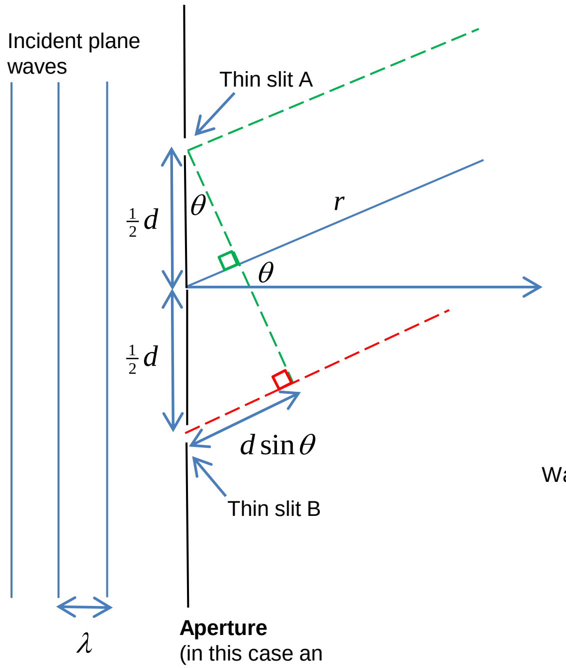
Wavenumber $k=\frac{2\pi}{\lambda}$ $\omega=2\pi f$ Frequency Wave speed $c=f\lambda$ $\omega=ck$ Wave power input $P\propto A^2\omega^2$

Diffraction Essentials

The ***Huygens-Fresnel Principle*** states: *"Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases)."*

Hence to determine the *wavefield* beyond an illuminated edge of a slit, we need to add up the effect of spherical wave sources in the vicinity of the slit or *aperture*.

Key geometrical idea from two infinitesimally thin slits ('Young's Slits')



Aperture
(in this case an opaque wall with two infinitesimally thin slits)

Spherical waves will emanate from the slits, and interfere with each other.

For distances such that: $r \gg \frac{d^2}{\lambda}$
(we call this the **Far Field**)
we can assume waves from each slit are **plane waves**, for any given observational angle θ.

Constructive interference occurs when the *phase difference* between the waves from slits A and B is an integer multiple of 2π radians.

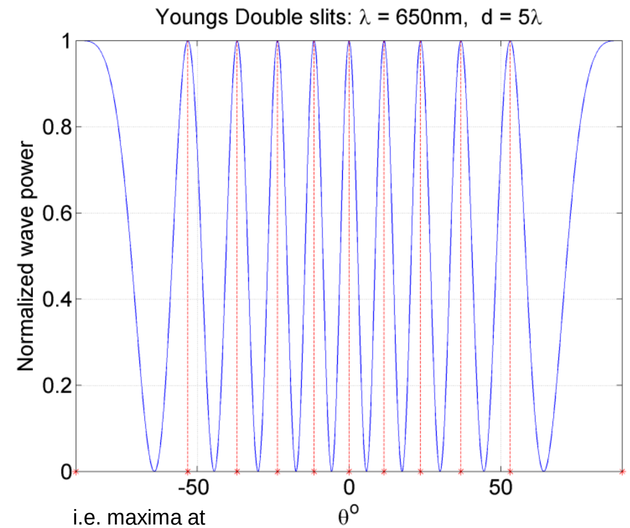
$$\frac{2\pi}{\lambda}d\sin\theta = 2\pi n$$

Wavenumber k — Path difference between waves from A and B — Integer n

Hence expect **maxima** in the resulting **Far Field Diffraction pattern** (e.g. spots of a laser on a wall) at angles

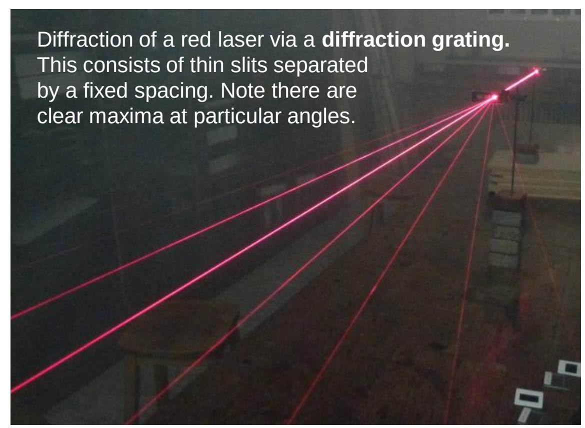
$$\sin\theta=\frac{n\lambda}{d}$$

Since the diffraction angle is *inversely* related to spacing d we can use **diffraction patterns** to *measure* small periodic structures (e.g. atomic layers, structure of DNA...) in the laboratory!



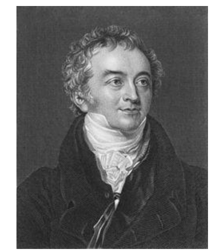
i.e. maxima at

$$\theta=\sin^{-1}\left(\tfrac{1}{5}n\right)=0^\circ,\pm11.5^\circ,\pm23.6^\circ,\pm36.9^\circ,\pm53.1^\circ,\pm90^\circ$$



Diffraction of a red laser via a **diffraction grating.** This consists of thin slits separated by a fixed spacing. Note there are clear maxima at particular angles.

Christiaan Huygens
1629-1695



Thomas Young
1773-1829

Augustin-Jean Fresnel
1788-1827

For a real diffraction grating we have a few more aspects to consider, but the essential idea for the double slit is the same. i.e. $\sin\theta = \frac{n\lambda}{d}$ But the meaning of this equation will change subtly!

ISSUE #1: The slits have a finite width

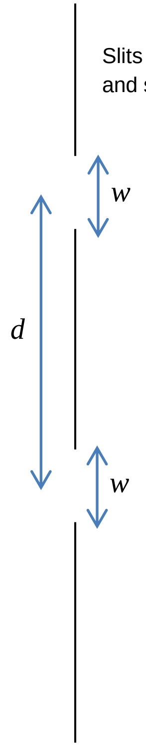
We shall model this by adding up the effect of an infinite number of thin slits, which make up the slit. This requires some Calculus, which we will not do here (see the full Diffraction note).

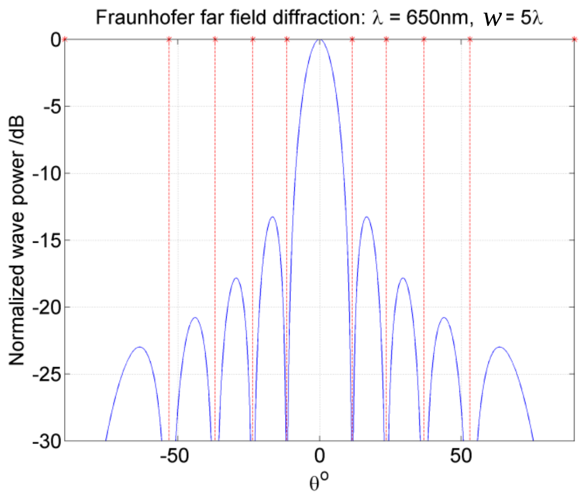
The end result is a envelope to the diffraction pattern which has **zeros** at

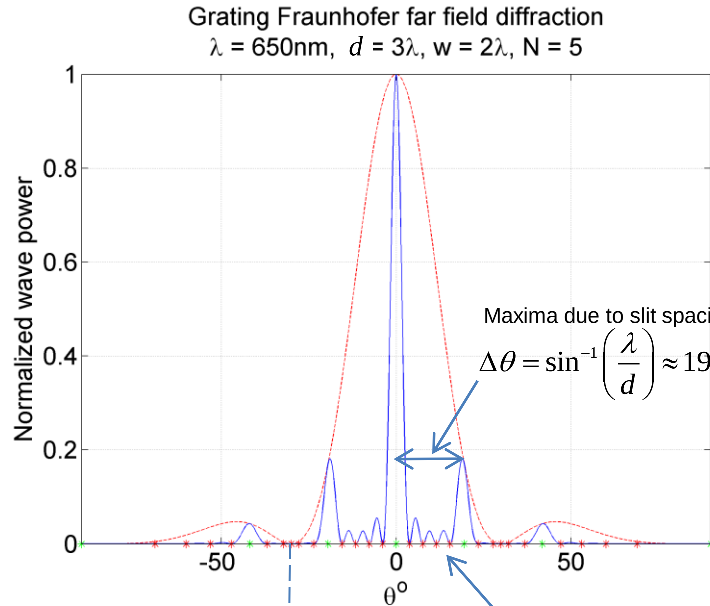
$$\sin\theta = \frac{m\lambda}{w}$$

m is any integer
w is the slit width

Slits of width w and spacing d







Maxima due to slit spacing d

$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) \approx 19.5^\circ$$

$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{w}\right) = 30^\circ$$

Minima of **envelope** due to finite slit width w

Fine structure due to overall size of aperture, which comprises of N = 5 slits in this case. The extra **minima** (see the red stars) are at

$$\theta = \sin^{-1}\left(\frac{p\lambda}{Nd}\right)$$

... but are **maxima** (green stars) when p/N is an integer.

ISSUE #2: There are N slits (i.e. not just one or two...)

This will result in a **fine structure** (i.e. lots of extra little maxima). The maxima due to the slit spacing will appear *sharper*, and their will be *additional zeros* when

$$\sin\theta = \frac{p\lambda}{Nd}$$

p is any integer
N slits of slit width d

Caveat: there is a **maximum** when p/N is an integer i.e. angles corresponding to the maxima due to the slit spacing.

Joseph von Fraunhofer
1787-1826

Decibels are a useful measure of power over a large range of values $\mathrm{dB}(x) = \log_{10} x$

Far-Field diffraction summary

This is the actual formula for the **diffraction pattern wave power**. It incorporates all the maxima and minima effects described above.

$$|\psi|^2 = \frac{A^2}{N^2 r^2}\left(\frac{\sin\left(\frac{\pi}{\lambda} w \sin\theta\right)}{\frac{\pi}{\lambda} w \sin\theta} \times \frac{\sin\left(\frac{\pi}{\lambda} N s \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda} s \sin\theta\right)}\right)^2$$

n, m, p are integers

Envelope due to finite slit width

Zeros at: $\theta = \sin^{-1}\left(\frac{n\lambda}{w}\right);\; n \neq 0$

Maxima due to slit spacing

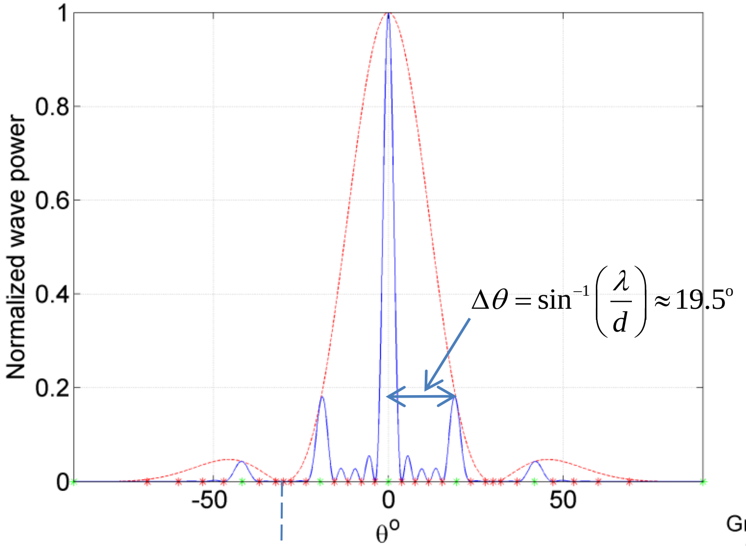
Maxima at: $\theta = \sin^{-1}\left(\frac{m\lambda}{s}\right)$

Fine structure due to number of slits
(i.e. overall size of aperture)

Zeros at: $\theta = \sin^{-1}\left(\frac{p\lambda}{Ns}\right)$

But *maxima* when $\frac{p}{N}$ integer m

Grating Fraunhofer far field diffraction
λ = 650nm, s = 3λ, w = 2λ, N = 5



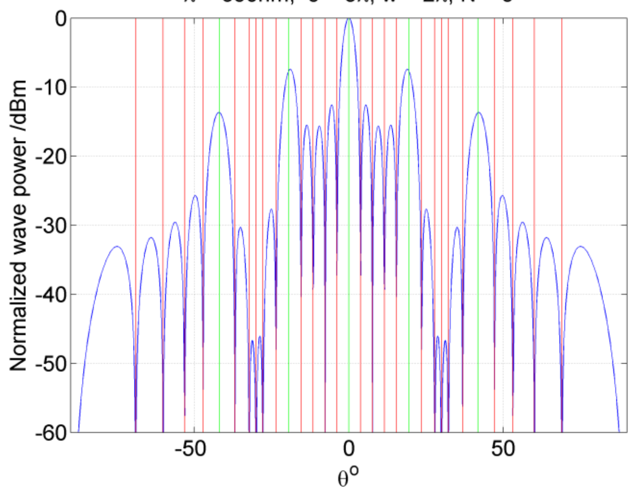
$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{w}\right) = 30^\circ$$

Almost all of these diffraction effects result in a *main lobe* of angular width (in radians)

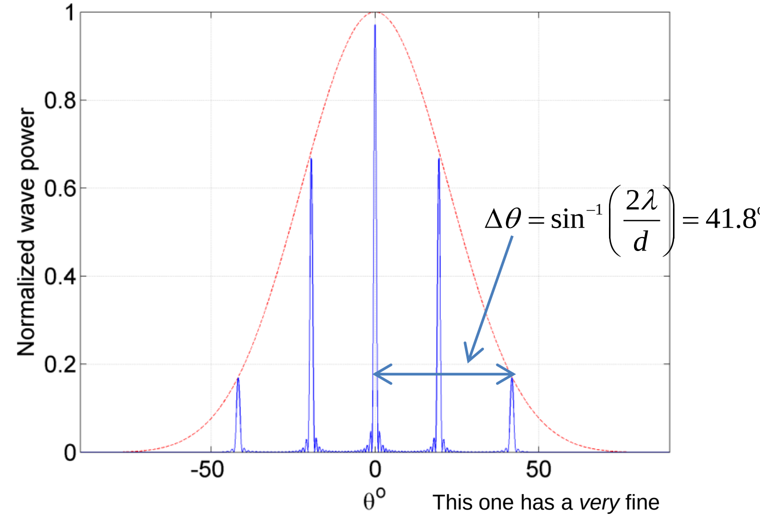
$$\Delta\theta \approx \frac{\lambda}{d}$$

where *d* is a characteristic length of the grating, slit etc. For any optical instrument, the 'resolving power' is likely to be diffraction limited. So this ratio gives *the minimum angular deviation* that two objects could be **resolved** via an optical system.

Grating Fraunhofer far field diffraction
λ = 650nm, s = 3λ, w = 2λ, N = 5

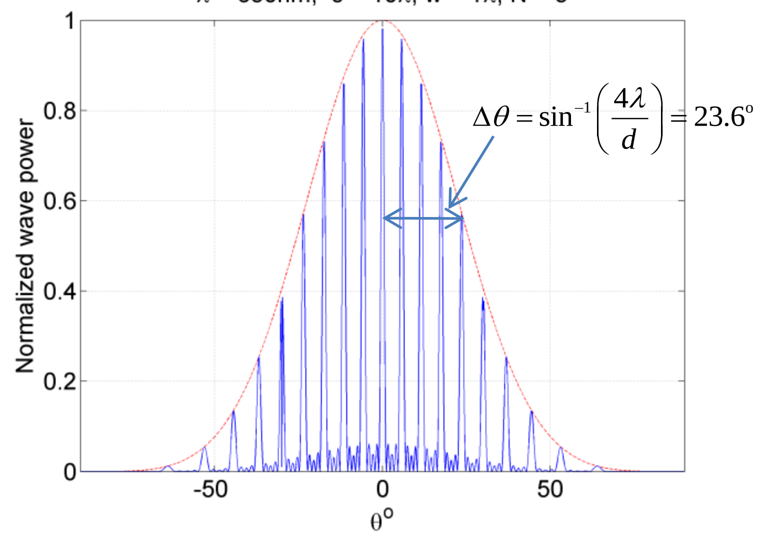


Grating Fraunhofer far field diffraction
λ = 650nm, s = 3λ, w = 1λ, N = 20



This one has a *very* fine structure since *N* is large

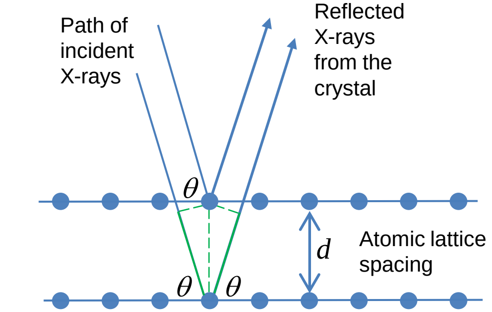
Grating Fraunhofer far field diffraction
λ = 650nm, s = 10λ, w = 1λ, N = 5

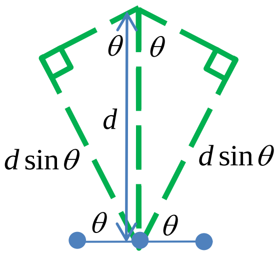


*1 arc-second is 1/3600 of a degree

e.g. for the James Web space telescope, *d* = 6.5m. So for Infra-Red light of wavelength 3.15μm, $\Delta\theta$ is about 0.1 arc-seconds*.

Bragg's law of X-ray diffraction from atoms in a crystal lattice

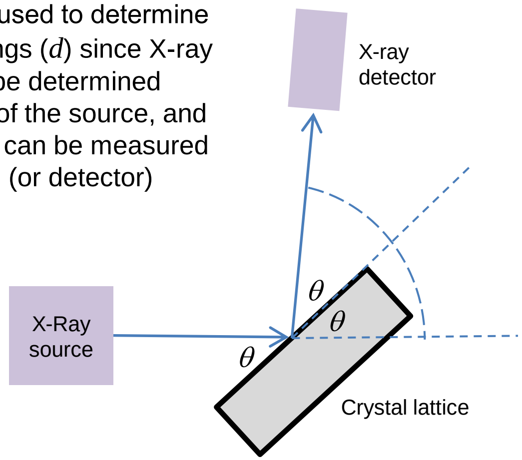


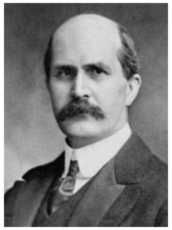


Reflected rays will constructively interfere when the *path difference* between the rays reflecting off nearby atomic layers is an integer multiple of the wavelength of the X-rays

$$2d\sin\theta = n\lambda$$

Bragg's law can be used to determine atomic lattice spacings (d) since X-ray wavelengths λ can be determined from the properties of the source, and the diffraction angle can be measured via rotating a crystal (or detector) as shown





Sir William Henry Bragg 1862-1942