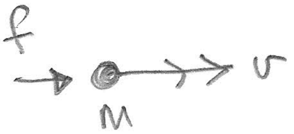
$$\boxed{E = mc^2} \quad \text{(1D example)}$$

Momentum $\boxed{p = \gamma m v}$



$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Force $f = \frac{dp}{dt} = \frac{m\, d(\gamma v)}{dt}$ (m constant)

$$= m\left(\gamma \frac{dv}{dt} + v\frac{d\gamma}{dt}\right)$$

$$\frac{d\gamma}{dt} = -\frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)^{-3/2} \cdot -\left(\frac{2v}{c^2}\right)\frac{dv}{dt}$$

$$* \quad \boxed{\frac{d\gamma}{dt} = \frac{v}{c^2}\gamma^3 \frac{dv}{dt}}$$

So $f = m\left(\gamma\frac{dv}{dt} + \frac{v^2}{c^2}\gamma^3\frac{dv}{dt}\right)$

$$f = m\gamma^3\frac{dv}{dt}\left(\frac{1}{\gamma^2} + \frac{v^2}{c^2}\right)$$

Now $\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2}$ so $\boxed{f = m\gamma^3\frac{dv}{dt}}$

Note only in this 1D case (if $\underline{f} \perp \underline{v}$ we have an extra term).

Now Work done $W = \int f\,dx$

$v = \frac{dx}{dt}$ so $dx = v\,dt$

$\therefore$ $W = \int f v\,dt$ (ie integral of power fv)

$\therefore$ $W = \int m\gamma^3 v\frac{dv}{dt}\,dt$

$$W = \int m\gamma^3 v\,dv$$

Now $\frac{d\gamma}{dt} = \frac{v}{c^2}\gamma^3\frac{dv}{dt}$

$$c^2 d\gamma = v\gamma^3 dv$$

$\therefore$ $W = mc^2\int_{\gamma_1}^{\gamma_2} d\gamma$

ie difference in total energy

$\therefore$ Work done $= (\gamma_2 - \gamma_1)mc^2$

So TOTAL ENERGY is

$$\boxed{E = \gamma mc^2}$$

Since when $v = 0$, $\gamma = 1$ $\Rightarrow \boxed{E_0 = mc^2}$

$$\boxed{KE = (\gamma - 1)mc^2}$$