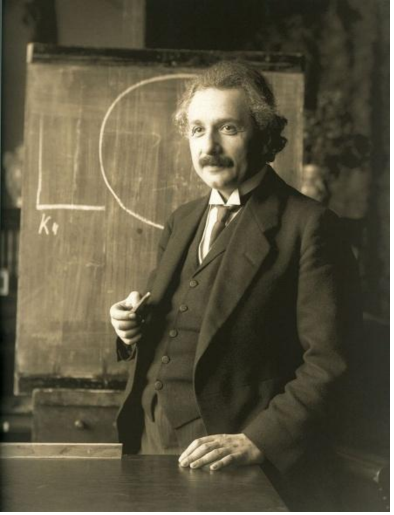
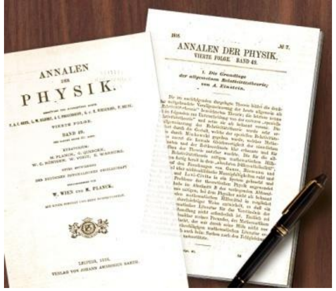
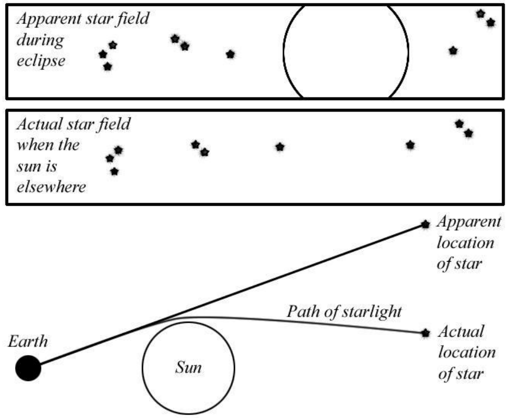
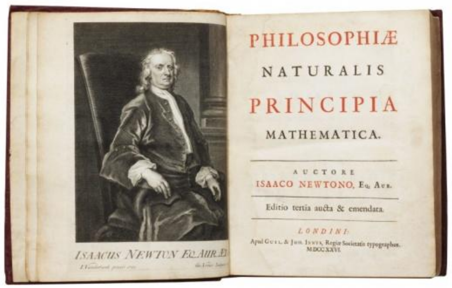
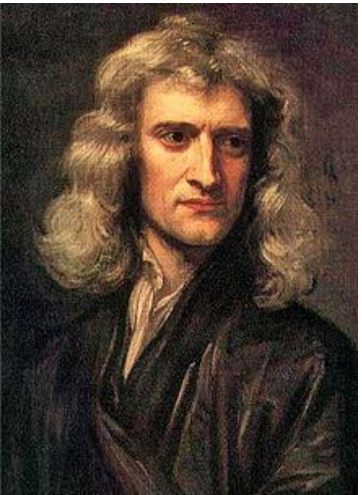






What is Science?

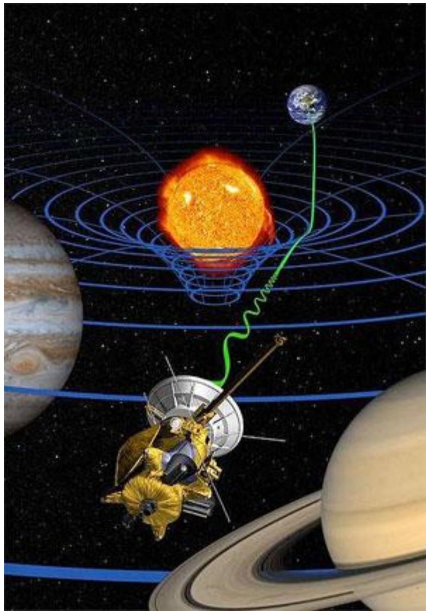




Universe by Numbers: Day 1

July 2016

Dr Andrew French



The Scientific Method

2 **Propose a theory**, involving things that can be measured

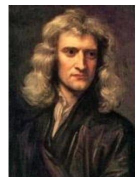
$$x = \frac{1}{2} g t^2$$

Distance fallen

Acceleration (i.e. the rate at which speed increases)

Time

On Earth!

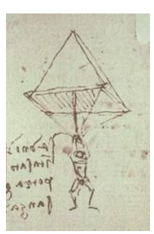


$g = 9.81 \mathrm{ms}^{-2}$

Galileo Galilei
1564-1642

1 **Make some observations**

"Falling objects seem to accelerate at the same rate... Independent of how massive they are!"

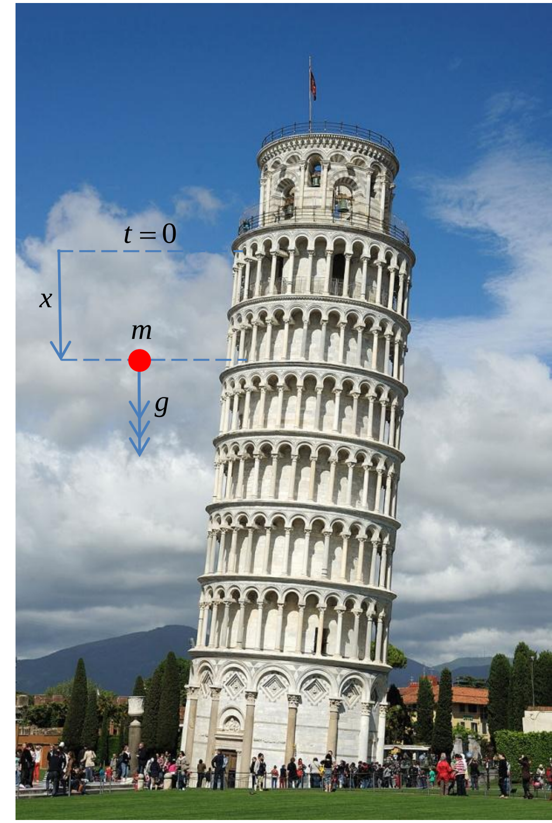


If we ignore air resistance!

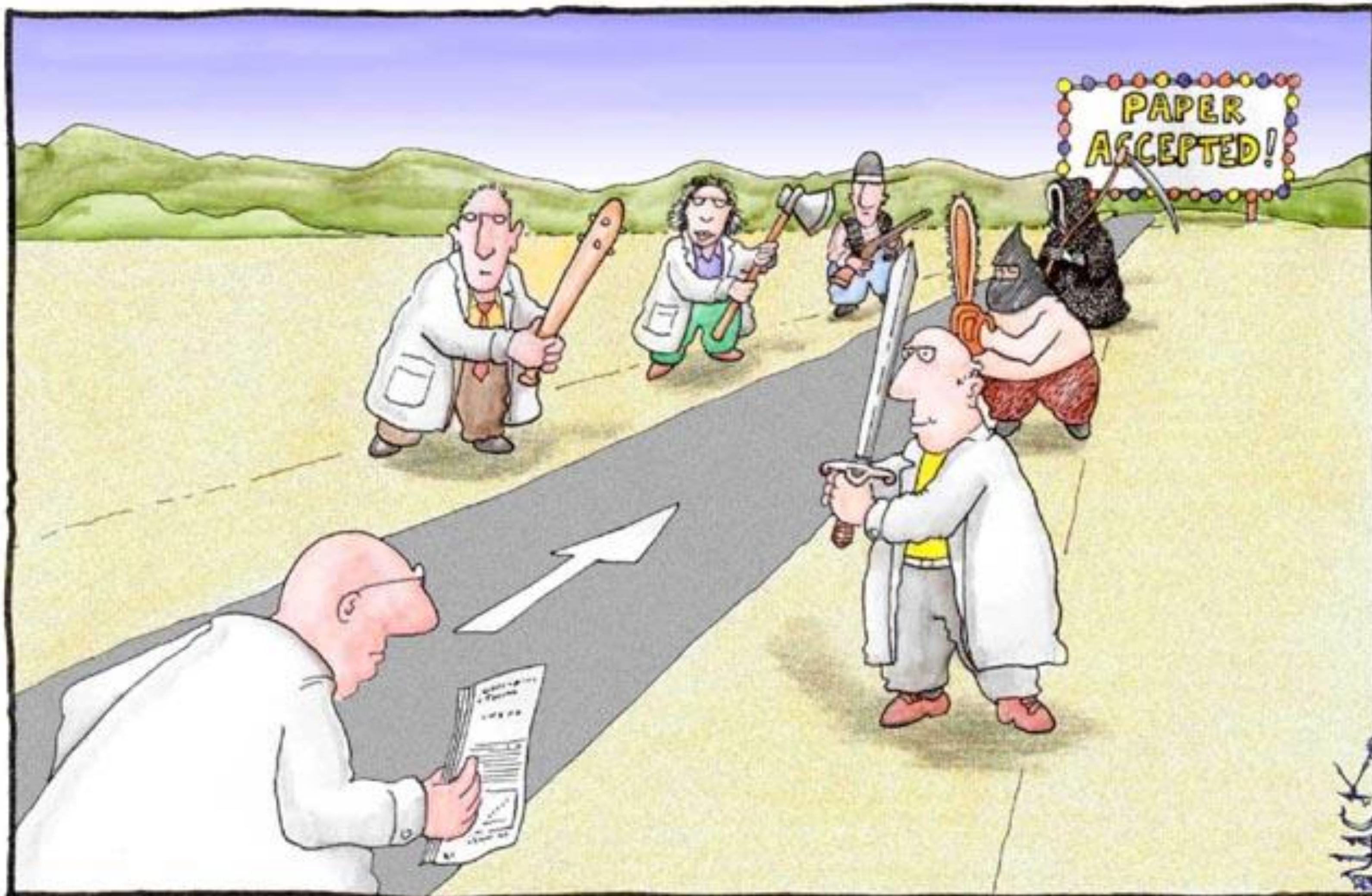
3 **Do an experiment**

Is there a match between theory and measured results?

Is the experiment repeatable?



4 **Write up your findings** and allow your peers to review it

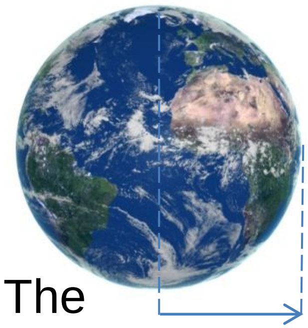


Most scientists regarded the new streamlined peer-review process as 'quite an improvement.'

http://www.freethunk.net/nickkim/nickkim.php

How can we quantify the Universe?

Length



The Earth

$$R = 6,371\text{km}$$

Surface area $$4\pi R^2 = 1.139\times 10^9\,\text{km}^2$$

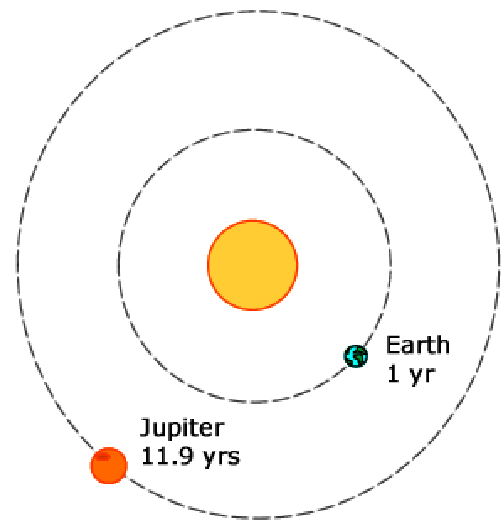
Volume $$\frac{4}{3}\pi R^3 = 1.083\times 10^{12}\,\text{km}^3$$

Time $$1\,\text{year} = 365\times 24\times 60\times 60\text{s} = 3.15\times 10^7\,\text{s} \approx \pi\times 10^7\,\text{s}$$

Mass

$$M = 5.972\times 10^{24}\,\text{kg}$$

This means 1 followed by twenty four zeros!



Charge, **energy**, **momentum**, **force**, **entropy** ….

Standard form

In Science, quantities can vary in scale from the very, very small to the truly enormous!

Distance light travels in one year
(a "light-year")

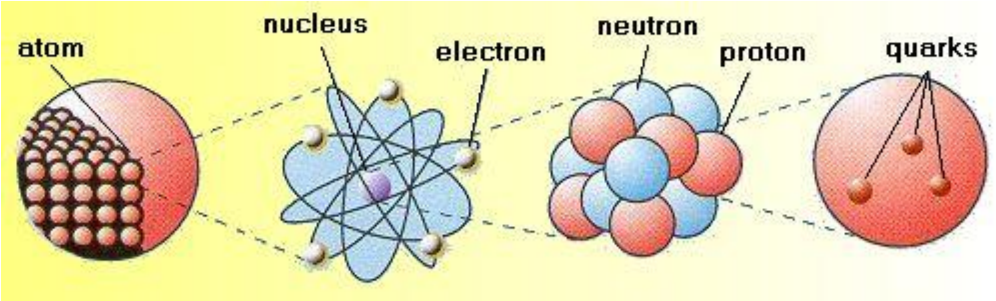
$$9.461 \times 10^{12}\ \text{km}$$

$$= 9{,}461{,}000{,}000{,}000\text{km}$$

Diameter of an atomic nucleus

$$10^{-15}\ \text{m}$$

$$= 0.000000000000001\text{m}$$



Using **powers of ten** is a much better way than writing all the zeros!

Proxima Centauri
4.25 light-years from Earth

Standard form

Using standard form is easy – the power is the number of zeros either side of the decimal point

$$1 = 10^0$$

$$10 = 10^1$$

$$100 = 10^2$$

$$1000 = 10^3$$

$$10,000 = 10^4$$

$$100,000 = 10^5$$

$$1,000,000 = 10^6$$

Million

$$\frac{1}{10} = 10^{-1}$$

$$\frac{1}{100} = 10^{-2}$$

$$\frac{1}{1000} = 10^{-3}$$

$$\frac{1}{10,000} = 10^{-4}$$

$$\frac{1}{100,000} = 10^{-5}$$

$$\frac{1}{1,000,000} = 10^{-6}$$

Millionth

$$1234 = \boxed{1.234 \times 10^4}$$

$$0.0005678$$

$$= \boxed{5.678 \times 10^{-4}}$$

For standard form this number is 1,2,3,4,5,6,7,8 or 9

i.e. *not* 0.8 or 19

The number of digits after the decimal point tell you how *precise* the number is

Write the following in standard form

$$10000000$$

$$314000000000$$

$$0.000000002718$$

$$1234 \times 10^{3}$$

$$56789 \times 10^{-10}$$

Write the following in standard form **Solutions**

$$10000000 = \boxed{10^7}$$

$$314000000000 = \boxed{3.14 \times 10^{11}}$$

$$0.000000002718 = \boxed{2.718 \times 10^{-9}}$$

$$1234 \times 10^3 = 1.234 \times 10^3 \times 10^3 = \boxed{1.234 \times 10^6}$$

$$56789 \times 10^{-10} = 5.6789 \times 10^4 \times 10^{-10}$$

$$= \boxed{5.6789 \times 10^{-6}}$$

Laws of Indices

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

$$10^a \times 10^b = 10^{a+b}$$

$$10^2 \times 10^{-6} = 10^{2-6} = 10^{4}$$

$$x^a x^b = x^{a+b}$$

These can be *any* numbers!

$$\frac{1}{10^4} = 10^{-4}$$

$$\therefore 10^6 \div 10^4 = \frac{10^6}{10^4} = 10^6 \times 10^{-4} = 10^2$$

$$10^a \div 10^b = 10^{a-b}$$

$$\frac{1}{x^a} = x^{-a}$$

Laws of Indices - powers

$$\left(10^2\right)^2 = 10^2 \times 10^2 = 10^{2\times 2} = 10^4$$

$$\left(10^2\right)^3 = 10^2 \times 10^2 \times 10^2 = 10^{3\times 2} = 10^6$$

$$\boxed{\left(10^a\right)^b = 10^{ab}}$$

$$\boxed{\left(x^a\right)^b = x^{ab}}$$

$$\left(2^{-3}\right)^{-2} = 2^{-3\times -2} = \boxed{2^6}$$

Laws of Indices – roots

$$4 = 4^{\frac{1}{2}} \times 4^{\frac{1}{2}}$$

$$\sqrt{4} = 4^{\frac{1}{2}}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$8 = 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}}$$

$$\sqrt[3]{8} = 8^{\frac{1}{3}}$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\sqrt[3]{343 \times 10^9} = \left(7^3 \times 10^9\right)^{\frac{1}{3}} = \boxed{7 \times 10^3}$$

Arithmetic for Science summary

Standard Form

$$123,000,000 = 1.23 \times 10^{8} \qquad 0.0000456 = 1.23 \times 10^{-5}$$

Laws of Indices

$$10^{2} \times 10^{10} = 10^{12}$$

$$x^{a} x^{b} = x^{a+b}$$

$$\frac{10^{2}}{10^{10}} = 10^{-8}$$

$$\frac{1}{x^{a}} = x^{-a}$$

$$\left(10^{2}\right)^{10} = 10^{20}$$

$$\left(x^{a}\right)^{b} = x^{ab}$$

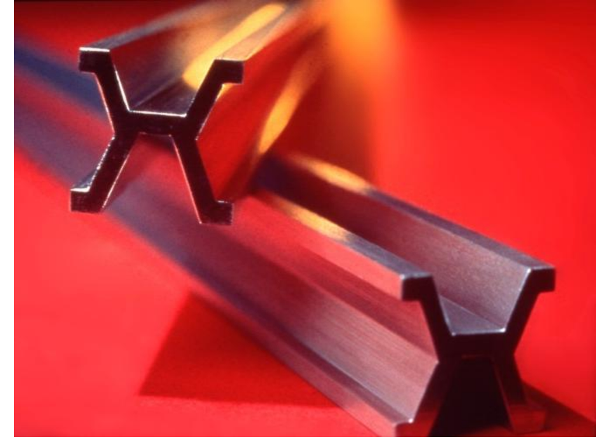
$$\sqrt[10]{10^{10}} = 10$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

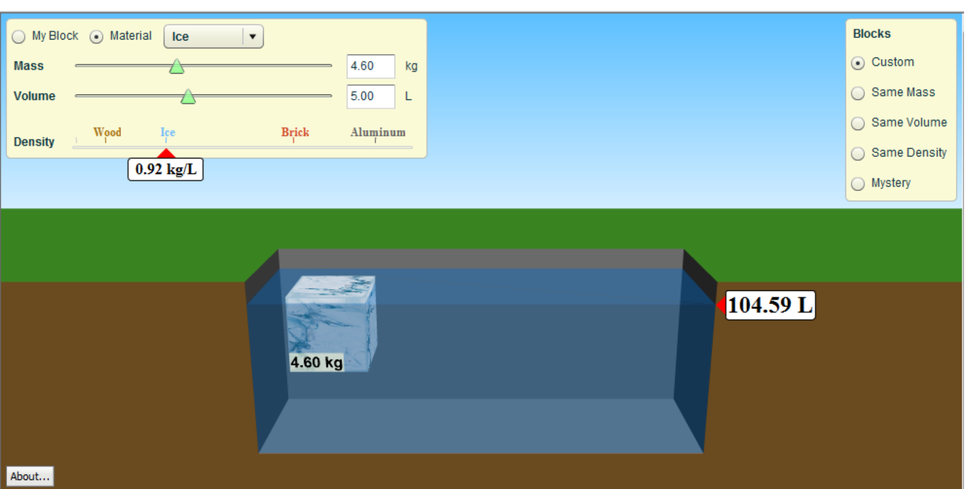
Note x, a and b just stand for 'any number'

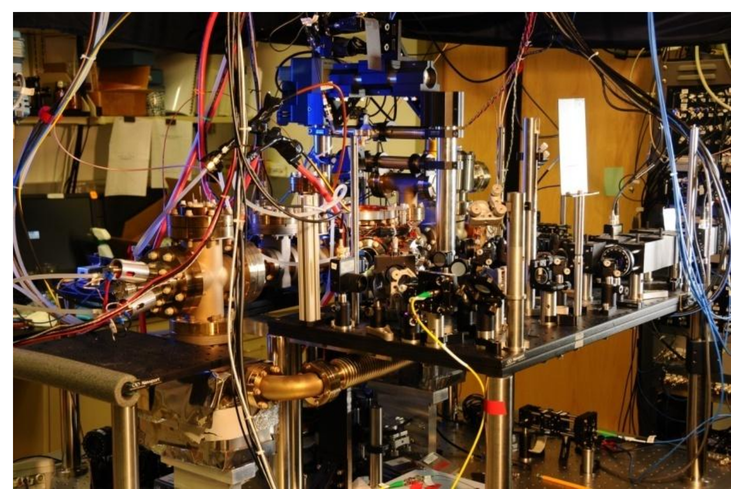
The *relationship* between them is the important idea.

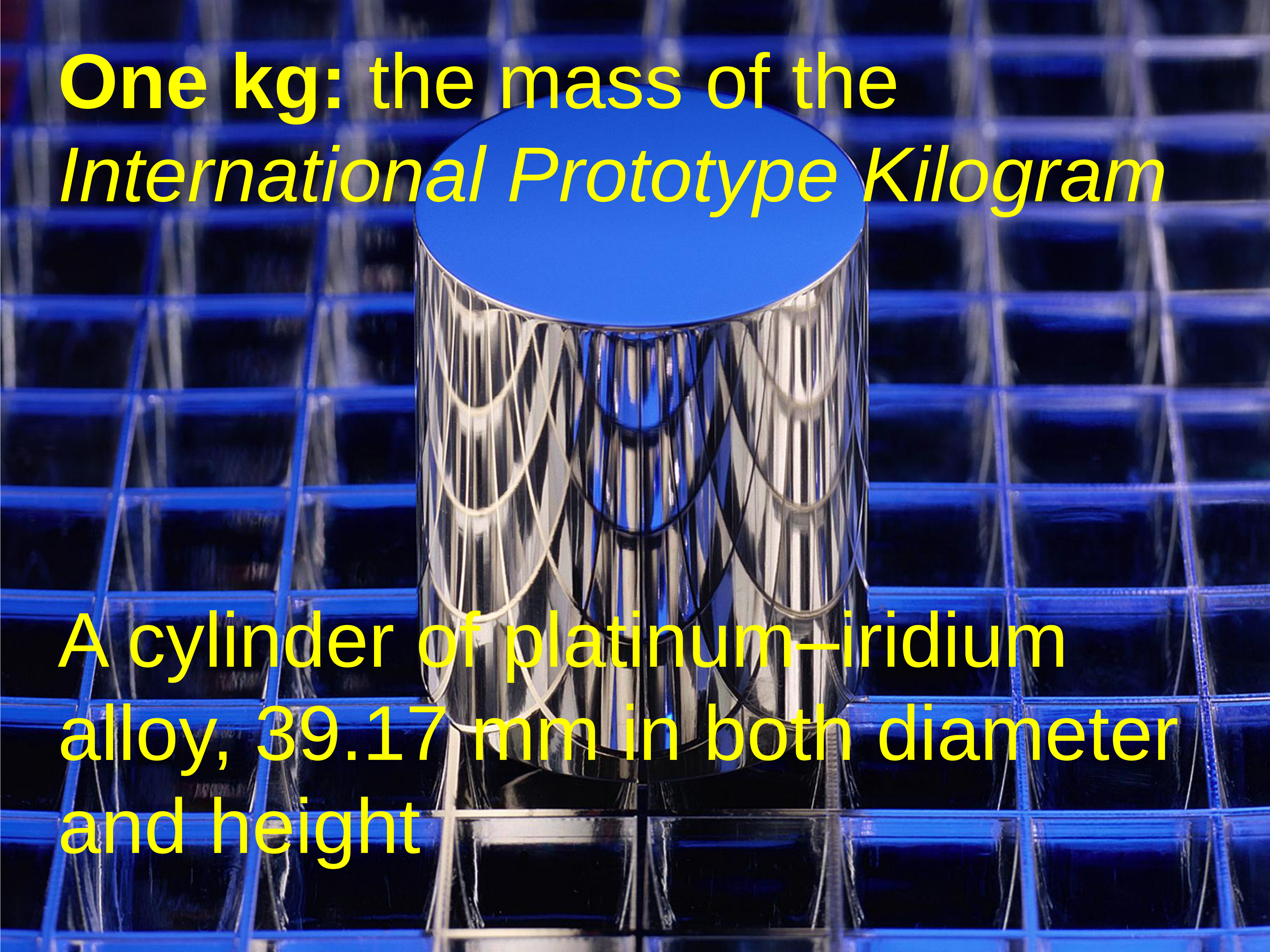
This is what **algebra** is all about!

Measuring distance, time and density

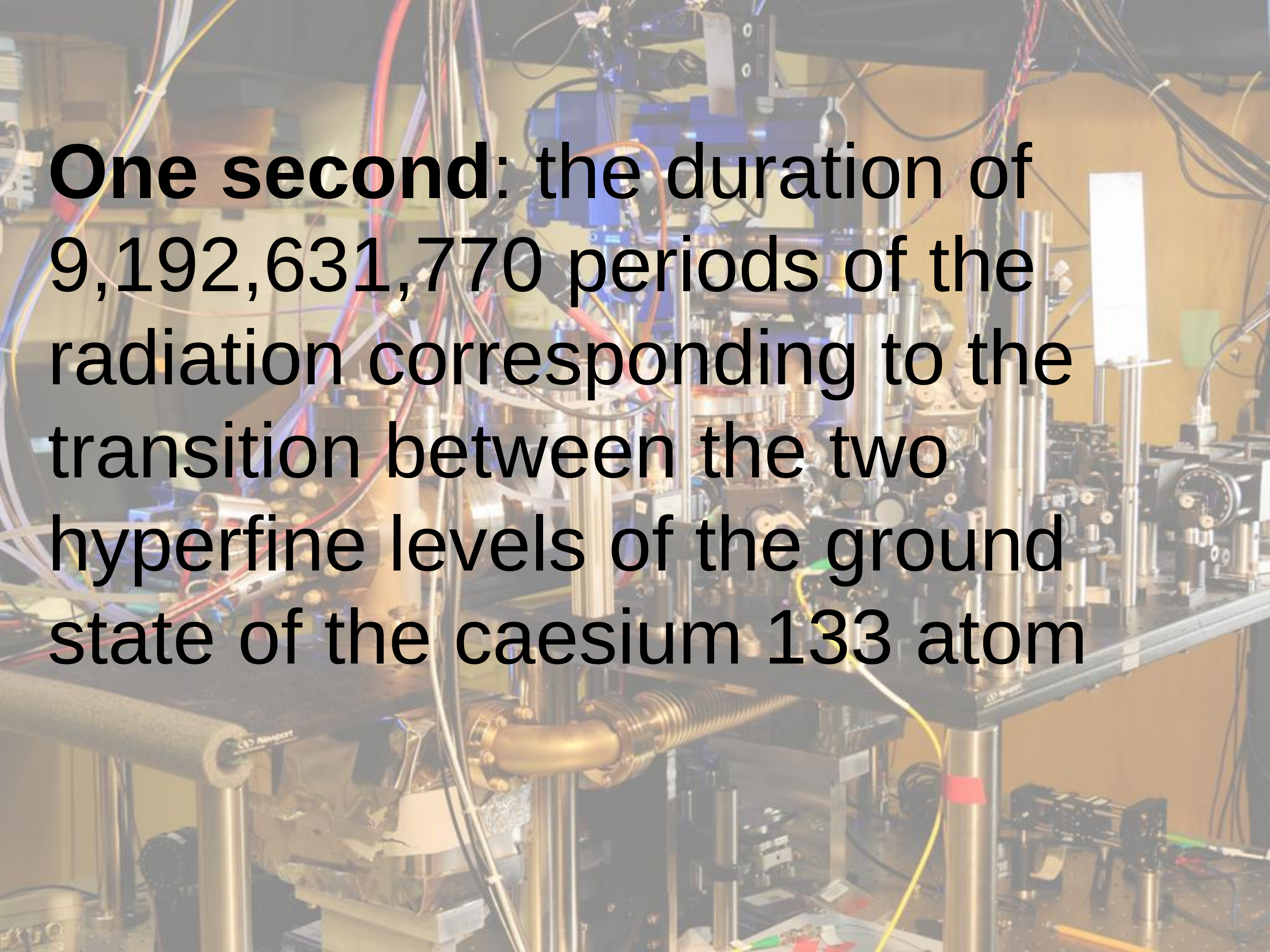






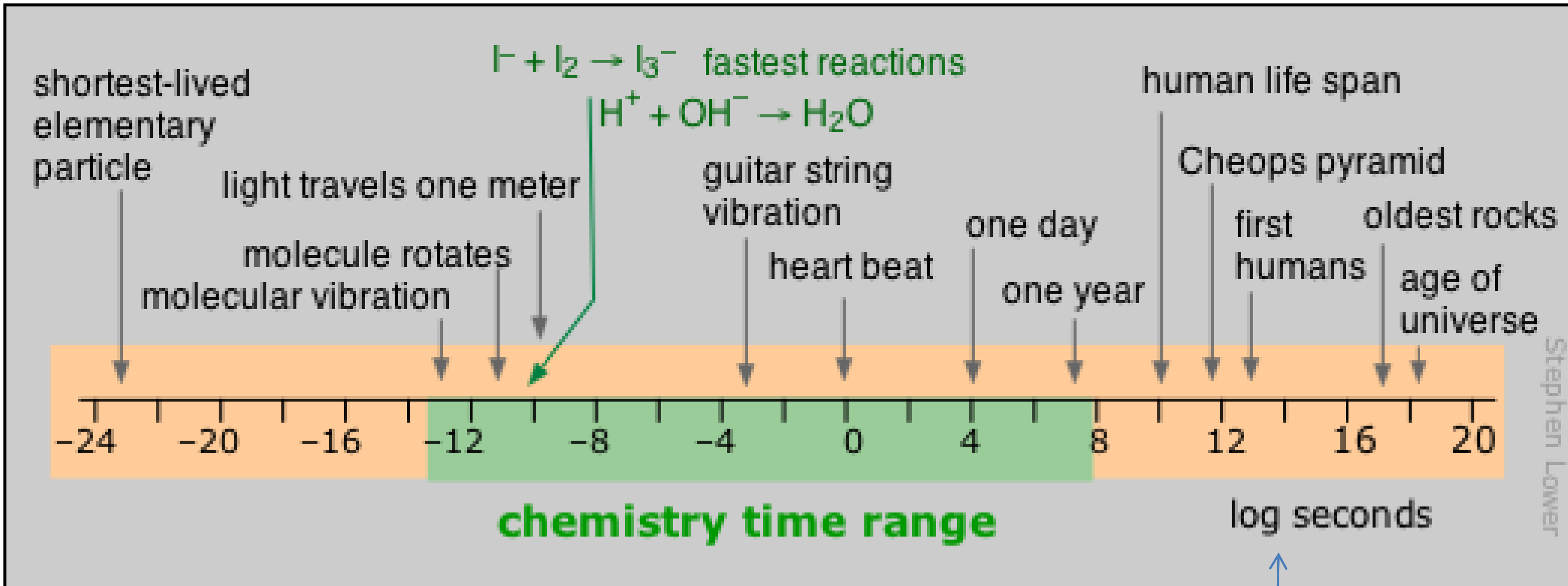
One kg: the mass of the *International Prototype Kilogram*

A cylinder of platinum–iridium alloy, 39.17 mm in both diameter and height



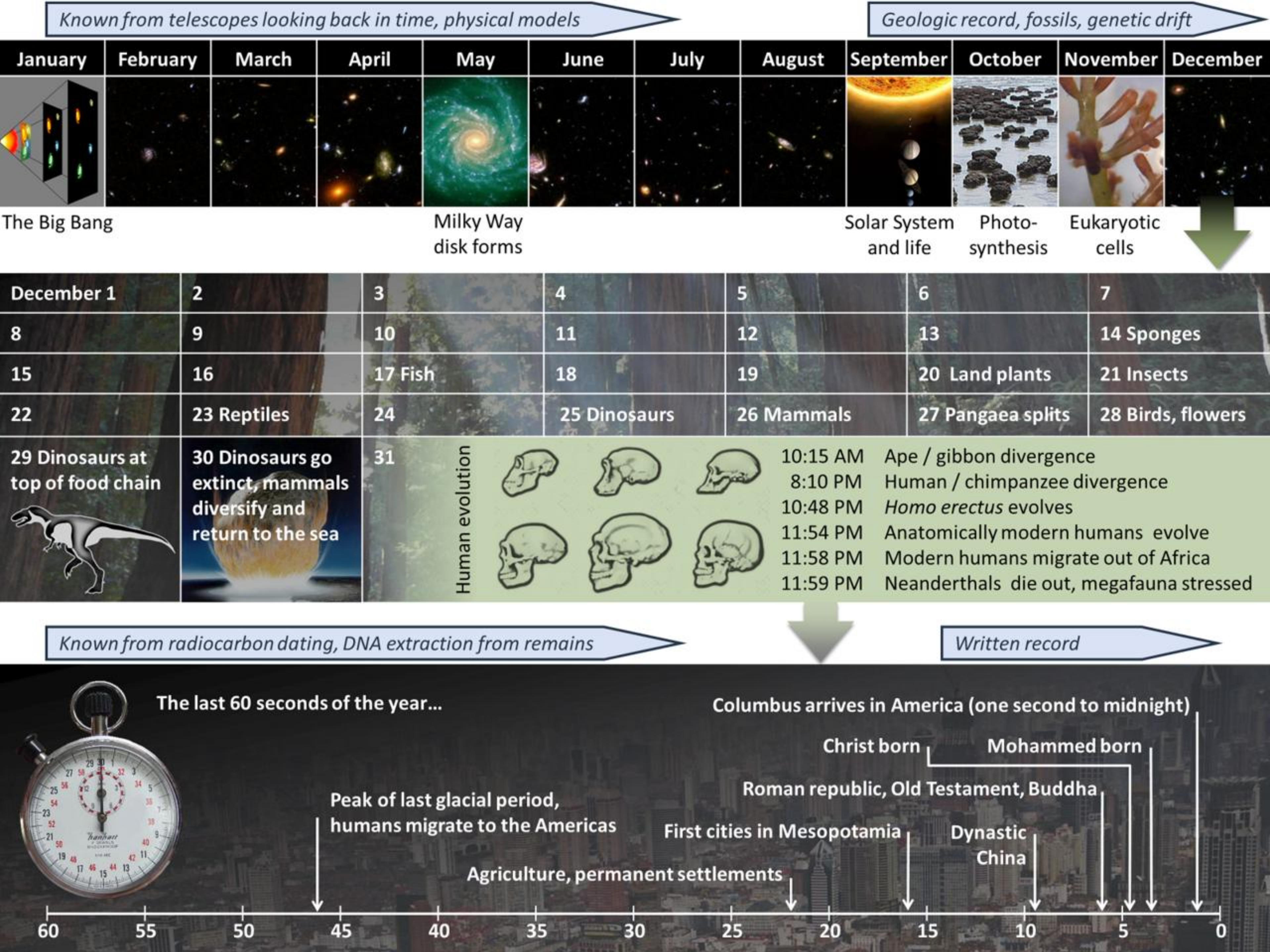
One second: the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom

Time scales in Chemistry

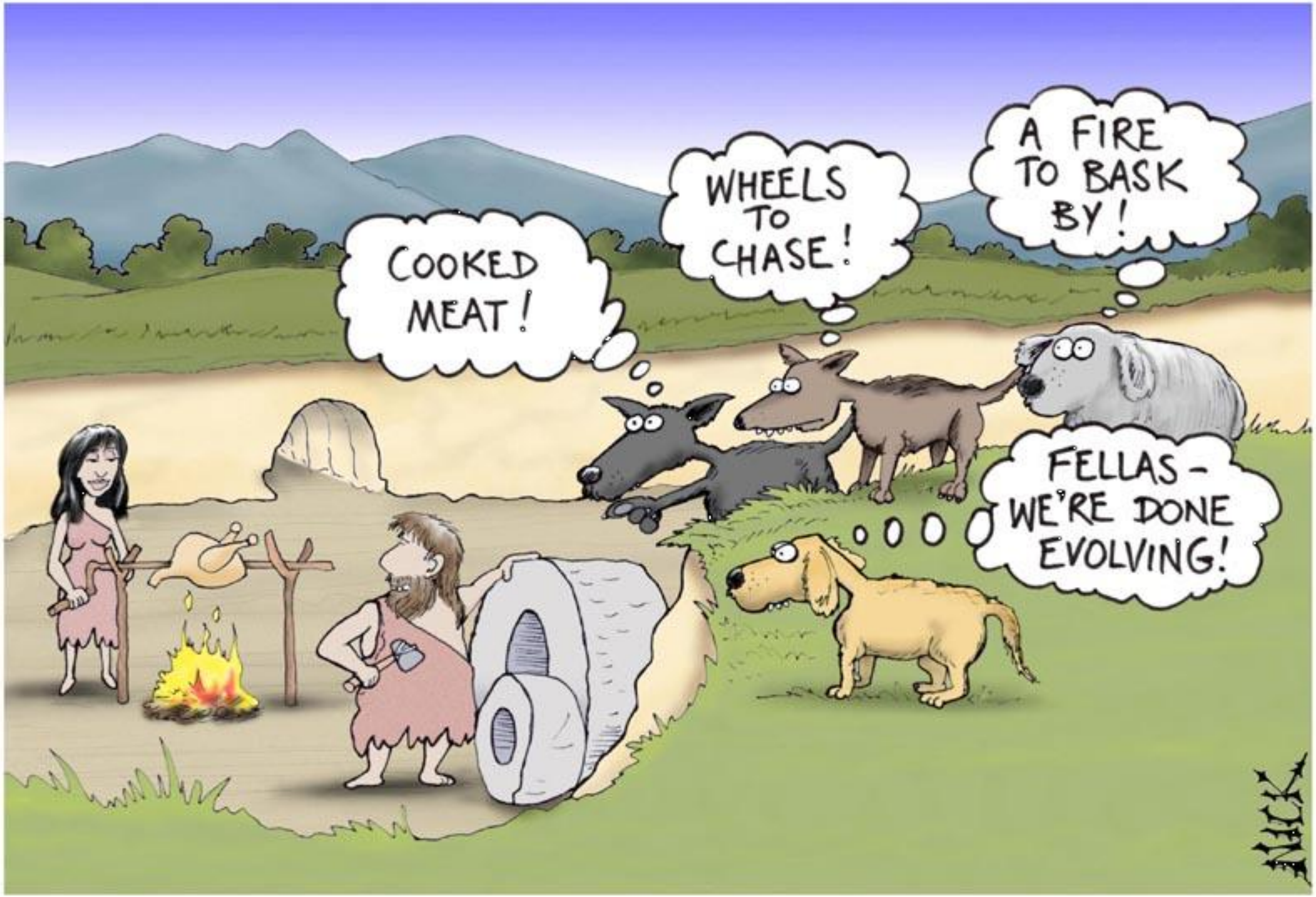


Essentially the 'power of ten'

Recall one year is about 3×10^7 s

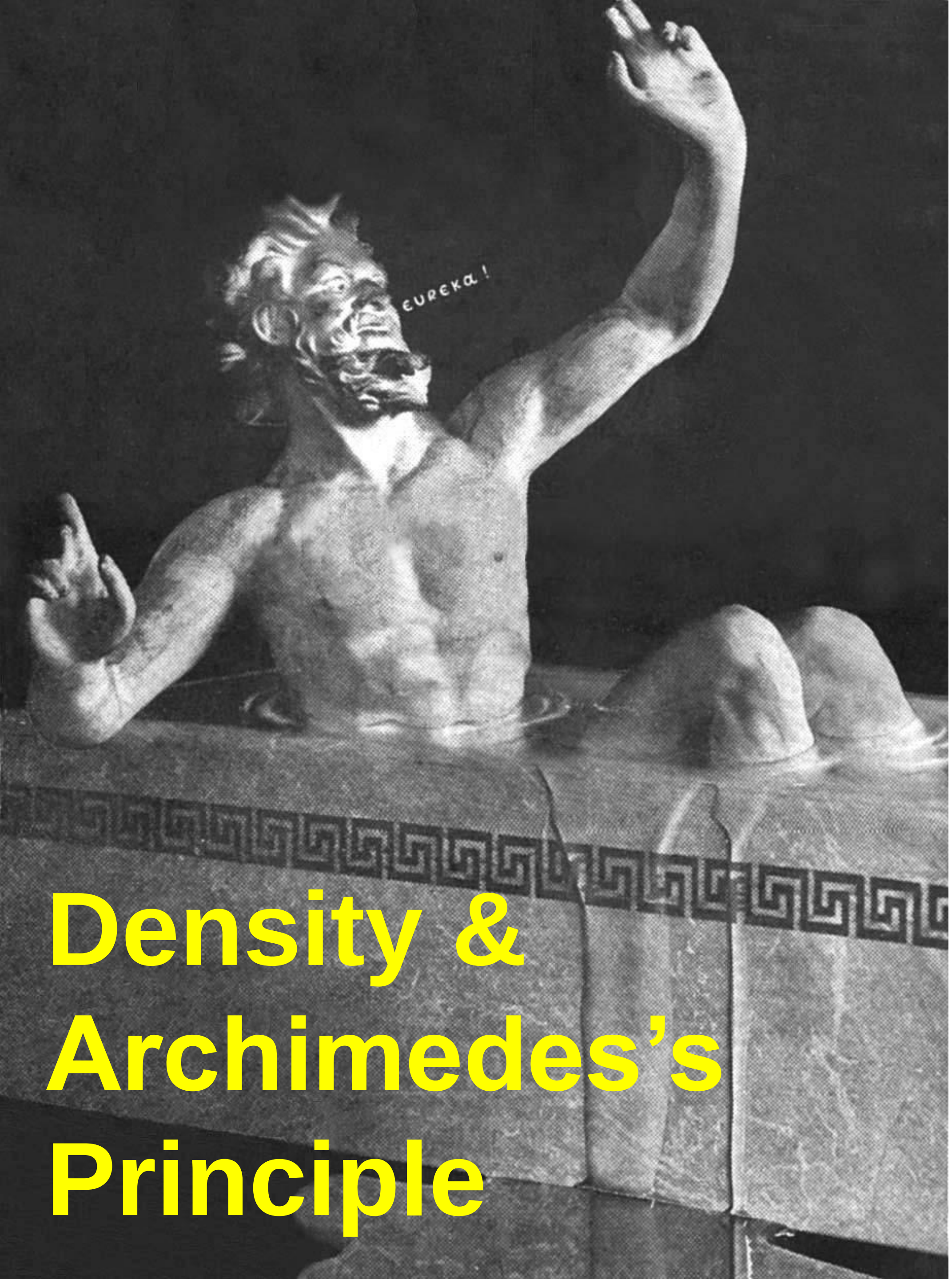


Known from telescopes looking back in time, physical models
Geologic record, fossils, genetic drift
January
February
March
April
May
June
July
August
September
October
November
December
The Big Bang
Milky Way disk forms
Solar System and life
Photo-synthesis
Eukaryotic cells
December 1
2
3
4
5
6
7
8
9
10
11
12
13
14 Sponges
15
16
17 Fish
18
19
20 Land plants
21 Insects
22
23 Reptiles
24
25 Dinosaurs
26 Mammals
27 Pangaea splits
28 Birds, flowers
29 Dinosaurs at top of food chain
30 Dinosaurs go extinct, mammals diversify and return to the sea
31
Human evolution
10:15 AM Ape / gibbon divergence
8:10 PM Human / chimpanzee divergence
10:48 PM Homo erectus evolves
11:54 PM Anatomically modern humans evolve
11:58 PM Modern humans migrate out of Africa
11:59 PM Neanderthals die out, megafauna stressed
Known from radiocarbon dating, DNA extraction from remains
Written record
The last 60 seconds of the year...
Columbus arrives in America (one second to midnight)
Christ born
Mohammed born
Roman republic, Old Testament, Buddha
Peak of last glacial period, humans migrate to the Americas
First cities in Mesopotamia
Dynastic China
Agriculture, permanent settlements
60
55
50
45
40
35
30
25
20
15
10
5
0



Forget the experts; domestication of the dog only took about 8 seconds.

One metre: The distance travelled by light in vacuum in 1/299,792,458 second



Density & Archimedes's Principle

Density is *mass per unit volume*

$$\rho = \frac{M}{V}$$

Mass

Volume

Density

Density is *mass per unit volume*

Note units!

$$\rho = \frac{M}{V}$$

Air is about 1.2 **kgm^{-3}**
Wood is about 0.5 gcm^{-3}
Water is about 1 gcm^{-3}
Aluminium is 2.7 gcm^{-3}
Iron is 7.8 gcm^{-3}
Copper is 8.9 gcm^{-3}
Mercury is 13.5 gcm^{-3}
Gold is 19.3 gcm^{-3}
Uranium is 19.1 gcm^{-3}

Converting density units

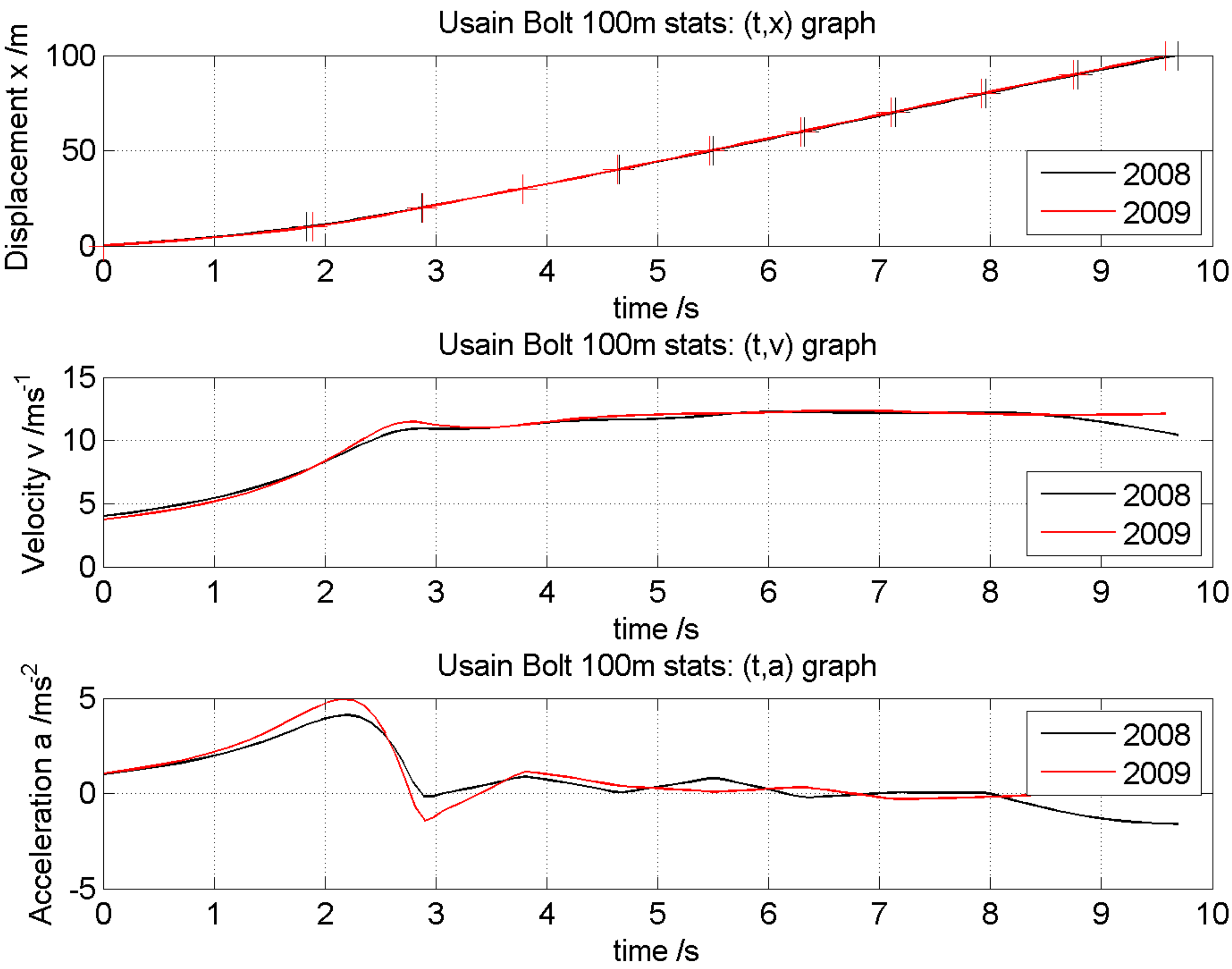
$$1\,\mathrm{kgm^{-3}} = \frac{1000\,\mathrm{g}}{(100\,\mathrm{cm})^3}$$

$$1\,\mathrm{kgm^{-3}} = \frac{10^3}{10^{2\times 3}}\,\mathrm{gcm^{-3}}$$

$$1\,\mathrm{kgm^{-3}} = 10^{-3}\,\mathrm{gcm^{-3}}$$

$$\rho_{air} = 1.2\ \mathrm{kgm^{-3}} = \boxed{1.2\times 10^{-3}\,\mathrm{gcm^{-3}}}$$

Units of speed and acceleration



Converting speeds

$$1\text{mile} = 1609.344\text{m}$$

$$1\text{km} = 1000\text{m}$$

$$1\text{hour} = 3600\text{s}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$1\text{kmh}^{-1} = \frac{1000\text{m}}{3600\text{s}}$$

$$1\text{kmh}^{-1} = \frac{1}{3.6}\text{ms}^{-1}$$

$$1\text{kmh}^{-1} = 0.2778\text{ms}^{-1}$$

$$1\text{ms}^{-1} = 3.6\,\text{kmh}^{-1}$$

$$1\text{mph} = \frac{1609.344\text{m}}{3600\text{s}}$$

$$1\text{mph} = 0.44704\text{ms}^{-1}$$

$$1\text{ms}^{-1} = \frac{1}{0.44704}\text{mph}$$

$$1\text{ms}^{-1} = 2.237\text{mph}$$

Usain Bolt's **maximum speed** is about 12 metres per second

$$1\,\mathrm{ms}^{-1} = 3.6\,\mathrm{kmh}^{-1}$$
$$1\,\mathrm{ms}^{-1} = 2.237\,\mathrm{mph}$$

$$v = 12\,\mathrm{ms}^{-1}$$
$$v = 12 \times 3.6\,\mathrm{kmh}^{-1}$$
$$v = 43.2\,\mathrm{kmh}^{-1}$$

$$v = 12\,\mathrm{ms}^{-1}$$
$$v = 12 \times 2.237\,\mathrm{mph}$$
$$v = 26.8\,\mathrm{mph}$$

By converting to km, (or miles) per hour we can compare Usain's top speed to that of a car

He would not quite break the UK residential speed limit!

Acceleration is the *rate of change of speed*

Objects that accelerate due to gravity **gain 9.81 ms^{-1}** *every second*
i.e. an ***acceleration* of 9.81ms^{-2}**

Usain Bolt has a maximum acceleration of about 5 ms^{-2}

$$5\text{ms}^{-2} = 5 \times \frac{\frac{1}{1000}\text{km}}{\left(\frac{1}{3600}\text{hours}\right)^2}$$

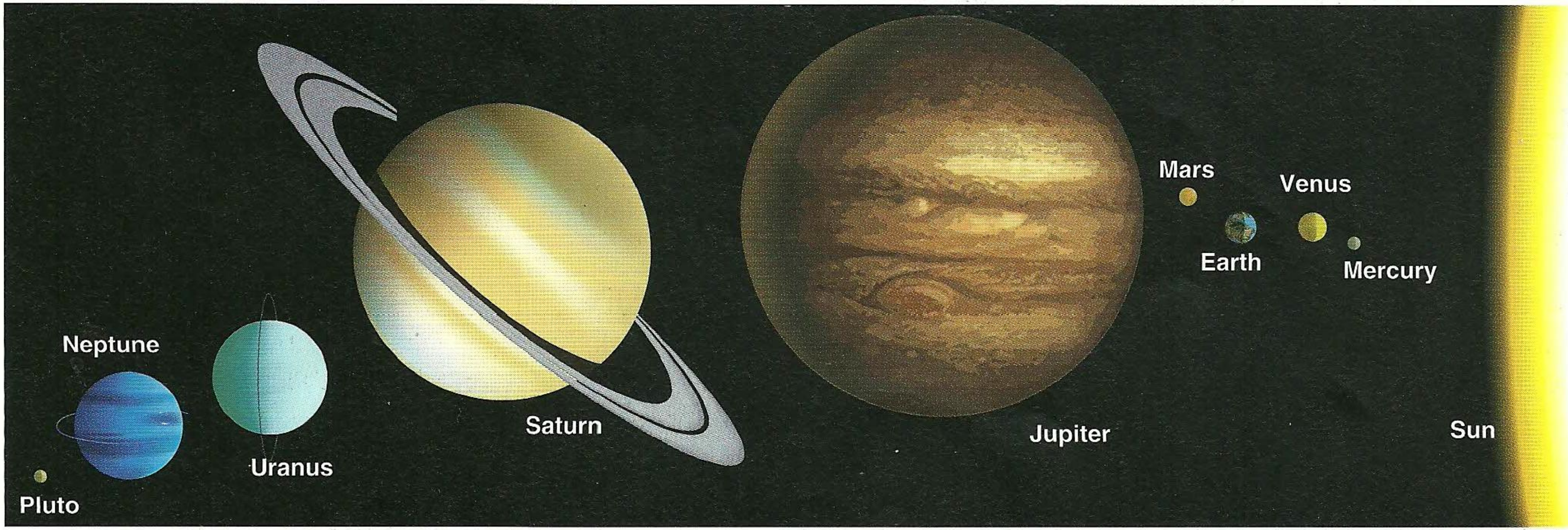
$$5\text{ms}^{-2} = 5 \times \frac{3600^2}{1000}\text{kmh}^{-2}$$

$$\boxed{5\text{ms}^{-2} = 64{,}800\,\text{kmh}^{-2}}$$

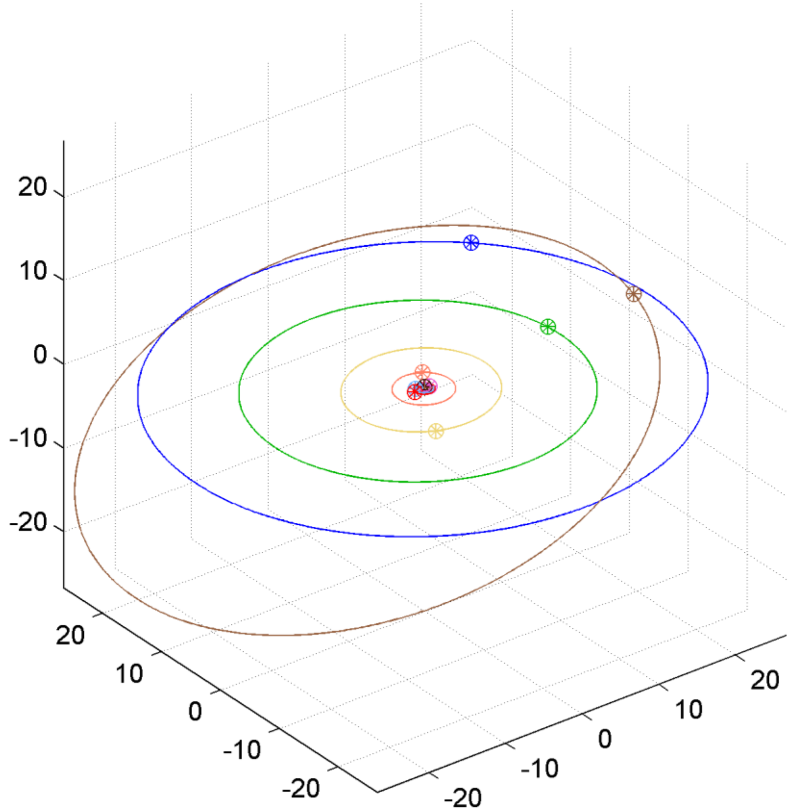
Although this is an equivalent unit, it is *not as useful* as 5ms^{-2} since Usain Bolt is only accelerating
for a few seconds!

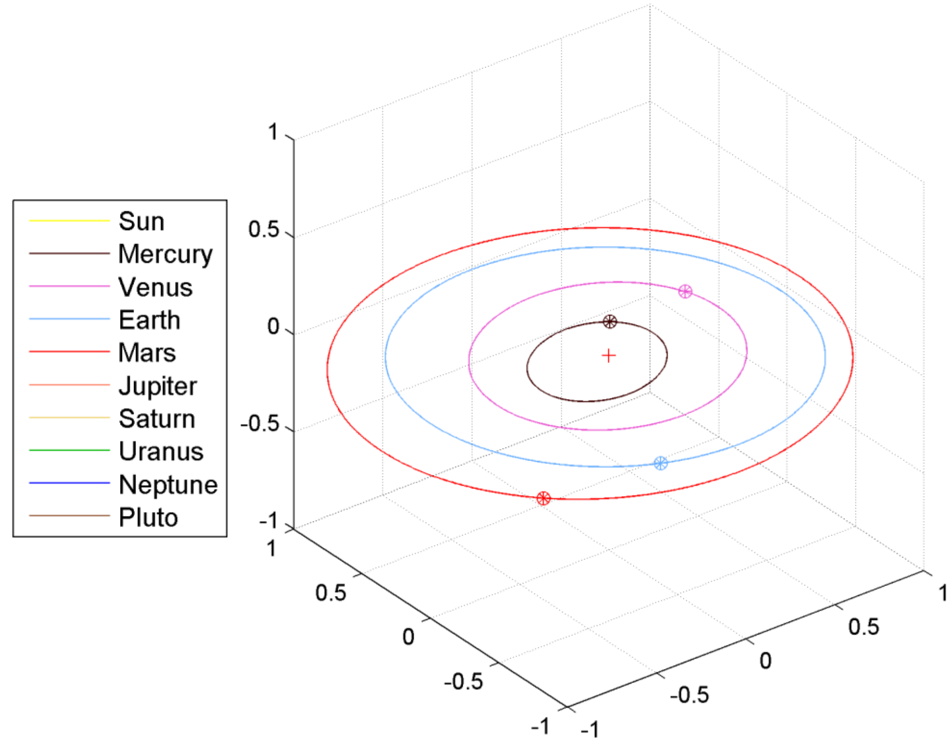
Top tip: use units appropriate to the problem
you are working with

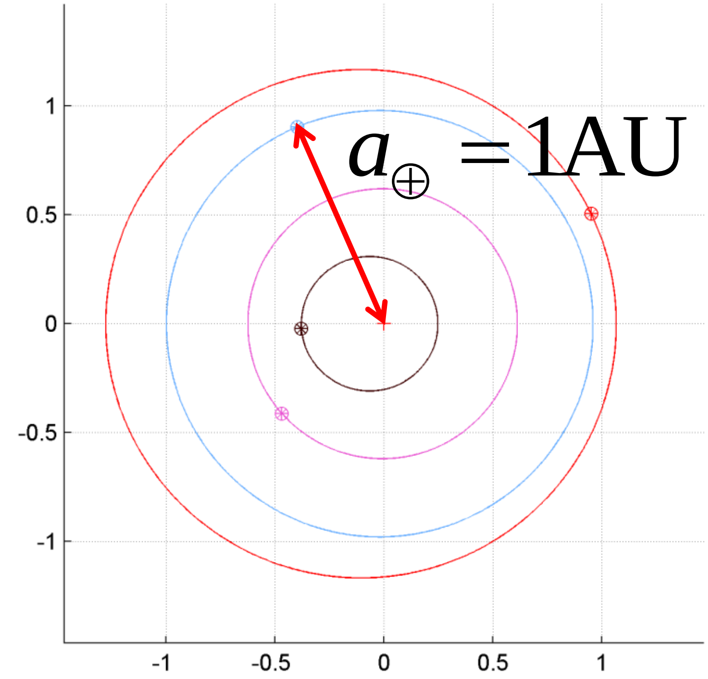
*Strictly speaking, the rate of change of *velocity*. The latter is a **vector** quantity (i.e. has *magnitude* and *direction*) whereas speed is a **scalar**. For movement down a 100m track, speed and velocity have the same meaning



Planet	*Diameter of planet*	*Average distance of planet from the Sun*	*Time taken to go round the Sun*	*Number of moons*	*Average temperature on sunny side*
Mercury	4900 km	58 million km	88 days	0	350°C
Venus	12 000 km	108 million km	225 days	0	480°C
Earth	12 800 km	150 million km	365¼ days	1	20°C
Mars	6800 km	228 million km	687 days	2	0°C
Jupiter	143 000 km	780 million km	12 years	14	−150°C
Saturn	120 000 km	1430 million km	29 years	24	−190°C
Uranus	52 000 km	2800 million km	84 years	15	−220°C
Neptune	49 000 km	4500 million km	165 years	3	−240°C
Pluto	3000 km	5900 million km	248 years	1	−240°C







Scale in astronomical units AU

$$1\text{AU} = 1.496 \times 10^{11}\,\text{m}$$

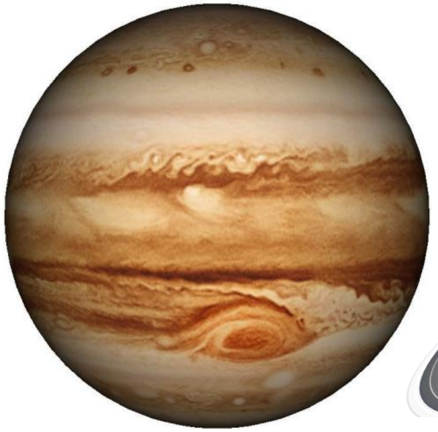
Object	$M/M_{\oplus}$	a /AU	$R/R_{\oplus}$	T_{rot} / days	P /Yr
Sun	332,837	-	109.123	-	-
Mercury	0.055	0.387	0.383	58.646	0.241
Venus†	0.815	0.723	0.949	243.018	0.615
Earth	1.000	1.000	1.000	0.997	1.000
Mars	0.107	1.523	0.533	1.026	1.881
Jupiter	317.85	5.202	11.209	0.413	11.861
Saturn	95.159	9.576	9.449	0.444	29.628
Uranus†	14.500	19.293	4.007	0.718	84.747
Neptune	17.204	30.246	3.883	0.671	166.344
Pluto†	0.003	39.509	0.187	6.387	248.348

Venus, Uranus and Pluto rotate clockwise about their internal axis
All other planets rotate anti-clockwise

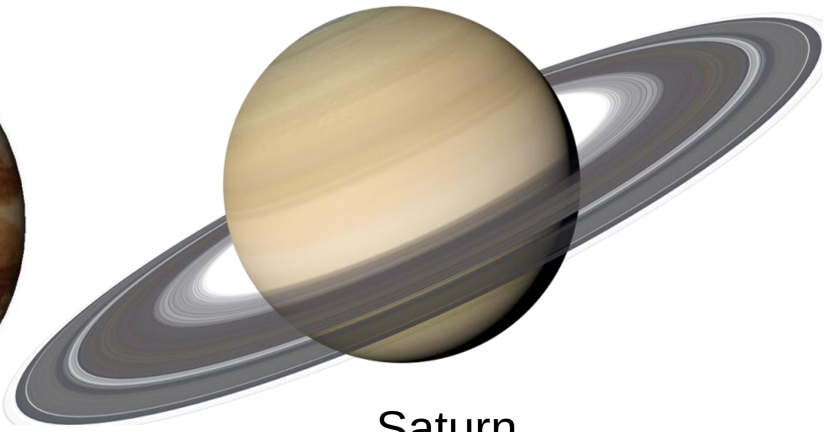
Earth parameters

$$M_{\oplus} = 5.972 \times 10^{24}\,\text{kg}$$

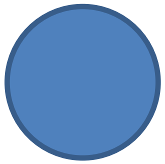
$$R_{\oplus} = 6.371 \times 10^{6}\,\text{m}$$



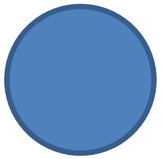
Jupiter



Saturn



Uranus

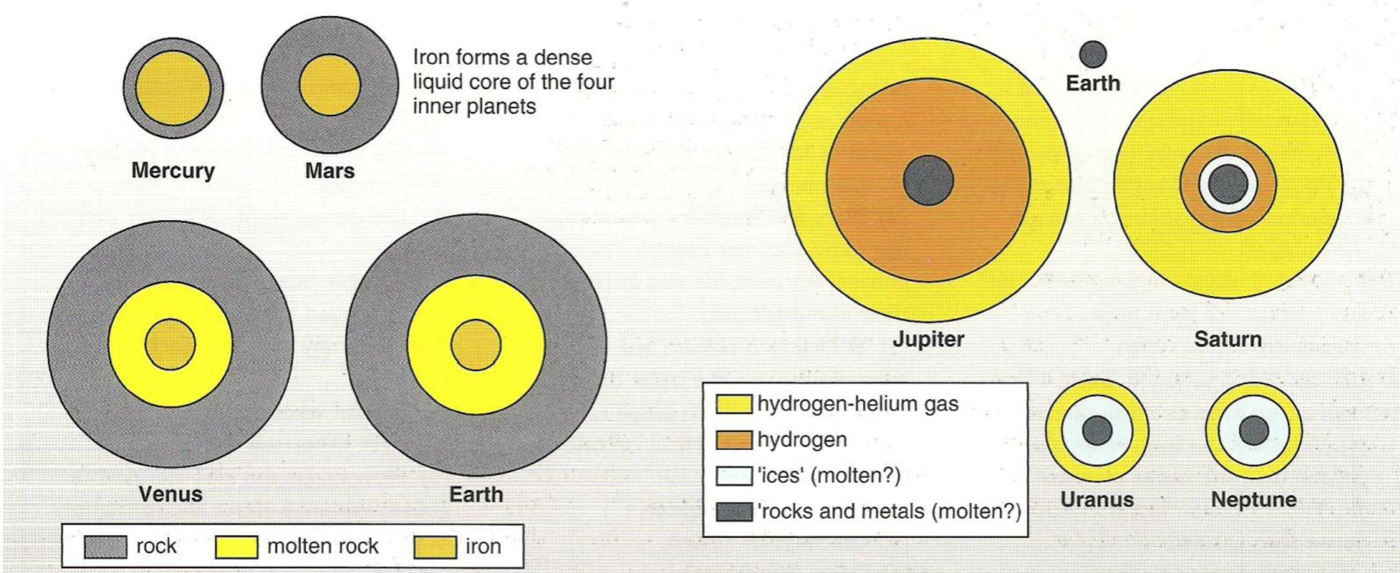


Neptune

Planet	Mass relative to Earth	Radius (Earth = 1)	Relative density (water = 1)	Distance from Sun in AU†	% Rocks	% Ice	% Gas	Main gases in atmosphere
Mercury	0.06	0.38	5.4	0.39	nearly all	–	–	none
Venus	0.82	0.95	5.2	0.72	nearly all	–	some in atmosphere	CO_2
Earth	1	1	5.5	1	nearly all	water in oceans, ice at poles	some in atmosphere	N_2, O_2
Mars	0.11	0.53	3.9	1.5	nearly all	ice at poles	some in atmosphere	CO_2
Jupiter	318	11.2	1.3	5.2		10% rock/ice	90%	H_2, He
Saturn	95	9.4	0.7	9.5		30% rock/ice	70%	H_2, He
Uranus	14.6	4.1	1.2	19.1		70% rock/ice	30%	H_2, He, CH_4
Neptune	17.2	3.9	1.7	30.1		70% rock/ice	30%	H_2, He, CH_4
Pluto	0.1?	0.4?	?	39.4		mostly rock/ice	?	none?

Table 1

† 1 Astronomical Unit of AU is the average Earth–Sun distance.
O_2 oxygen, N_2 nitrogen, CH_4 methane, CO_2 carbon dioxide.



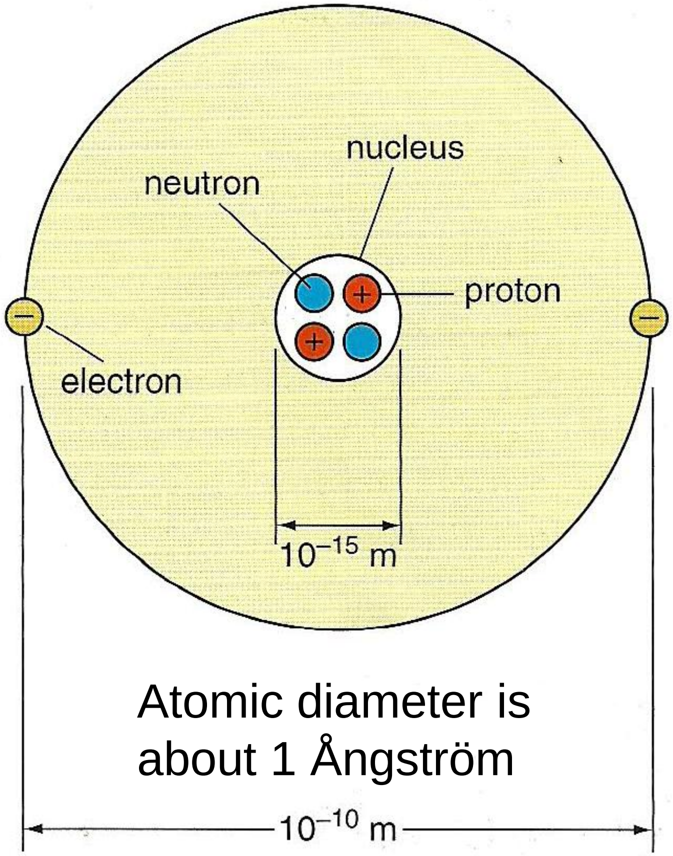
The size of an atom

Earth diameter = 12,756km

Marble diameter = 3.6cm

$$\left(\frac{1.2756\times10^{7}}{3.6\times10^{-2}}\right)^{3}\approx\boxed{4.4\times10^{25}}$$

Volume of Earth in marbles



Atomic diameter is about 1 Ångström

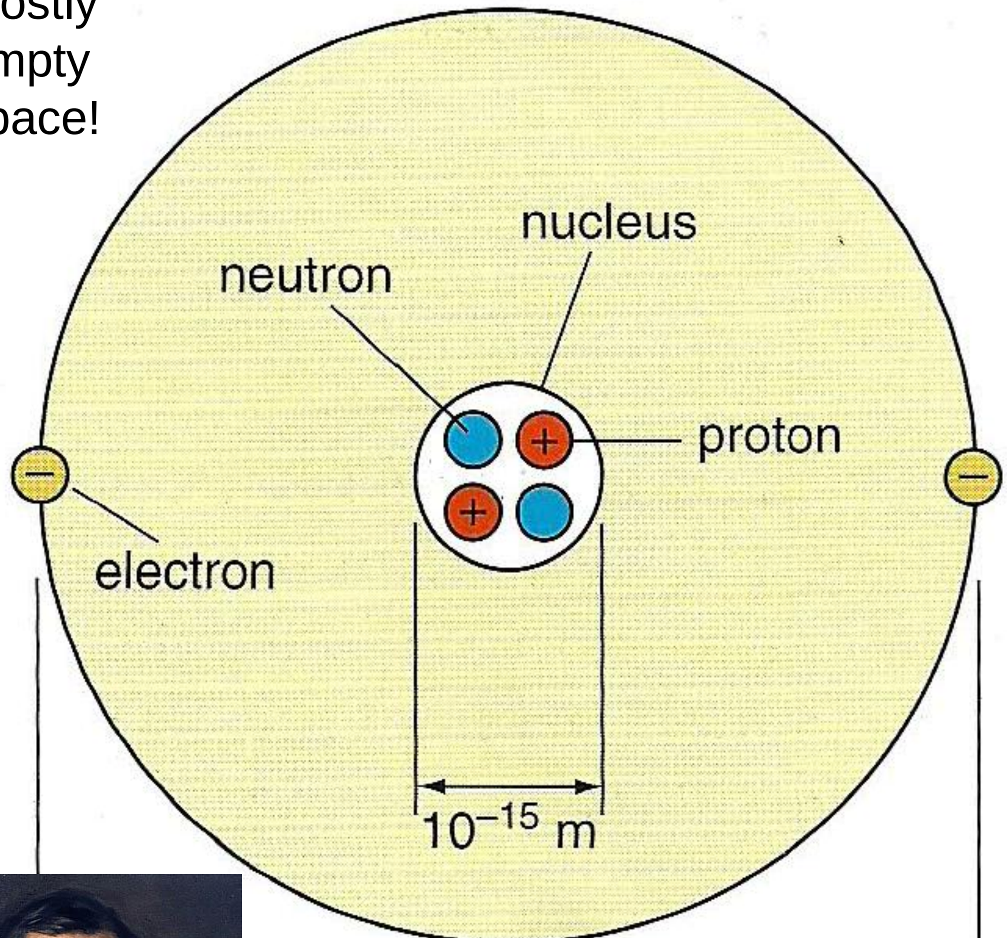
$$\left(\frac{3.6\times10^{-2}}{1\times10^{-10}}\right)^{3}\approx\boxed{4.7\times10^{25}}$$

Number of atoms in a marble

There are as many atoms in a marble as an Earth made of marbles!

Atomic mass and density

Mostly empty space!



This is one Ångström

Anders Ångström
1814-1874

$$m_e = 9.109 \times 10^{-31}\,\text{kg}$$

$$m_p = 1.673 \times 10^{-27}\,\text{kg}$$

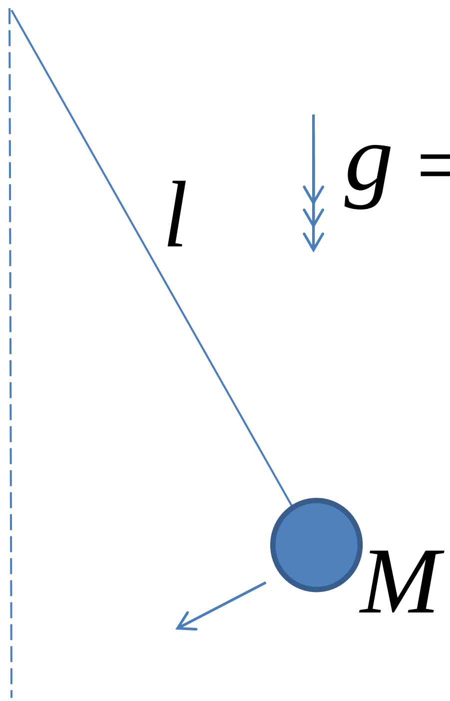
$$m_n = 1.675 \times 10^{-27}\,\text{kg}$$

$$\rho \approx \frac{1.673 \times 10^{-27}\,\text{kg}}{\frac{4}{3}\pi \times \left(\frac{1}{2} \times 10^{-10}\right)^3}$$

$$\rho \approx \boxed{3{,}200\,\text{kgm}^{-3}}$$

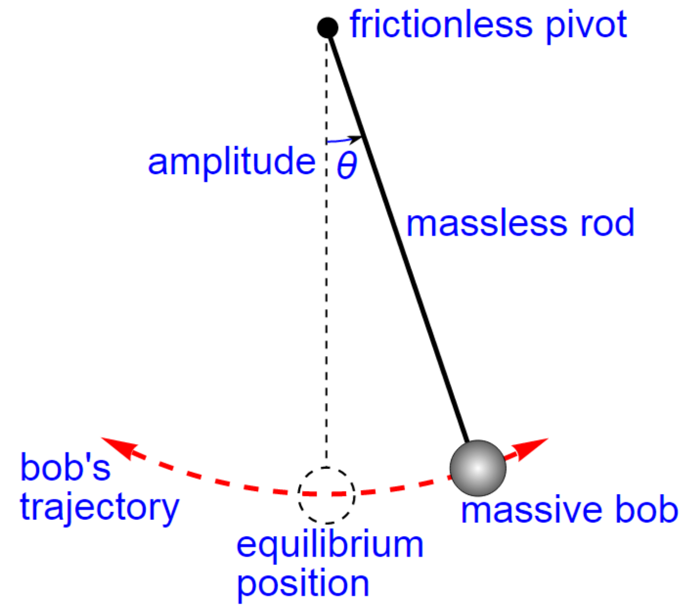
$$\rho_{water} \approx 1{,}000\,\text{kgm}^{-3}$$

Dimensional analysis



$$g = 9.81\text{ms}^{-2}$$

We an work out a formula for the **period** of a swinging pendulum based upon units only!



dimensionless constant

units

$$T = k \times M^a l^b g^c$$

Guess the **period** depends on *mass*, *length* and the *acceleration due to gravity*

$$\text{s} = \text{kg}^a \text{m}^b \left(\text{ms}^{-2}\right)^c$$

$$\text{s}^1 = \text{kg}^a \text{m}^{b+c} \text{s}^{-2c}$$

$$\boxed{a = 0}$$

$$\boxed{c = -\tfrac{1}{2}}$$

$$b + c = 0 \Rightarrow \boxed{b = \tfrac{1}{2}}$$

$$\boxed{T = k\sqrt{\frac{l}{g}}}$$

So the period *cannot depend* on the bob mass

Topics to reflect on:

The Learning Cycle. Have an experience. Capture it (ideally in writing!) Reflect upon it. Conclude from it. Plan your next step.

The Scientific method. Make some observations. Propose a theory. Turn it into Mathematics, i.e. *equations* which relate *measureable quantities*. Conduct an experiment. Does the theory match what you measured?* Write up your ideas and have them peer reviewed.

‘**Scientific arithmetic.’ Standard Form, Laws of Indices.**

Units of **length**, **mass**, **time**. Finding **density**, **speed** and **acceleration**. **Converting** between units.

Dimensional analysis

Depending on your course, we may not cover all of these. Review the topics you did meet. If you have time to spare, read on!

*Note you can *never prove your theory is correct*, only *disprove it* if there is insufficient match to experimental results.