

## Derivatives and integrals of hyperbolic functions

$\frac{dy}{dx}$	$y = f(x)$	$\int f(x)dx$
$\frac{dy}{dx} = a \sinh ax$	$y = \cosh ax$	$\int \sinh ax dx = \frac{1}{a} \sinh ax + c$
$\frac{dy}{dx} = a \cosh ax$	$y = \sinh ax$	$\int \sinh ax dx = \frac{1}{a} \cosh ax + c$
$\frac{dy}{dx} = a \operatorname{sech}^2 ax$	$y = \tanh ax$	$\int \tanh ax dx = \frac{1}{a} \ln(\cosh ax) + c$
$\frac{dy}{dx} = -a \operatorname{sech} ax \times \tanh ax$	$y = \operatorname{sech} ax$	$\int \operatorname{sech} ax dx = \frac{2}{a} \tan^{-1}(e^{ax}) + c$
$\frac{dy}{dx} = -a \operatorname{cosech} ax \times \coth ax$	$y = \operatorname{cosech} ax$	$\int \operatorname{cosech} ax dx = \frac{1}{a} \ln\left(\tanh \frac{1}{2} ax\right) + c$
$\frac{dy}{dx} = -a \operatorname{cosech}^2 ax$	$y = \coth ax$	$\int \coth ax dx = \frac{1}{a} \ln(\sinh ax) + c$
$\frac{dy}{dx} = \frac{a}{\sqrt{a^2 x^2 - 1}}$	$y = \cosh^{-1} ax$	$\int \cosh^{-1} ax dx = x \cosh^{-1} ax - \frac{1}{a} \sqrt{ax-1} \sqrt{ax+1} + c$
$\frac{dy}{dx} = \frac{a}{\sqrt{a^2 x^2 + 1}}$	$y = \sinh^{-1} ax$	$\int \sinh^{-1} ax dx = x \sinh^{-1} ax - \frac{1}{a} \sqrt{a^2 x^2 + 1} + c$
$\frac{dy}{dx} = \frac{a}{1 - a^2 x^2}$	$y = \tanh^{-1} ax$	$\int \tanh^{-1} ax dx = x \tanh^{-1} ax + \frac{1}{2a} \ln(1 - a^2 x^2) + c$

$$\begin{aligned}\cosh x &= \frac{1}{2}(e^x + e^{-x}) \\ \sinh x &= \frac{1}{2}(e^x - e^{-x}) \\ \tanh x &= \frac{\sinh x}{\cosh x}\end{aligned}$$

$$\begin{aligned}\operatorname{sech} x &= \frac{1}{\cosh x} \\ \operatorname{cosech} x &= \frac{1}{\sinh x} \\ \coth x &= \frac{1}{\tanh x}\end{aligned}$$

$$\begin{aligned}\sinh^{-1} x &= \ln(x + \sqrt{x^2 + 1}) \\ \cosh^{-1} x &= \ln(x + \sqrt{x^2 - 1}) \\ \tanh^{-1} x &= \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)\end{aligned}$$

$$\begin{aligned}\operatorname{sech}^{-1} x &= \cosh^{-1} \frac{1}{x} \\ \operatorname{cosech}^{-1} x &= \sinh^{-1} \frac{1}{x} \\ \coth^{-1} x &= \tanh^{-1} \frac{1}{x}\end{aligned}$$

$$\begin{aligned}\cosh^2 x - \sinh^2 x &= 1 \\ \operatorname{sech}^2 x + \tanh^2 x &= 1 \\ \coth^2 x - \operatorname{cosech}^2 x &= 1\end{aligned}$$

$$\begin{aligned}\sinh 2x &= 2 \sinh x \cosh x \\ \cosh 2x &= \cosh^2 x + \sinh^2 x\end{aligned}$$

$$\begin{aligned}\sinh(A \pm B) &= \sinh A \cosh B \pm \cosh A \sinh B \\ \cosh(A \pm B) &= \cosh A \cosh B \pm \sinh A \sinh B \\ \tanh(A \pm B) &= \frac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}\end{aligned}$$

$$\begin{aligned}\cosh^2 x &= \frac{1}{2}(\cosh 2x + 1) \\ \sinh^2 x &= \frac{1}{2}(\cosh 2x - 1) \\ \cosh^3 x &= \frac{1}{4}(3 \cosh x + \cosh 3x) \\ \sinh^3 x &= \frac{1}{4}(-3 \sinh x + \sinh 3x)\end{aligned}$$

$$y = \cosh^{-1} ax$$

$$\cosh y = ax$$

$$\sinh y = a \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{a}{\sinh y}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\therefore \sinh y = \sqrt{\cosh^2 y - 1} = \sqrt{a^2 x^2 - 1}$$

$$\frac{d}{dx} \cosh^{-1} ax = \frac{a}{\sqrt{a^2 x^2 - 1}}$$

$$\int \cosh^{-1} ax dx = \int (\cosh^{-1} ax \times 1) dx$$

$$\int \cosh^{-1} ax dx = x \cosh^{-1} ax - \int \left( x \times \frac{d}{dx} \cosh^{-1} ax \right) dx$$

$$\int \cosh^{-1} ax dx = x \cosh^{-1} ax - \int \frac{ax}{\sqrt{a^2 x^2 - 1}} dx$$

$$\frac{d}{dx} \sqrt{a^2 x^2 - 1} = \frac{1}{2} \frac{2xa^2}{\sqrt{a^2 x^2 - 1}} = \frac{a^2 x}{\sqrt{a^2 x^2 - 1}}$$

$$\therefore \int \frac{ax}{\sqrt{a^2 x^2 - 1}} dx = \frac{1}{a} \sqrt{a^2 x^2 - 1} = \frac{1}{a} \sqrt{ax-1} \sqrt{ax+1}$$

$$\int \cosh^{-1} ax dx = x \cosh^{-1} ax - \frac{1}{a} \sqrt{ax-1} \sqrt{ax+1} + c$$

$$\int \tanh ax dx = \int \frac{\sinh ax}{\cosh ax} dx$$

$$\int \tanh ax dx = \frac{1}{a} \int \frac{ae^{ax} - ae^{-ax}}{e^{ax} + e^{-ax}} dx$$

$$\int \tanh ax dx = \frac{1}{a} \ln |e^{ax} + e^{-ax}| + c$$

$$\int \tanh ax dx = \frac{1}{a} \ln |\cosh ax| + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

Use integration by parts

$$\int (uv) dx = u \int v dx - \int \left( \int v dx \times \frac{du}{dx} \right) dx$$

$$u = \tan^{-1} x, \quad v = 1$$

$$\int \operatorname{sech} ax dx = \int \frac{1}{\frac{1}{2}(e^{ax} + e^{-ax})} dx = \int \frac{2e^{ax}}{(e^{ax})^2 + 1} dx$$

$$z = e^{ax} \therefore \frac{dz}{dx} = ae^{ax} \therefore dx = \frac{dz}{ae^{ax}} = \frac{1}{a} \frac{dz}{z}$$

$$\int \operatorname{sech} ax dx = \int \frac{2z}{z^2 + 1} \frac{1}{a} \frac{dz}{z} = \frac{2}{a} \int \frac{1}{1 + z^2} dz = \frac{2}{a} \tan^{-1} z$$

$$\int \operatorname{sech} ax dx = \frac{2}{a} \tan^{-1} e^{ax} + c$$

$$\int \operatorname{cosech} ax dx = \int \frac{1}{\frac{1}{2}(e^{ax} - e^{-ax})} dx = \int \frac{2e^{ax}}{(e^{ax})^2 - 1} dx$$

$$z = e^{ax} \therefore \frac{dz}{dx} = ae^{ax} \therefore dx = \frac{dz}{ae^{ax}} = \frac{1}{a} \frac{dz}{z}$$

$$\int \operatorname{cosech} ax dx = \int \frac{2z}{z^2 - 1} \frac{1}{a} \frac{dz}{z} = \frac{2}{a} \int \frac{1}{z^2 - 1} dz$$

$$\frac{1}{z^2 - 1} = \frac{1}{2} \frac{1}{z-1} - \frac{1}{2} \frac{1}{z+1}$$

$$\frac{2}{a} \int \frac{1}{z^2 - 1} dz = \frac{1}{a} \int \frac{1}{z-1} dz - \frac{1}{a} \int \frac{1}{z+1} dz = \frac{1}{a} \ln \left( \frac{z-1}{z+1} \right)$$

$$\int \operatorname{cosech} ax dx = \frac{1}{a} \ln \left( \frac{e^{ax} - 1}{e^{ax} + 1} \right) = \frac{1}{a} \ln \left( \frac{e^{\frac{1}{2}ax} (e^{\frac{1}{2}ax} - e^{-\frac{1}{2}ax})}{e^{\frac{1}{2}ax} (e^{\frac{1}{2}ax} + e^{-\frac{1}{2}ax})} \right)$$

$$\int \operatorname{cosech} ax dx = \frac{1}{a} \ln \left( \tanh \frac{1}{2} ax \right) + c$$

$$\int \coth ax dx = \int \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}} dx = \frac{1}{a} \int \frac{a(e^{ax} - e^{-ax})}{e^{ax} + e^{-ax}} dx$$

$$\frac{d}{dx} (e^{ax} + e^{-ax}) = a(e^{ax} - e^{-ax})$$

$$\int \coth ax dx = \frac{1}{a} \ln (e^{ax} - e^{-ax}) = \frac{1}{a} \ln (\sinh ax) + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$