Momentum is defined as the product of mass and velocity. It is therefore a vector quantity. A more general version of Newton's Second Law is that ***force is the rate of change of momentum***. In the absence of any external force, the ***total momentum*** in a system is therefore ***constant***. The 'conservation of momentum' in a force-neutral system is one of the most basic laws of Physics.

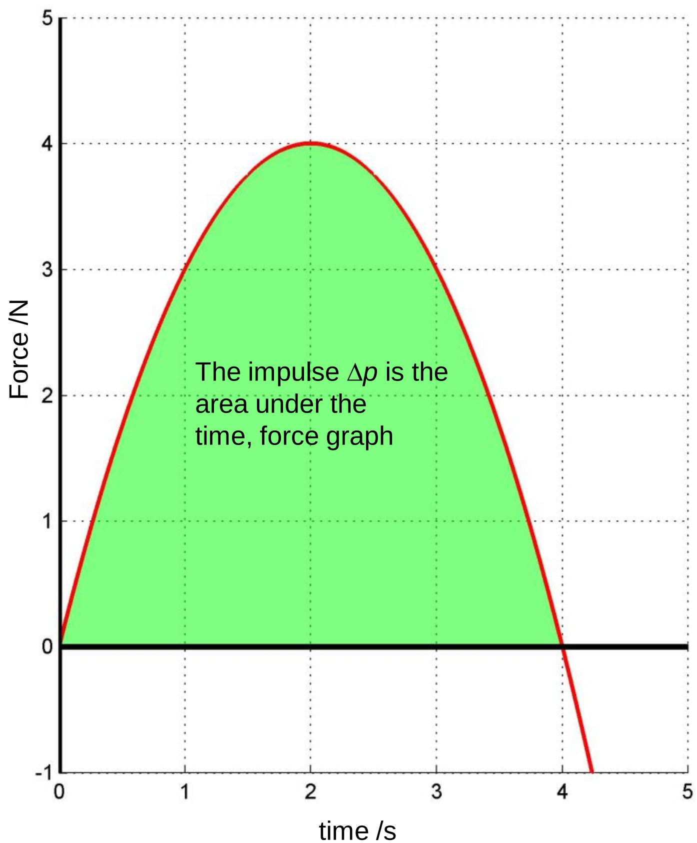
$$\boxed{\begin{array}{ll} \mathbf{p} = m\mathbf{v} & \text{Momentum} \\ \mathbf{f} = \dfrac{d\mathbf{p}}{dt} & \text{Newton II} \end{array}}$$

If mass m is constant $\mathbf{f} = \dfrac{d\mathbf{p}}{dt} = \dfrac{d}{dt}(m\mathbf{v}) = m\dfrac{d\mathbf{v}}{dt} = m\mathbf{a}$ i.e. force = mass x acceleration

+ve

Total momentum is *conserved* in this collision

If a time-varying force is acting upon an object (in a particular direction) the area under the (time, force) graph will correspond to a momentum change.



A momentum change caused by the application of a force is called an **impulse** $\Delta p = \int f dt$

In the example of the left, the force f is related to time t by

$$f(t) = -t^2 + 4t$$

$$\therefore \Delta p = \int_0^4 \left(-t^2 + 4t\right) dt$$

$$\Delta p = \left[-\tfrac{1}{3}t^3 + 2t^2\right]_0^4$$

$$\Delta p = \left(-\tfrac{1}{3}64 + 32\right) - (0) = 32 \times \left(-\tfrac{2}{3} + 1\right)$$

$$\Delta p = 10\tfrac{2}{3}\,\mathrm{N}$$

Note we must be careful about writing $\mathbf{f} = \dfrac{d\mathbf{p}}{dt}$

If mass is not constant:

$$\mathbf{f} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}$$

would seem like a natural thing to write, but it is in fact **not correct**, since transforming to a moving (bit not accelerating) frame of reference will change the force. This violates the principle of (Galilean) relativity. i.e. appropriate when velocities are much less than the speed of light.

The correct extension of Newton's Second Law when *mass is varying* is:

$$\boxed{m\frac{d\mathbf{v}}{dt} = \mathbf{f} + \mathbf{u}_R\frac{dm}{dt}}$$

$\mathbf{u}_R$ is the *relative* velocity of the ejected mass

For a space-rocket, there is *no* external force acting. If the relative velocity of propellant ejected out the back of the rocket is C and mass ejection rate is μ, Newton's Second Law becomes:

$$\left(m_r + m_f - \mu t\right)\frac{dv}{dt} = \mu C$$

m_r Rocket mass

m_f Propellant mass when $t = 0$

$$v = \int_0^t \frac{\mu C}{m_r + m_f - \mu t}dt$$

$$v = \left[-C\ln\left(m_r + m_f - \mu t\right)\right]_0^t$$

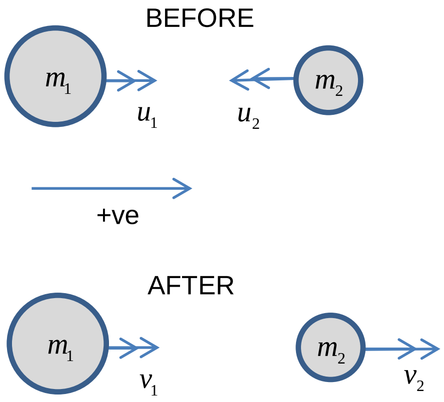
$$\boxed{v = C\ln\left(\frac{m_r + m_f}{m_r + m_f - \mu t}\right)}$$

This is called the *Tsiolkovsky rocket equation*

Note maximum burn time is $t_{\max} = m_f / \mu$

In a **collision**, although momentum will always be *conserved*, the kinetic energy of the colliding objects may not. Frictional losses, the deformation of one of the bodies following collision etc will extract energy. A ***elastic*** collision is defined as when no kinetic energy is lost. To model this, and also ***inelastic*** collisions (where kinetic energy *is* lost) we shall define a parameter called the **coefficient of restitution**. This is defined as the **speed of separation / speed of approach.**

Collisions in a straight line



By conservation of momentum

$$\boxed{m_1u_1 - m_2u_2 = m_1v_1 + m_2v_2}$$

Define the coefficient of restitution

$$\boxed{C = \frac{v_2 - v_1}{u_2 + u_1}}$$

Hence $\boxed{v_2 = v_1 + C(u_2 + u_1)}$

We can now solve for the velocities post-collision:

$$m_1u_1 - m_2u_2 = m_1v_1 + m_2\left(v_1 + C(u_2 + u_1)\right)$$

$$m_1u_1 - m_2u_2 - m_2C(u_2 + u_1) = v_1(m_1 + m_2)$$

$$u_1(m_1 - m_2\varepsilon) - m_2u_2(1 + C) = v_1(m_1 + m_2)$$

$$v_1 = \frac{u_1(m_1 - m_2C) + u_2(-m_2 - Cm_2)}{m_1 + m_2}$$

$$v_2 = \frac{u_1(m_1 - m_2C) + u_2(-m_2 - Cm_2)}{m_1 + m_2} + C(u_2 + u_1)$$

$$v_2 = \frac{u_1(m_1 - m_2C) + u_2(-m_2 - Cm_2) + (m_1 + m_2)C(u_2 + u_1)}{m_1 + m_2}$$

$$v_2 = \frac{u_1(m_1 - m_2C + m_1C + m_2C) + u_2(-m_2 - Cm_2 + m_1C + m_2C)}{m_1 + m_2}$$

$$v_2 = \frac{u_1m_1(1 + \varepsilon) + u_2\left\{C(m_1 + m_2) - m_2(1 + C)\right\}}{m_1 + m_2}$$

$$\boxed{\begin{aligned} v_1 &= \frac{u_1(m_1 - m_2C) - m_2u_2(1 + C)}{m_1 + m_2} \\ v_2 &= \frac{u_1m_1(1 + C) + u_2\left\{C(m_1 + m_2) - m_2(1 + C)\right\}}{m_1 + m_2} \end{aligned}}$$

Special case: Elastic Collisions $C = 1$

$$\boxed{\begin{aligned} v_1 &= \frac{u_1(m_1 - m_2) - 2m_2u_2}{m_1 + m_2} \\ v_2 &= \frac{2u_1m_1 + u_2(m_1 - m_2)}{m_1 + m_2} \end{aligned}}$$

It is possible, if somewhat tedious, to show that kinetic energy is conserved i.e.

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

Special case: Inelastic Collisions $C = 0$

$$\boxed{\begin{aligned} v_1 &= \frac{m_1u_1 - m_2u_2}{m_1 + m_2} \\ v_2 &= \frac{m_1u_1 - m_2u_2}{m_1 + m_2} \end{aligned}}$$

i.e. both colliding bodies move with the same velocity. They are stuck together!

In this case the total kinetic energy post-collision is

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}\frac{(m_1u_1 - m_2u_2)^2}{m_1 + m_2}$$

The kinetic energy loss is

$$\boxed{\Delta E = \tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 - \tfrac{1}{2}\frac{(m_1u_1 - m_2u_2)^2}{m_1 + m_2} = \tfrac{1}{2}\frac{m_1(u_1 - u_2)^2}{1 + \frac{m_1}{m_2}}}$$

Example 1: Find the mass M, and then calculate the amount of kinetic energy lost in the collision.

BEFORE

+ve

Note the coefficient of restitution is C = 0.5 in this case.

AFTER

By conservation of momentum

$$2M - 2 = M + 3$$

$$\boxed{M = 5}$$

The amount of kinetic energy lost is

$$\Delta E = \tfrac{1}{2}(5)(2^2) + \tfrac{1}{2}(1)(2^2) - \tfrac{1}{2}(5)(1) - \tfrac{1}{2}(1)(3^2)$$

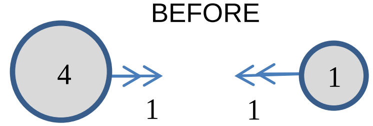
$$\Delta E = \tfrac{1}{2}\left(20 + 4 - 5 - 9\right)$$

$$\Delta E = \tfrac{1}{2}\left(20 + 4 - 5 - 9\right)$$

$$\boxed{\Delta E = 5\text{J}}$$

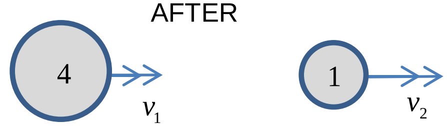
Example 2: Find the velocities post-collision
Assume the collision is elastic. Masses are in kg

BEFORE



+ve

AFTER



By conservation of momentum

$$4v_1 + v_2 = 4(1) - (1)(1)$$

$$4v_1 + v_2 = 3$$

Since collision is elastic

$$v_2 - v_1 = 2$$

Subtracting these equations eliminates v_2

$$5v_1 = 1$$

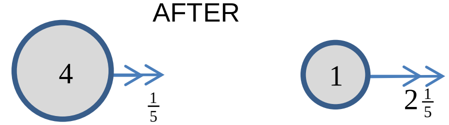
$$\boxed{v_1 = \tfrac{1}{5}}$$

Hence

$$v_2 = 2 + v_1$$

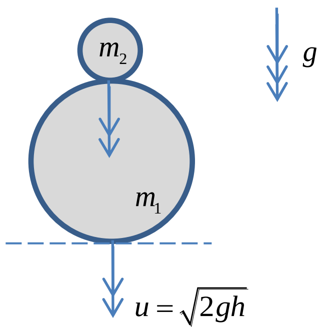
$$\boxed{v_2 = 2\tfrac{1}{5}}$$

AFTER

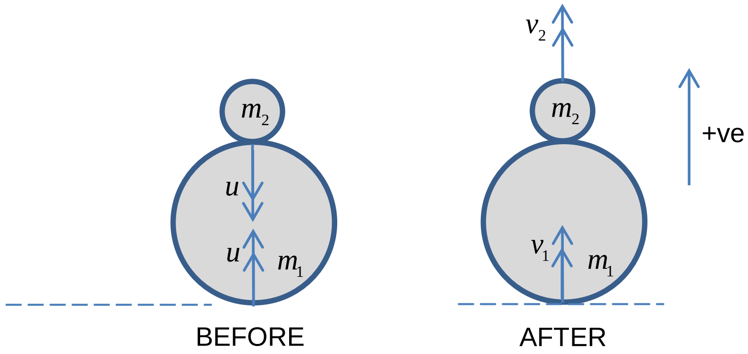


Interesting scenario: two balls dropped together

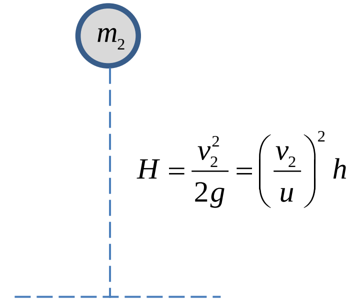
$m_1 \geq m_2$



Both balls are dropped from height h. The lower (and more massive one) collides elastically with a hard floor



The balls then collide



Upper ball rises to height H

By conservation of momentum

$$m_1 u - m_2 u = m_1 v_1 + m_2 v_2$$

Coefficient of restitution is defined in this case as:

$$C = \frac{v_2 - v_1}{2u} \quad \therefore \boxed{v_1 = v_2 - 2uC}$$

Hence:

$$m_1 u - m_2 u = m_1 (v_2 - 2uC) + m_2 v_2$$

$$m_1 u - m_2 u + 2Cm_1 u = v_2 (m_1 + m_2)$$

$$v_2 = \frac{(2C+1)m_1 - m_2}{m_1 + m_2} u$$

$$\boxed{v_2 = \frac{(2C+1) - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} u}$$

If the collisions are elastic:

$$\boxed{v_2 = \frac{3 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} u}$$

$$v_1 = \frac{3 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} u - 2u = \frac{3 - \frac{m_2}{m_1} - 2\left(1 + \frac{m_2}{m_1}\right)}{1 + \frac{m_2}{m_1}} u$$

$$\boxed{v_1 = \frac{1 - \frac{3m_2}{m_1}}{1 + \frac{m_2}{m_1}} u}$$

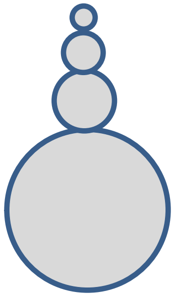
If the collisions are elastic and $m_1 \gg m_2$

$v_2 = 3u$

Hence $\boxed{H = 9h}$ is the maximum height the upper ball will rise.
This can be quite a startling demonstration!
For best classroom results, use a basketball and a tennis ball.

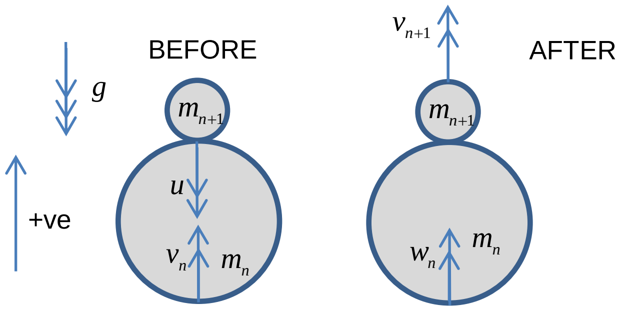
If the collisions are elastic *and* $m_1 = m_2$

$v_2 = u$ Hence $\boxed{H = h}$



Now imagine a stack of *N* balls dropped together. For brevity, let us assume all collisions are elastic.

Let the ball masses be in a geometric ratio $\dfrac{m_n}{m_{n+1}} = k$



Note $v_1 = u$

By conservation of momentum

$$m_{n+1}v_{n+1} + m_n w_n = -m_{n+1}u + m_n v_n$$

$$\therefore v_{n+1} = \frac{m_n\left(v_n - w_n\right) - m_{n+1}u}{m_{n+1}} = k\left(v_n - w_n\right) - u$$

Since collision is elastic

$$C = 1 = \frac{v_{n+1} - w_n}{v_n + u} \quad \therefore w_n = v_{n+1} - v_n - u$$

$$v_{n+1} = k\left(v_n - \left\{v_{n+1} - v_n - u\right\}\right) - u$$

$$v_{n+1} = -kv_{n+1} + 2kv_n + ku - u$$

$$v_{n+1}\left(k+1\right) = 2kv_n + u\left(k-1\right)$$

$$v_{n+1} = \frac{2k}{k+1}v_n + \frac{k-1}{k+1}u = av_n + b$$

$$v_2 = au + b$$

$$v_3 = av_2 + b = a\left(au+b\right) + b = a^2u + ab + b$$

$$v_4 = av_3 + b = a^3u + a^2b + ab + b$$

.....

$$v_N = a^{N-1}u + b\sum_{i=1}^{N-2} a^i$$

$$v_N = a^{N-1}u + b\frac{a^{N-1}-1}{a-1}$$ Using the sum of a geometric series

$$a = \frac{2k}{k+1}$$

$$b = \frac{k-1}{k+1}u$$

$$\therefore a - 1 = \frac{2k}{k+1} - \frac{k+1}{k+1} = \frac{k-1}{k+1}$$

$$\therefore \frac{b}{a-1} = u$$

$$\therefore v_N = a^{N-1}u + ua^{N-1} - u$$

The recoil velocity of the n[th] mass can now be determined

$$\boxed{\frac{v_N}{u} = 2\left(\frac{2k}{k+1}\right)^{N-1} - 1}$$

If one repeats the above analysis taking into account a coefficient of restitution

$$\boxed{\frac{v_N}{u} = \left(\frac{k\left(1+C\right)}{k+1}\right)^{N-1}\left(1+C\right) - 1}$$

The Irish Moonshot (!)

A rather fun extension to this is to calculate how many balls are required to cause the upper one to escape from Earth (!). Let us assume *k* = 2 and all collisions are elastic, i.e. *C=1*

$$k = 2$$

$$\frac{v_n}{u} = 2\left(\frac{4}{3}\right)^{n-1} - 1$$

$$v_n = \sqrt{\frac{2GM_\oplus}{R_\oplus}}$$

$$u = \sqrt{2gh}$$

$$2\left(\frac{4}{3}\right)^{n-1} - 1 = \sqrt{\frac{GM_\oplus}{R_\oplus gh}}$$

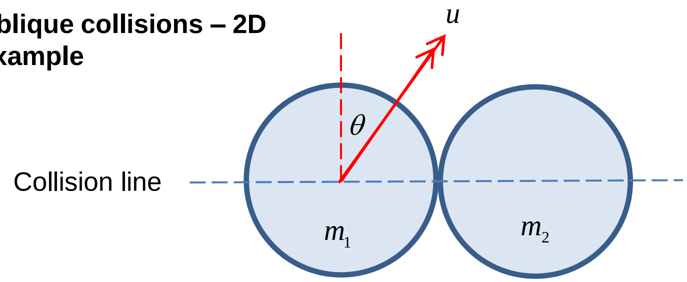
$$n = \frac{\log\left(\frac{1}{2}\sqrt{\frac{GM_\oplus}{R_\oplus gh}} + \frac{1}{2}\right)}{\log 4 - \log 3} + 1$$

$$n \approx \frac{\log\left(\frac{1}{2}\sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{6.38\times10^{6}\times9.81\times1}} + \frac{1}{2}\right)}{\log 4 - \log 3} + 1$$

$$\boxed{n \approx 26}$$

The latter step assumes the 26 ball system is dropped from 1 metre. Realistic? Well if the top ball is 1kg, the bottom ball is 2^{25} kg, i.e. 33.6 million tonnes! So perhaps the elastic collision assumption may not be a good one!

Oblique collisions – 2D
Example



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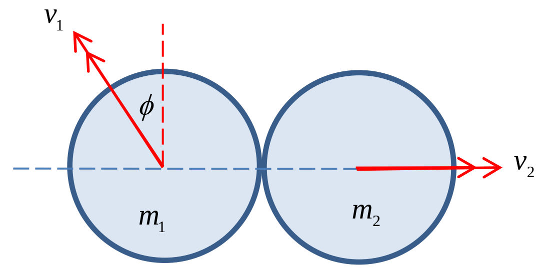


AFTER

In an oblique collision between particles, the only actual collision is along a ***collision line*** joining the particle centres.

Velocity components *perpendicular* to this line are **unchanged**.

Perpendicular velocities unchanged

$$u\cos\theta = v_1\cos\phi \quad \therefore v_1 = \frac{u\cos\theta}{\cos\phi} \qquad (1)$$

Conservation of momentum along collision line

$$m_1 u\sin\theta = m_2 v_2 - m_1 v_1 \sin\phi \qquad (2)$$

Restitution (along collision line)

$$C = \frac{v_1\sin\phi + v_2}{u\sin\theta}$$

$$m_1 C u\sin\theta = m_1 v_1 \sin\phi + m_1 v_2 \qquad (3)$$

$$m_1(1+C)u\sin\theta = (m_1+m_2)v_2 \qquad (2) + (3)$$

$$v_2 = \frac{m_1(1+C)u\sin\theta}{m_1+m_2}$$

$$m_1 u\sin\theta = m_2 v_2 - m_1 v_1 \sin\phi$$

$$m_1 u\sin\theta = m_2\left(\frac{m_1(1+C)u\sin\theta}{m_1+m_2}\right) - m_1\left(\frac{u\cos\theta}{\cos\phi}\right)\sin\phi \qquad v_1 = \frac{u\cos\theta}{\cos\phi} \qquad v_2 = \frac{m_1(1+C)u\sin\theta}{m_1+m_2}$$

$$1 = \frac{m_2(1+C)}{m_1+m_2} - \frac{\tan\phi}{\tan\theta}$$

$$\tan\phi = \left(\frac{m_2(1+C)}{m_1+m_2} - 1\right)\tan\theta$$

$$\boxed{\begin{aligned} v_2 &= \frac{m_1(1+C)u\sin\theta}{m_1+m_2} \\ \tan\phi &= \left(\frac{m_2(1+C)}{m_1+m_2} - 1\right)\tan\theta \\ v_1 &= \frac{u\cos\theta}{\cos\phi} \end{aligned}}$$

Example:

$\theta = 60^{\circ}$

$u = \sqrt{3}$

$m_1 = 1$

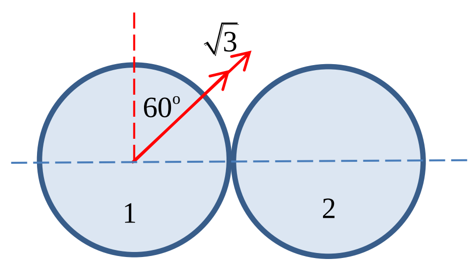
$m_2 = 2$

$C = 1$

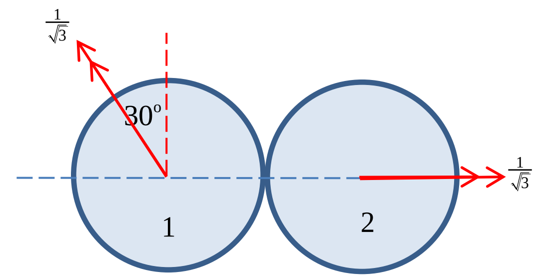
$v_2 = \frac{1}{\sqrt{3}}$

$\tan\phi = \frac{1}{\sqrt{3}} \quad \therefore \phi = 30^{\circ}$

$v_1 = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

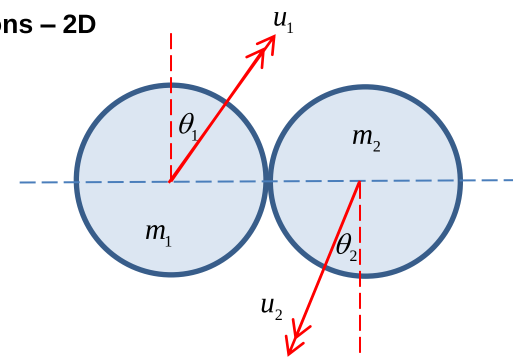


BEFORE



AFTER

Oblique collisions – 2D Generalized!



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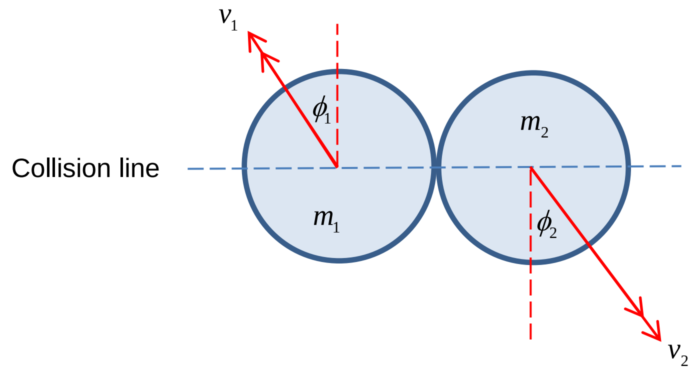


AFTER

In an oblique collision between particles, the only actual collision is along a ***collision line*** joining the particle centres.

Velocity components *perpendicular* to this line are **unchanged**.

Perpendicular velocities unchanged

$$u_1 \cos\theta_1 = v_1 \cos\phi_1$$
$$u_2 \cos\theta_2 = v_2 \cos\phi_2$$

Conservation of momentum along collision line

$$m_1 u_1 \sin\theta_1 - m_2 u_2 \sin\theta_2 = m_2 v_2 \sin\phi_2 - m_1 v_1 \sin\phi_1 \qquad (1)$$

Restitution (along collision line)

$$C = \frac{v_1 \sin\phi_1 + v_2 \sin\phi_2}{u_1 \sin\theta_1 + u_2 \sin\theta_2} \qquad (2)$$

$$\frac{u_1 \cos\theta_1}{\cos\phi_1} = v_1 \qquad \frac{u_2 \cos\theta_2}{\cos\phi_2} = v_2 \qquad (3)$$

(3) in (1)

$$m_1 u_1 \sin\theta_1 - m_2 u_2 \sin\theta_2 = m_2 u_2 \cos\theta_2 \tan\phi_2 - m_1 u_1 \cos\theta_1 \tan\phi_1$$

$$(2) \quad m_1 C\left(u_1 \sin\theta_1 + u_2 \sin\theta_2\right) = m_1 u_1 \cos\theta_1 \tan\phi_1 + m_1 u_2 \cos\theta_2 \tan\phi_2$$

$$m_1 C\left(u_1 \sin\theta_1 + u_2 \sin\theta_2\right) + m_1 u_1 \sin\theta_1 - m_2 u_2 \sin\theta_2$$
$$= \left(m_1 + m_2\right) u_2 \cos\theta_2 \tan\phi_2$$

$$m_1 u_1 \left(1 + C\right) \sin\theta_1 - \left(m_2 - m_1 C\right) u_2 \sin\theta_2 = \left(m_1 + m_2\right) u_2 \cos\theta_2 \tan\phi_2$$

$$\boxed{\phi_2 = \tan^{-1}\left(\frac{m_1 u_1 \left(1 + C\right)\sin\theta_1 - \left(m_2 - m_1 C\right) u_2 \sin\theta_2}{\left(m_1 + m_2\right) u_2 \cos\theta_2}\right)}$$

$$m_1 u_1 \sin\theta_1 - m_2 u_2 \sin\theta_2 = m_2 u_2 \cos\theta_2 \tan\phi_2 - m_1 u_1 \cos\theta_1 \tan\phi_1 \qquad \text{(3) in (1)}$$

$$m_2 C\left(u_1 \sin\theta_1 + u_2 \sin\theta_2\right) = m_2 u_1 \cos\theta_1 \tan\phi_1 + m_2 u_2 \cos\theta_2 \tan\phi_2 \qquad (2)$$

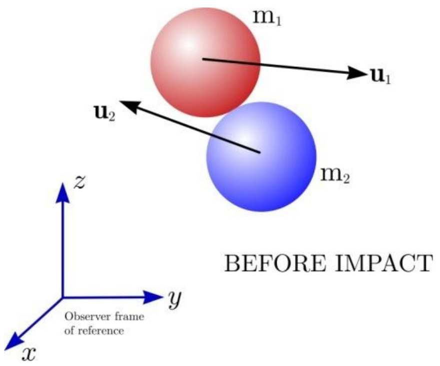
$$m_1 u_1 \sin\theta_1 - m_2 u_2 \sin\theta_2 - m_2 C\left(u_1 \sin\theta_1 + u_2 \sin\theta_2\right) = -(m_1 + m_2) u_1 \cos\theta_1 \tan\phi_1$$

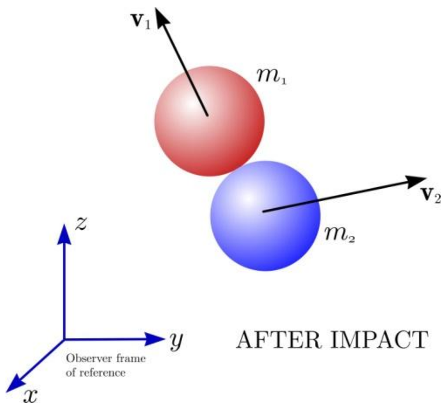
$$\left(m_1 - m_2 C\right) u_1 \sin\theta_1 - m_2 \left(1 + C\right) u_2 \sin\theta_2 = -(m_1 + m_2) u_1 \cos\theta_1 \tan\phi_1$$

$$\boxed{\phi_1 = \tan^{-1}\left(\frac{m_2 \left(1 + C\right) u_2 \sin\theta_2 - \left(m_1 - m_2 C\right) u_1 \sin\theta_1}{(m_1 + m_2) u_1 \cos\theta_1}\right)}$$

$$\boxed{\frac{u_1 \cos\theta_1}{\cos\phi_1} = v_1 \qquad \frac{u_2 \cos\theta_2}{\cos\phi_2} = v_2}$$

Oblique collisions – 3D If collisions are *not* in a straight line, a vector analysis is required to work out the post-collision velocities





By conservation of momentum

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

Define the coefficient of restitution

$$C = \frac{|\mathbf{v}_2 - \mathbf{v}_1|}{|\mathbf{u}_2 - \mathbf{u}_1|}$$

However, this doesn't help us isolate $\mathbf{v}_1$ and $\mathbf{v}_2$
to do this requires the *Zero Momentum Frame* (ZMF)

To transform to the ZMF, we shall subtract a velocity $\mathbf{V}$ from both masses such that the total momentum is zero

$$m_1(\mathbf{u}_1 - \mathbf{V}) + m_2(\mathbf{u}_2 - \mathbf{V}) = \mathbf{0}$$

$$\therefore \mathbf{V}(m_1 + m_2) = m_1\mathbf{u}_1 + m_2\mathbf{u}_2$$

$$\therefore \boxed{\mathbf{V} = \frac{m_1\mathbf{u}_1 + m_2\mathbf{u}_2}{m_1 + m_2}}$$

In the ZMF, the masses are now colliding in a straight line. Hence we can now write down the post-collision velocities in terms of the original velocities and the coefficient of restitution, C

$$\mathbf{v}_1 = C(\mathbf{V} - \mathbf{u}_1) + \mathbf{V}$$

$$\mathbf{v}_2 = C(\mathbf{V} - \mathbf{u}_2) + \mathbf{V}$$

In terms of C and the masses, this becomes

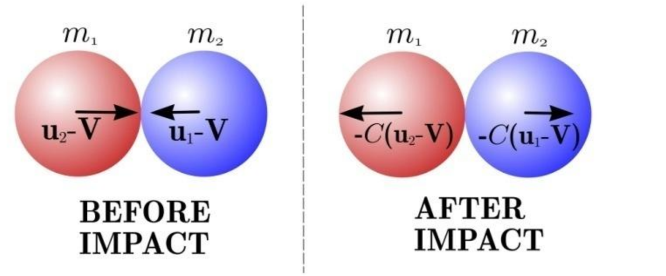
$$\boxed{\begin{aligned} \mathbf{v}_1 &= \mathbf{u}_1\left\{\frac{m_1 - Cm_2}{m_1 + m_2}\right\} + \mathbf{u}_2\left\{\frac{m_2(1+C)}{m_1 + m_2}\right\} \\ \mathbf{v}_2 &= \mathbf{u}_1\left\{\frac{m_1(1+C)}{m_1 + m_2}\right\} + \mathbf{u}_2\left\{\frac{m_2 - Cm_1}{m_1 + m_2}\right\} \end{aligned}}$$

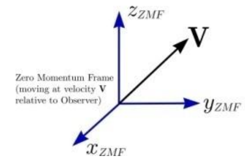
If $C = 0$, i.e. an inelastic collision, one can show the loss in kinetic energy is given by

$$\Delta E = \tfrac{1}{2}m_1|\mathbf{u}_1|^2 + \tfrac{1}{2}m_2|\mathbf{u}_2|^2 - \tfrac{1}{2}m_1|\mathbf{v}_1|^2 - \tfrac{1}{2}m_2|\mathbf{v}_2|^2$$

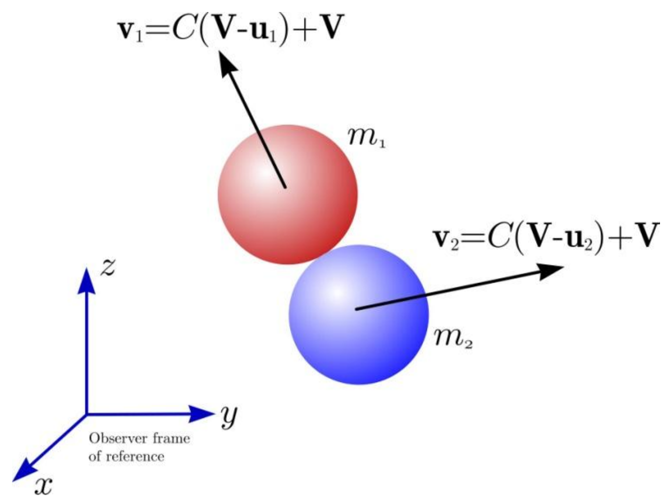
$$\boxed{\Delta E = \frac{\tfrac{1}{2}m_1|\mathbf{u}_1 - \mathbf{u}_2|^2}{1 + \frac{m_1}{m_2}}}$$

Zero Momentum Frame





C is the coefficient of restitution

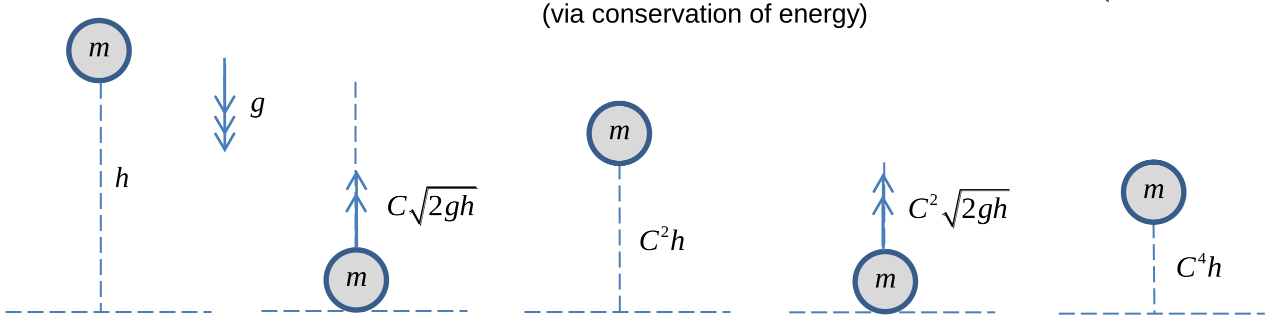


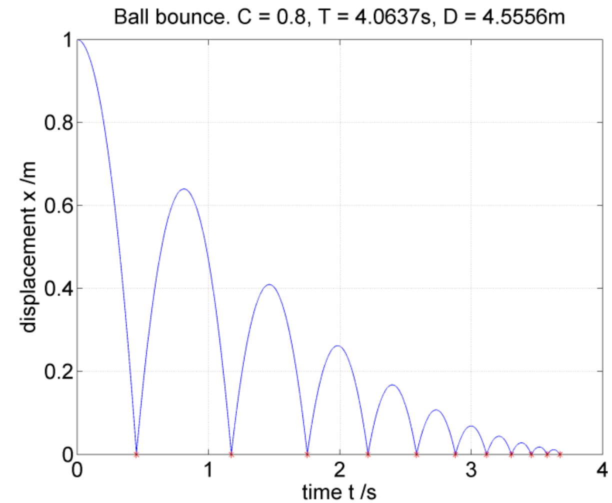
Ball bouncing on a horizontal surface

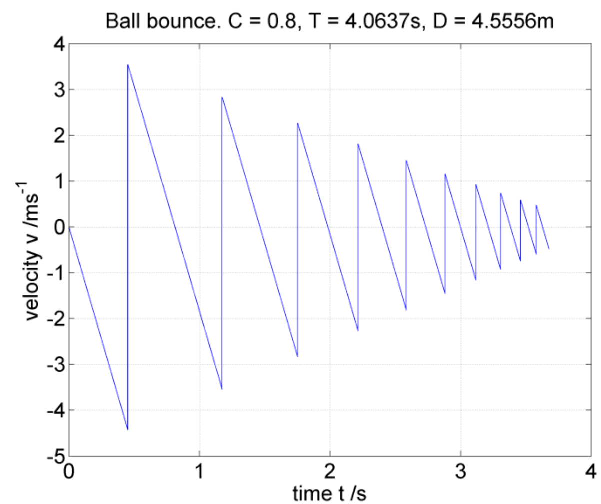
A ball is dropped from rest from vertical height h onto a horizontal floor. The impact velocity is $\sqrt{2gh}$ (via conservation of energy)

The ball-floor collision has a coefficient of restitution of C.

The ball therefore leaves the floor with velocity $C\sqrt{2gh}$







By conservation of energy, the ball rises to new height

$$mgh' = \tfrac{1}{2}m\left(C\sqrt{2gh}\right)^2$$

$$h' = C^2h$$

And therefore the impact velocity is

$$\sqrt{2gh'} = C\sqrt{2gh}$$

And the rebound velocity is $C^2\sqrt{2gh}$

The fall time before the first bounce is

$$h = \tfrac{1}{2}gt^2$$

$$\therefore t = \sqrt{\frac{2h}{g}}$$

Between the first and second bounce the time interval is

$$\Delta t = 2\sqrt{\frac{2h'}{g}} = 2C\sqrt{\frac{2h}{g}}$$

To generalize, the distance travelled after n bounces is

$$D = h + 2C^2h + 2C^4h + ... + 2\left(C^2\right)^{n-1}h$$

$$\frac{D}{2h} + \tfrac{1}{2} = 1 + C^2 + \left(C^2\right)^2 + ... + \left(C^2\right)^{n-1}$$

Geometric progression

$$\frac{D}{2h} + \tfrac{1}{2} = \frac{1-C^{2n}}{1-C^2}$$

$$D = 2h\left(\frac{1-C^{2n}}{1-C^2} - \tfrac{1}{2}\right) \qquad \therefore \boxed{D_\infty = 2h\left(\frac{1}{1-C^2} - \tfrac{1}{2}\right)}$$

The time travelled after n bounces is

$$T = \sqrt{\frac{2h}{g}} + 2C\sqrt{\frac{2h}{g}} + 2C^2\sqrt{\frac{2h}{g}} + ... + 2C^{n-1}\sqrt{\frac{2h}{g}}$$

$$\frac{T}{2}\sqrt{\frac{g}{2h}} + \tfrac{1}{2} = 1 + C + C^2 + ... + C^{n-1}$$

Geometric progression

$$\frac{T}{2}\sqrt{\frac{g}{2h}} + \tfrac{1}{2} = \frac{1-C^n}{1-C}$$

$$T = 2\sqrt{\frac{2h}{g}}\left(\frac{1-C^n}{1-C} - \tfrac{1}{2}\right) \qquad \therefore \boxed{T_\infty = 2\sqrt{\frac{2h}{g}}\left(\frac{1}{1-C} - \tfrac{1}{2}\right)}$$