**Why primes are interesting**

**Prime numbers** are *integers that have no other factor than themselves, or one.*

Prime numbers are the 'building blocks' of all other integers, since *any integer can be expressed in terms of prime factors*. This is a very useful process for simplifying divisions, working out *lowest common multiples*, *highest common factors* etc.

But prime numbers are also *mysterious*. There *is no known formula for predicting the $n^{th}$ prime number!* Since *algorithms* involving the product of two large primes is the basis of **RSA cryptography** that is used extensively for secure internet connections, this feature of prime numbers helps maintain our security.

The distribution of primes follows a fairly random pattern, which can be seen by plotting a **Ulam spiral** or **number spiral plot**. Or simply a grid with the primes coloured.

**Mersenne Primes** are of the form: $M_p = 2^p - 1$ **= 3,7,31,127,8191,** 131071, 524287 ...

where $p$ is also prime.

Note *not all primes p will result in a Mersenne prime*.

$$2^{11} - 1 = 23 \times 89$$

**Perfect numbers** can be found from the formula (which has a Mersenne prime factor)

$$P_p = 2^{p-1}\left(2^p - 1\right) \qquad p = 2,3,5,7,13,17,19,31,61,89,107,127...$$

Primes that make Mersenne primes

Perfect numbers are numbers where the sum of all possible factors (excluding the number itself) = the number.

```
PERFECT #1: 2^1 * ( 2^2 - 1 ) = 6 = 1+2+3
PERFECT #2: 2^2 * ( 2^3 - 1 ) = 28 = 1+2+4+7+14
PERFECT #3: 2^4 * ( 2^5 - 1 ) = 496 = 1+2+4+8+16+31+62+124+248
PERFECT #4: 2^6 * ( 2^7 - 1 ) = 8128 =
1+2+4+8+16+32+64+127+254+508+1016+2032+4064
PERFECT #5: 2^12 * ( 2^13 - 1 ) = 33550336 =
1+2+4+8+16+32+64+128+256+512+1024+2048+4096+8191+16382+32764+65528
+131056+26211+524224+1048448+2096896+4193792+8387584+16775168
```

At the time of writing, **only 47 Perfect numbers have been found**. And all of these are ***even*** …

First *fifty million* primes!
https://primes.utm.edu/lists/small/millions/

```
1 is ODD, is a SQUARE, is a CUBE, and prime factors are 1
2 is EVEN, is PRIME and generates MERSENNE PRIME 2^2 - 1 = 3
3 is ODD, is a MERSENNE PRIME and generates MERSENNE PRIME 2^3 - 1 = 7
4 is EVEN, is a SQUARE, and prime factors are 2 x 2
5 is ODD, is PRIME and generates MERSENNE PRIME 2^5 - 1 = 31
6 is EVEN, and prime factors are 2 x 3. *PERFECT*
7 is ODD, is a MERSENNE PRIME and generates MERSENNE PRIME 2^7 - 1 = 127
8 is EVEN, is a CUBE, and prime factors are 2 x 2 x 2
9 is ODD, is a SQUARE, and prime factors are 3 x 3
10 is EVEN, and prime factors are 2 x 5
```

Marin Mersenne
1588-1648

$$\frac{4321}{5678} = \frac{29 \times 149}{2 \times 17 \times 167}$$

This can't be simplified since both numerator and denominator have no common prime factors

$$\frac{3210}{642} = \frac{2 \times 3 \times 5 \times 107}{2 \times 3 \times 107} = 5$$

This fraction *can* be simplified since common factors of $2 \times 3 \times 107$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 144 | 143 | 142 | 141 | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 |
| 101 | 100 | 99 | 98 | 97 | 96 | 95 | 94 | 93 | 92 | 91 | 132 |
| 102 | 65 | 64 | 63 | 62 | 61 | 60 | 59 | 58 | 57 | 90 | 131 |
| 103 | 66 | 37 | 36 | 35 | 34 | 33 | 32 | 31 | 56 | 89 | 130 |
| 104 | 67 | 38 | 17 | 16 | 15 | 14 | 13 | 30 | 55 | 88 | 129 |
| 105 | 68 | 39 | 18 | 5 | 4 | 3 | 12 | 29 | 54 | 87 | 128 |
| 106 | 69 | 40 | 19 | 6 | 1 | 2 | 11 | 28 | 53 | 86 | 127 |
| 107 | 70 | 41 | 20 | 7 | 8 | 9 | 10 | 27 | 52 | 85 | 126 |
| 108 | 71 | 42 | 21 | 22 | 23 | 24 | 25 | 26 | 51 | 84 | 125 |
| 109 | 72 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 83 | 124 |
| 110 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 123 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 |

**Ulam sprial**

**Primes**

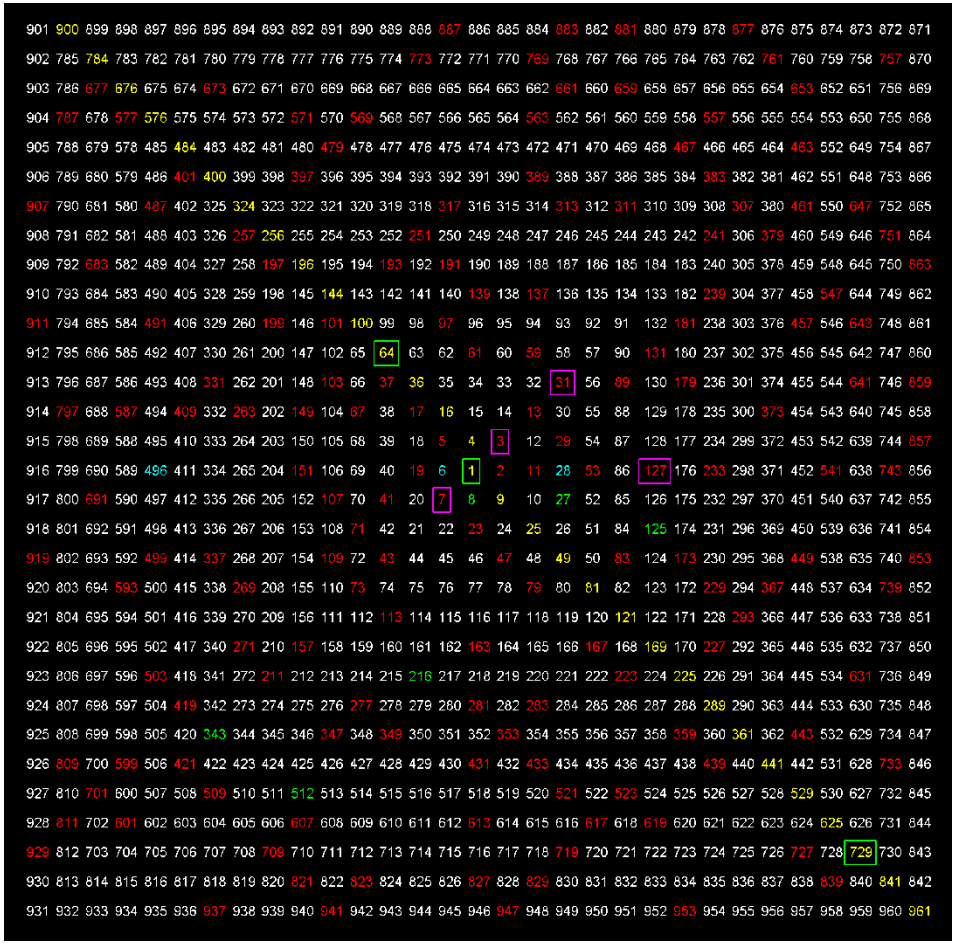
**Mersenne Primes**

**Perfect**

**Cubes**

**Squares**

**Cube and square**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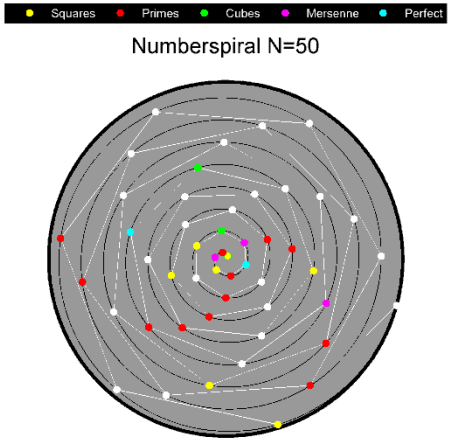
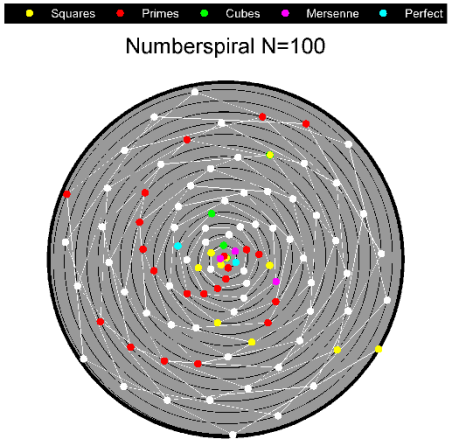
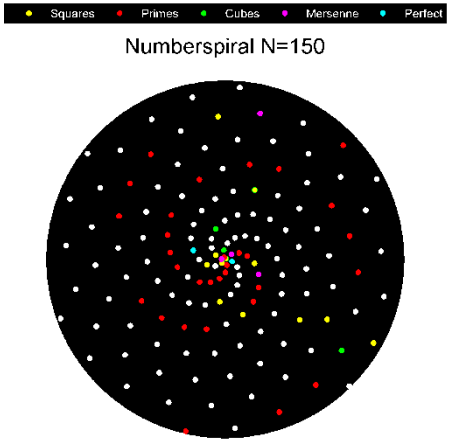
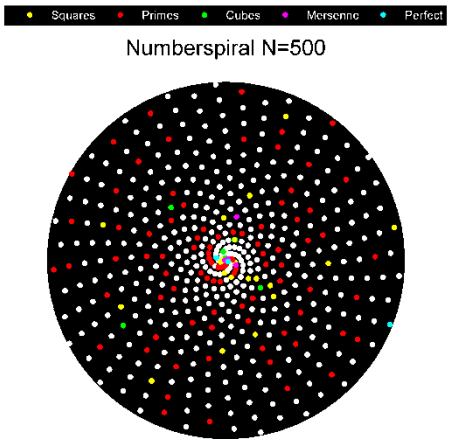
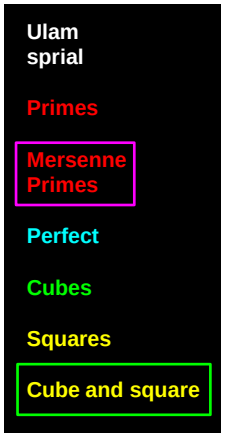
902 785 784 783 782 781 780 779 778 777 776 775 774 773 772 771 770 769 768 767 766 765 764 763 762 761 760 759 758 757 870
903 786 677 676 675 674 673 672 671 670 669 668 667 666 665 664 663 662 661 660 659 658 657 656 655 654 653 652 651 756 869
904 787 678 577 576 575 574 573 572 571 570 569 568 567 566 565 564 563 562 561 560 559 558 557 556 555 554 553 650 755 868
905 788 679 578 485 484 483 482 481 480 479 478 477 476 475 474 473 472 471 470 469 468 467 466 465 464 463 552 649 754 867
906 789 680 579 486 401 400 399 398 397 396 395 394 393 392 391 390 389 388 387 386 385 384 383 382 381 462 551 648 753 866
907 790 681 580 487 402 325 324 323 322 321 320 319 318 317 316 315 314 313 312 311 310 309 308 307 380 461 550 647 752 865
908 791 682 581 488 403 326 257 256 255 254 253 252 251 250 249 248 247 246 245 244 243 242 241 306 379 460 549 646 751 864
909 792 683 582 489 404 327 258 197 196 195 194 193 192 191 190 189 188 187 186 185 184 183 240 305 378 459 548 645 750 863
910 793 684 583 490 405 328 259 198 145 144 143 142 141 140 139 138 137 136 135 134 133 182 239 304 377 458 547 644 749 862
911 794 685 584 491 406 329 260 199 146 101 100 99 98 97 96 95 94 93 92 91 132 181 238 303 376 457 546 643 748 861
912 795 686 585 492 407 330 261 200 147 102 65 64 63 62 61 60 59 58 57 90 131 180 237 302 375 456 545 642 747 860
913 796 687 586 493 408 331 262 201 148 103 66 37 36 35 34 33 32 31 56 89 130 179 236 301 374 455 544 641 746 859
914 797 688 587 494 409 332 263 202 149 104 67 38 17 16 15 14 13 30 55 88 129 178 235 300 373 454 543 640 745 858
915 798 689 588 495 410 333 264 203 150 105 68 39 18 5 4 3 12 29 54 87 128 177 234 299 372 453 542 639 744 857
916 799 690 589 496 411 334 265 204 151 106 69 40 19 6 1 2 11 28 53 86 127 176 233 298 371 452 541 638 743 856
917 800 691 590 497 412 335 266 205 152 107 70 41 20 7 8 9 10 27 52 85 126 175 232 297 370 451 540 637 742 855
918 801 692 591 498 413 336 267 206 153 108 71 42 21 22 23 24 25 26 51 84 125 174 231 296 369 450 539 636 741 854
919 802 693 592 499 414 337 268 207 154 109 72 43 44 45 46 47 48 49 50 83 124 173 230 295 368 449 538 635 740 853
920 803 694 593 500 415 338 269 208 155 110 73 74 75 76 77 78 79 80 81 82 123 172 229 294 367 448 537 634 739 852
921 804 695 594 501 416 339 270 209 156 111 112 113 114 115 116 117 118 119 120 121 122 171 228 293 366 447 536 633 738 851
922 805 696 595 502 417 340 271 210 157 158 159 160 161 162 163 164 165 166 167 168 169 170 227 292 365 446 535 632 737 850
923 806 697 596 503 418 341 272 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 291 364 445 534 631 736 849
924 807 698 597 504 419 342 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 363 444 533 630 735 848
925 808 699 598 505 420 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 443 532 629 734 847
926 809 700 599 506 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 531 628 733 846
927 810 701 600 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 627 732 845
928 811 702 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 731 844
929 812 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 843
930 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842
931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961

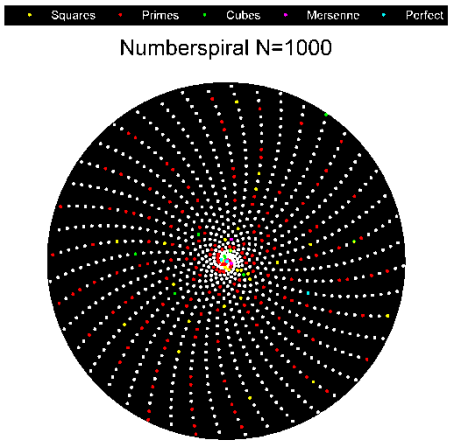

Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=50

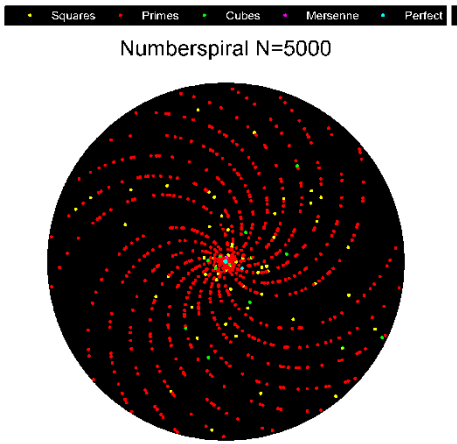

Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=100

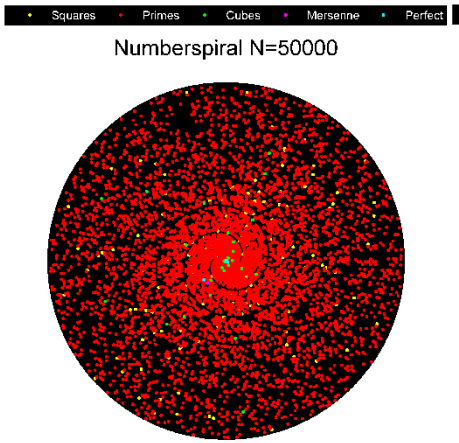

Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=150

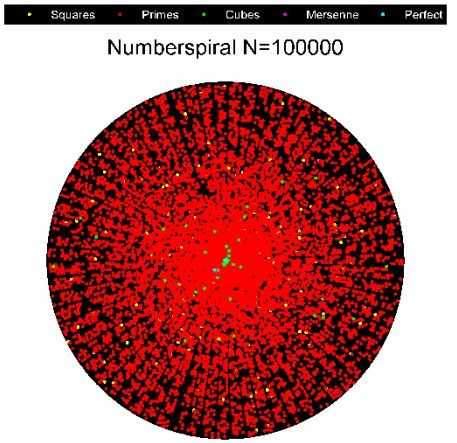

Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=500


Ulam sprial
Primes
Mersenne Primes
Perfect
Cubes
Squares
Cube and square

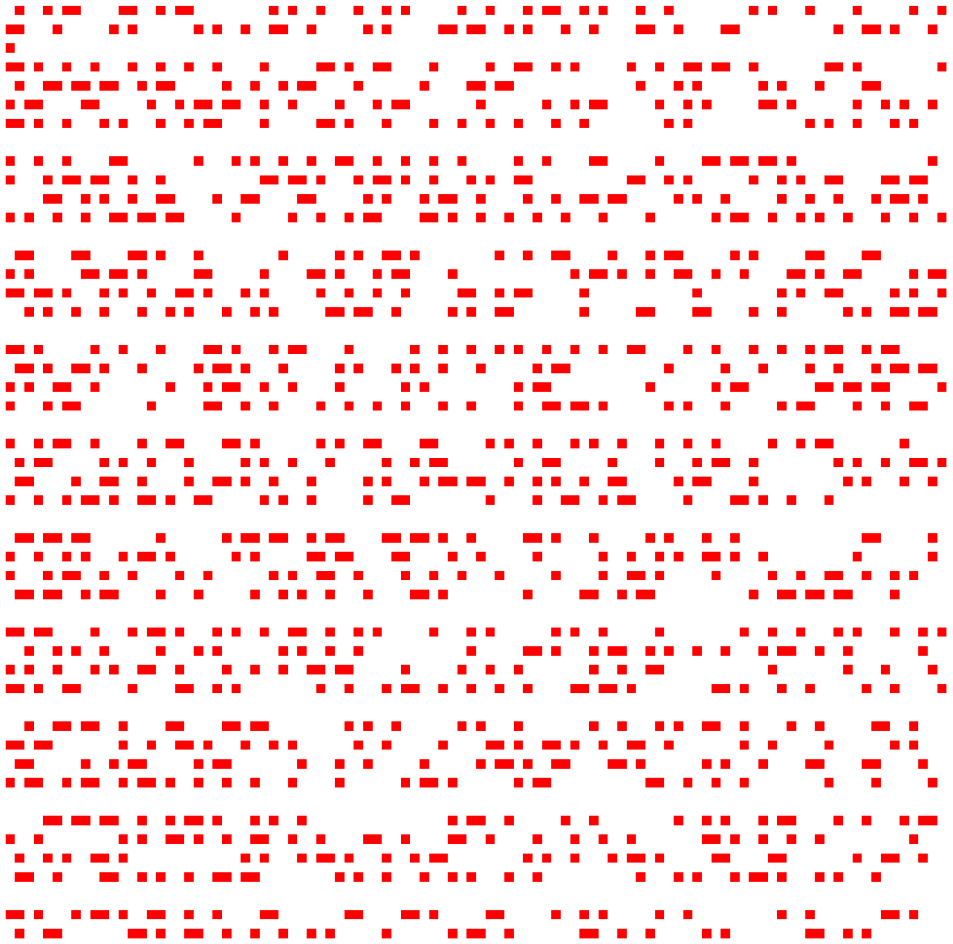

Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=1000


Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=5000


Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=50000


Squares
Primes
Cubes
Mersenne
Perfect
Numberspiral N=100000

**Number grid, with prime numbers only shaded red**



First 10,000 primes

First 1,000,000 primes

1 is ODD, is a SQUARE, is a CUBE, and prime factors are 1
2 is EVEN, is PRIME and generates MERSENNE PRIME 2^2 - 1 = 3
3 is ODD, is a MERSENNE PRIME and generates MERSENNE PRIME 2^3 - 1 = 7
4 is EVEN, is a SQUARE, and prime factors are 2 x 2
5 is ODD, is PRIME and generates MERSENNE PRIME 2^5 - 1 = 31
6 is EVEN, and prime factors are 2 x 3. *PERFECT*
7 is ODD, is a MERSENNE PRIME and generates MERSENNE PRIME 2^7 - 1 = 127
8 is EVEN, is a CUBE, and prime factors are 2 x 2 x 2
9 is ODD, is a SQUARE, and prime factors are 3 x 3
10 is EVEN, and prime factors are 2 x 5
11 is ODD, is PRIME
12 is EVEN, and prime factors are 2 x 2 x 3
13 is ODD, is PRIME and generates MERSENNE PRIME 2^13 - 1 = 8191
14 is EVEN, and prime factors are 2 x 7
15 is ODD, and prime factors are 3 x 5
16 is EVEN, is a SQUARE, and prime factors are 2 x 2 x 2 x 2
17 is ODD, is PRIME and generates MERSENNE PRIME 2^17 - 1 = 131071
18 is EVEN, and prime factors are 2 x 3 x 3
19 is ODD, is PRIME and generates MERSENNE PRIME 2^19 - 1 = 524287
20 is EVEN, and prime factors are 2 x 2 x 5
21 is ODD, and prime factors are 3 x 7
22 is EVEN, and prime factors are 2 x 11
23 is ODD, is PRIME
24 is EVEN, and prime factors are 2 x 2 x 2 x 3
25 is ODD, is a SQUARE, and prime factors are 5 x 5
26 is EVEN, and prime factors are 2 x 13
27 is ODD, is a CUBE, and prime factors are 3 x 3 x 3
28 is EVEN, and prime factors are 2 x 2 x 7. *PERFECT*
29 is ODD, is PRIME
30 is EVEN, and prime factors are 2 x 3 x 5
31 is ODD, is a MERSENNE PRIME and generates MERSENNE PRIME 2^31 - 1 = 2147483647
32 is EVEN, and prime factors are 2 x 2 x 2 x 2 x 2
33 is ODD, and prime factors are 3 x 11
34 is EVEN, and prime factors are 2 x 17
35 is ODD, and prime factors are 5 x 7
36 is EVEN, is a SQUARE, and prime factors are 2 x 2 x 3 x 3
37 is ODD, is PRIME
38 is EVEN, and prime factors are 2 x 19
39 is ODD, and prime factors are 3 x 13
40 is EVEN, and prime factors are 2 x 2 x 2 x 5
41 is ODD, is PRIME
42 is EVEN, and prime factors are 2 x 3 x 7
43 is ODD, is PRIME
44 is EVEN, and prime factors are 2 x 2 x 11
45 is ODD, and prime factors are 3 x 3 x 5
46 is EVEN, and prime factors are 2 x 23
47 is ODD, is PRIME
48 is EVEN, and prime factors are 2 x 2 x 2 x 2 x 3
49 is ODD, is a SQUARE, and prime factors are 7 x 7
50 is EVEN, and prime factors are 2 x 5 x 5
51 is ODD, and prime factors are 3 x 17
52 is EVEN, and prime factors are 2 x 2 x 13
53 is ODD, is PRIME
54 is EVEN, and prime factors are 2 x 3 x 3 x 3
55 is ODD, and prime factors are 5 x 11
56 is EVEN, and prime factors are 2 x 2 x 2 x 7
57 is ODD, and prime factors are 3 x 19
58 is EVEN, and prime factors are 2 x 29
59 is ODD, is PRIME
60 is EVEN, and prime factors are 2 x 2 x 3 x 5
61 is ODD, is PRIME
62 is EVEN, and prime factors are 2 x 31
63 is ODD, and prime factors are 3 x 3 x 7
64 is EVEN, is a SQUARE, is a CUBE, and prime factors are 2 x 2 x 2 x 2 x 2 x 2
65 is ODD, and prime factors are 5 x 13
66 is EVEN, and prime factors are 2 x 3 x 11
67 is ODD, is PRIME
68 is EVEN, and prime factors are 2 x 2 x 17
69 is ODD, and prime factors are 3 x 23
70 is EVEN, and prime factors are 2 x 5 x 7
71 is ODD, is PRIME
72 is EVEN, and prime factors are 2 x 2 x 2 x 3 x 3
73 is ODD, is PRIME
74 is EVEN, and prime factors are 2 x 37
75 is ODD, and prime factors are 3 x 5 x 5
76 is EVEN, and prime factors are 2 x 2 x 19
77 is ODD, and prime factors are 7 x 11
78 is EVEN, and prime factors are 2 x 3 x 13
79 is ODD, is PRIME
80 is EVEN, and prime factors are 2 x 2 x 2 x 2 x 5
81 is ODD, is a SQUARE, and prime factors are 3 x 3 x 3 x 3
82 is EVEN, and prime factors are 2 x 41
83 is ODD, is PRIME
84 is EVEN, and prime factors are 2 x 2 x 3 x 7
85 is ODD, and prime factors are 5 x 17
86 is EVEN, and prime factors are 2 x 43
87 is ODD, and prime factors are 3 x 29
88 is EVEN, and prime factors are 2 x 2 x 2 x 11
89 is ODD, is PRIME
90 is EVEN, and prime factors are 2 x 3 x 3 x 5
91 is ODD, and prime factors are 7 x 13
92 is EVEN, and prime factors are 2 x 2 x 23
93 is ODD, and prime factors are 3 x 31
94 is EVEN, and prime factors are 2 x 47
95 is ODD, and prime factors are 5 x 19
96 is EVEN, and prime factors are 2 x 2 x 2 x 2 x 2 x 3
97 is ODD, is PRIME
98 is EVEN, and prime factors are 2 x 7 x 7
99 is ODD, and prime factors are 3 x 3 x 11
100 is EVEN, is a SQUARE, and prime factors are 2 x 2 x 5 x 5

**Prime factorization and number type of the first 100 integers.**