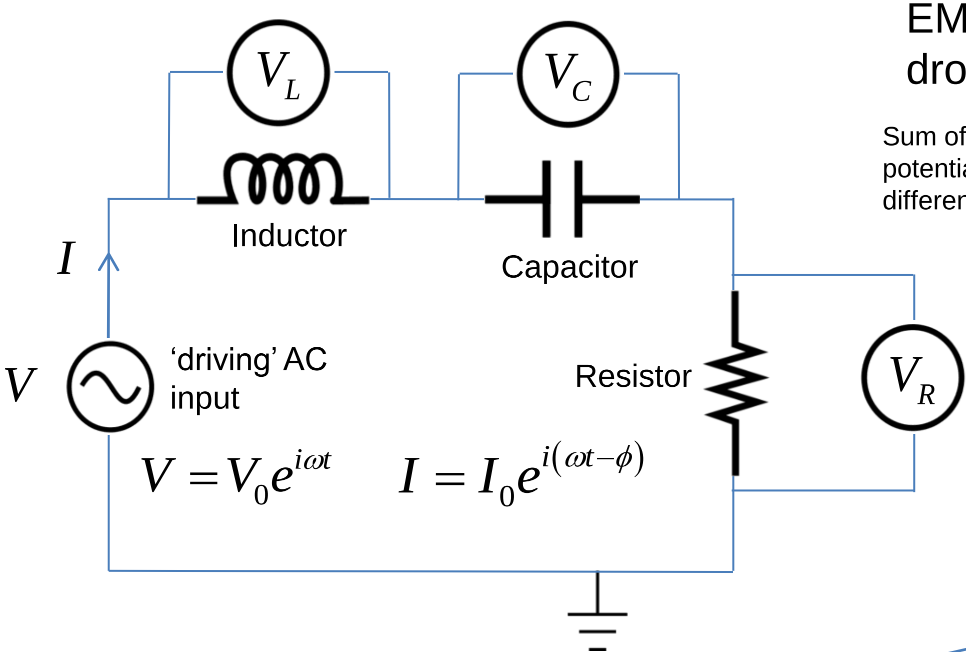
$$\phi = \tan^{-1}\left(\frac{kz}{1-z^2}\right)$$

Note $f_0 C V_0$
is the average current
when the maximum amount
of charge stored in the capacitor
is discharged over one complete
period at frequency f_0



Magnitude of current I
f0 = 7117.6254 Hz, fmax = 6366.1977 Hz, RC = 1e-005s, alpha = 0.071176
L = 0.05 H, C = 1e-008 F, R = 1000 ohms
x 10-3
magnitude of I /amps
Frequency /Hz
$I = \frac{-2\pi z f_0 C V_0}{\sqrt{(1-z^2)^2 + k^2 z^2}} \sin(\omega t - \phi)$
$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$
$z = \frac{f}{f_0}$
$k = 2\pi f_0 RC$
Phase of current I
phase of I /rads
Frequency /Hz
$\phi = \tan^{-1}\left(\frac{kz}{1-z^2}\right)$

Complex *impedance*



Let current *I* flow through the circuit. The net EMF $V - V_L$ must equal the sum of the potential drops across each electrical component.

Sum of potential differences → $V_R + V_C = V - V_L$ ← EMF – 'back EMF' due to induction in the coil

$$V_R = IR$$

$$V_C = \frac{1}{C}\int I dt \quad \leftarrow \quad Q = CV$$

$$I = \frac{dQ}{dt} \therefore CV = \int I dt$$

$$V_L = L\frac{dI}{dt}$$

$$IR + \frac{1}{C}\int I dt = V_0 e^{i\omega t} - L\frac{dI}{dt}$$

$$IR + \frac{1}{C}\int I dt + L\frac{dI}{dt} = V_0 e^{i\omega t}$$

$$I_0 \, e^{i(\omega t - \phi)}\left(R + \frac{1}{i\omega C} + i\omega L\right) = V_0 e^{i\omega t}$$

$$Z = R + \frac{1}{i\omega C} + i\omega L$$

$$\therefore V = I\left|Z\right| e^{i\arg(Z)}$$

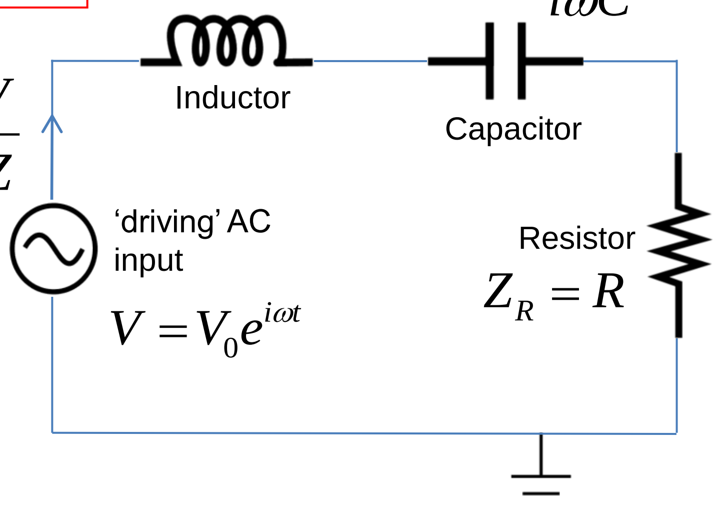
$$\phi = \arg(Z)$$

$$\therefore V = IZ$$

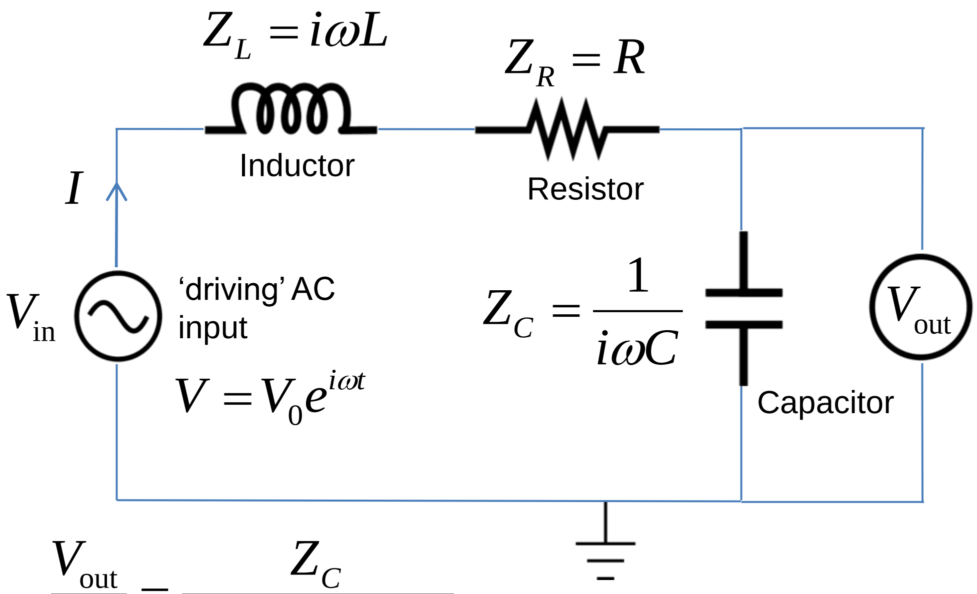
← Ohm's Law, generalized for AC

$$Z = Z_L + Z_C + Z_R \qquad Z_L = i\omega L \qquad Z_C = \frac{1}{i\omega C}$$

$$I = \frac{V}{Z}$$



Impedances for L,C,R components



$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_L + Z_C + Z_R}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{i\omega C}}{R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{iRC\omega - \left(\omega^2 LC - 1\right)}$$

$$x = \frac{f}{f_0} \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$\tau = RC \quad \alpha = f_0 \tau$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2\pi f \tau + i\left(\frac{4\pi^2 f^2}{4\pi^2 f_0^2} - 1\right)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2\pi x f_0 \tau + i\left(x^2 - 1\right)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2\pi x \alpha + i\left(x^2 - 1\right)}$$