

Powers of 2	Powers of 3	Powers of 5	Squares	Cubes
$2^0 = 1$	$3^0 = 1$	$5^0 = 1$	$1^2 = 1$	$1^3 = 1$
$2^1 = 2$	$3^1 = 3$	$5^1 = 5$	$2^2 = 4$	$2^3 = 8$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$	$3^2 = 9$	$3^3 = 27$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$	$4^2 = 16$	$4^3 = 64$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$	$5^2 = 25$	$5^3 = 125$
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$	$6^2 = 36$	$6^3 = 216$
$2^6 = 64$	$3^6 = 729$		$7^2 = 49$	$7^3 = 343$
$2^7 = 128$	$3^7 = 2187$		$8^2 = 64$	$8^3 = 512$
$2^8 = 256$	$3^8 = 6561$		$9^2 = 81$	$9^3 = 729$
$2^9 = 512$			$10^2 = 100$	$10^3 = 1000$
$2^{10} = 1024$			$11^2 = 121$	$11^3 = 1331$
$2^{11} = 2048$			$12^2 = 144$	$12^3 = 1728$
$2^{12} = 4096$			$13^2 = 169$	$13^3 = 2197$
$2^{13} = 8192$			$14^2 = 196$	$14^3 = 2744$
$2^{14} =$ 16,384			$15^2 = 225$	$15^3 = 3375$
$2^{14} =$ 32,768			$16^2 = 256$	$16^3 = 3375$
$2^{16} =$ 65,536			$17^2 = 289$	$17^3 = 4913$
			$18^2 = 324$	$18^3 = 5832$
			$19^2 = 361$	$19^3 = 6859$
$2^{24} =$ 16,777,216			$20^2 = 400$	$20^3 = 8000$
$2^{32} =$ 4,294,967,296				

**Learn these
squares,
cubes
& powers!**

Laws of indices

$$10^3 = 10 \times 10 \times 10 = 1,000 \quad \text{powers}$$

$$10^{-4} = \frac{1}{10^4} = \frac{1}{10,000} \quad \text{meaning of negative powers}$$

$$10^2 \times 10^5 = 10^{2+5} = 10^7 = 10,000,000$$

power add

$$10^2 \div 10^5 = \frac{10^2}{10^5} = 10^2 \times 10^{-5} = 10^{-3} = \frac{1}{1,000}$$

power subtract

$$(10^3)^4 = 10^{3 \times 4} = 10^{12} \quad \text{power multiply}$$

$$\sqrt{10^4} = (10^4)^{\frac{1}{2}} = 10^2 = 100 \quad \text{roots or 'surds'}$$

$$\sqrt[3]{10^{18}} = (10^{18})^{\frac{1}{3}} = 10^6 = 1,000,000$$

$$b^x \times b^y = b^{x+y}$$

$$(b^x)^y = b^{xy}$$

$$b^{-x} = \frac{1}{b^x}$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

Generalize the laws
of indices for *any*
numbers b, x, y

n must be > 0

multiply grid for 1 : 20. Rule = cube.
Number of elements obeying rule = 12 (3%)

20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

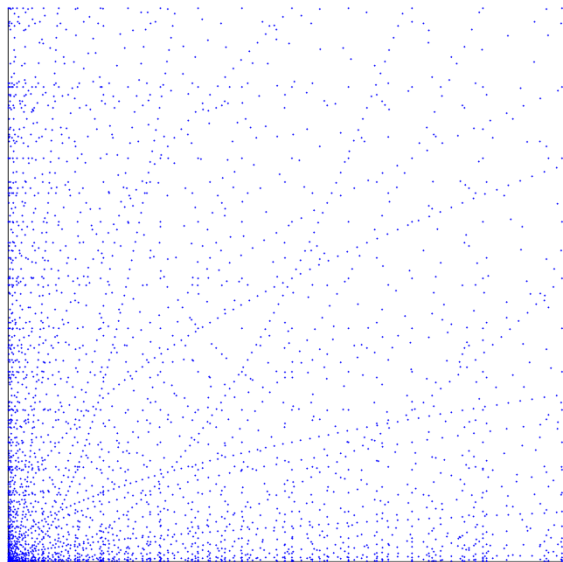
multiply grid for 1 : 20. Rule = square.
Number of elements obeying rule = 42 (10.5%)

20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

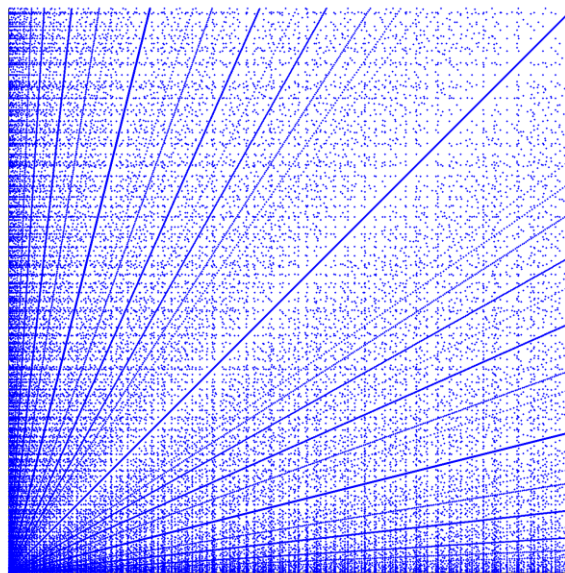
SQUARES, CUBES ...

n	n ²	n ³	n ⁴	n ⁵
0	0	0	0	0
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1,024
5	25	125	625	3,125
6	36	216	1,296	7,776
7	49	343	2,401	
8	64	512	4,096	
9	81	729	6,561	
10	100	1,000	10,000	
11	121	1,331		
12	144	1,728		
13	169	2,197		
14	196	2,744		
15	225	3,375		
16	256	4,096		
17	289	4,913		
18	324	5,832		
19	361	6,859		
20	400	8,000		

multiply grid for 1 : 8000. Rule = cube highlighted
Number of elements obeying rule = 3154 (0.0049281%)



multiply grid for 1 : 8000. Rule = square highlighted
Number of elements obeying rule = 44872 (0.070112%)

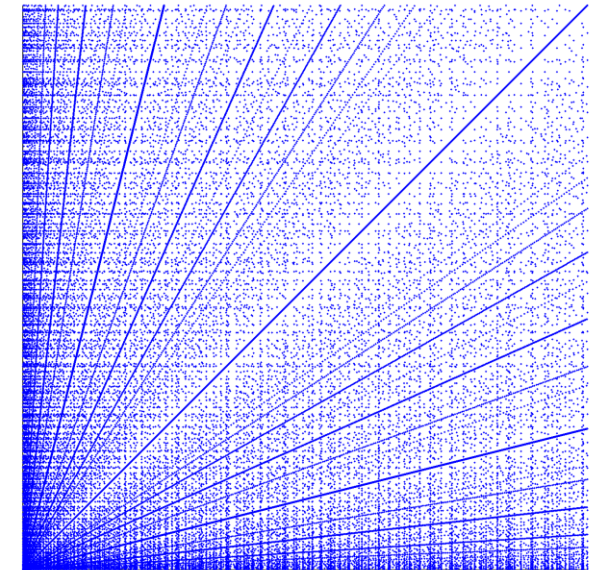


8100 90^2	8281 91^2	8464 92^2	8649 93^2	8836 94^2	9025 95^2	9216 96^2	9409 97^2	9604 98^2	9801 99^2
6400 80^2	6561 81^2	6724 82^2	6889 83^2	7056 84^2	7225 85^2	7396 86^2	7569 87^2	7744 88^2	7921 89^2
4900 70^2	5041 71^2	5184 72^2	5329 73^2	5476 74^2	5625 75^2	5776 76^2	5929 77^2	6084 78^2	6241 79^2
3600 60^2	3721 61^2	3844 62^2	3969 63^2	4096 64^2	4225 65^2	4356 66^2	4489 67^2	4624 68^2	4761 69^2
2500 50^2	2601 51^2	2704 52^2	2809 53^2	2916 54^2	3025 55^2	3136 56^2	3249 57^2	3364 58^2	3481 59^2
1600 40^2	1681 41^2	1764 42^2	1849 43^2	1936 44^2	2025 45^2	2116 46^2	2209 47^2	2304 48^2	2401 49^2
900 30^2	961 31^2	1024 32^2	1089 33^2	1156 34^2	1225 35^2	1296 36^2	1369 37^2	1444 38^2	1521 39^2
400 20^2	441 21^2	484 22^2	529 23^2	576 24^2	625 25^2	676 26^2	729 27^2	784 28^2	841 29^2
100 10^2	121 11^2	144 12^2	169 13^2	196 14^2	225 15^2	256 16^2	289 17^2	324 18^2	361 19^2
x^2 squares	1 1^2	4 2^2	9 3^2	16 4^2	25 5^2	36 6^2	49 7^2	64 8^2	81 9^2

multiply grid for 1 : 20. Rule = square.
Number of elements obeying rule = 42 (10.5%)

20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

multiply grid for 1 : 8000. Rule = square highlighted
Number of elements obeying rule = 44872 (0.070112%)

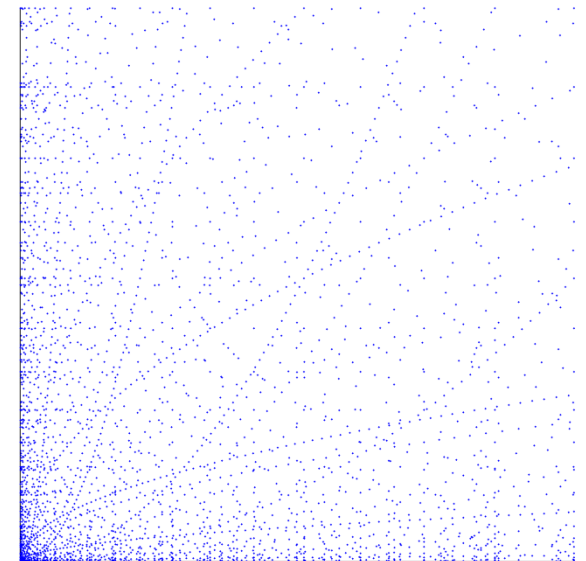


729000 90^3	<u>753571</u> 91^3	778688 92^3	<u>804357</u> 93^3	830584 94^3	<u>857375</u> 95^3	884736 96^3	<u>912673</u> 97^3	941192 98^3	<u>970299</u> 99^3
512000 80^3	$729^2=9^6$ <u>531441</u> 81^3	551368 82^3	<u>571787</u> 83^3	592704 84^3	<u>614125</u> 85^3	636056 86^3	<u>658503</u> 87^3	681472 88^3	<u>704969</u> 89^3
343000 70^3	<u>357911</u> 71^3	373248 72^3	<u>389017</u> 73^3	405224 74^3	<u>421875</u> 75^3	438976 76^3	<u>456533</u> 77^3	474552 78^3	<u>493039</u> 79^3
216000 60^3	<u>226981</u> 61^3	238328 62^3	<u>250047</u> 63^3	$512^2=8^6$ 262144 64^3	<u>274625</u> 65^3	287496 66^3	<u>300763</u> 67^3	314432 68^3	<u>328509</u> 69^3
125000 50^3	<u>132651</u> 51^3	140608 52^3	<u>148877</u> 53^3	157464 54^3	<u>166375</u> 55^3	175616 56^3	<u>185193</u> 57^3	195112 58^3	<u>205379</u> 59^3
64000 40^3	<u>68921</u> 41^3	74088 42^3	<u>79507</u> 43^3	85184 44^3	<u>91125</u> 45^3	97336 46^3	<u>103823</u> 47^3	110592 48^3	$343^2=7^6$ <u>117649</u> 49^3
27000 30^3	<u>29791</u> 31^3	32768 32^3	<u>35937</u> 33^3	39304 34^3	<u>42875</u> 35^3	$216^2=6^6$ 46656 36^3	<u>50653</u> 37^3	54872 38^3	<u>59319</u> 39^3
8000 20^3	<u>9261</u> 21^3	10648 22^3	<u>12167</u> 23^3	13824 24^3	$125^2=5^6$ <u>15625</u> 25^3	17576 26^3	<u>19683</u> 27^3	21952 28^3	<u>24389</u> 29^3
1000 10^3	<u>1331</u> 11^3	1728 12^3	<u>2197</u> 13^3	2744 14^3	<u>3375</u> 15^3	$64^2=4^6$ 4096 16^3	<u>4913</u> 17^3	5832 18^3	<u>6859</u> 19^3
x^3 cubes	$1^2=1^6$ <u>1</u> 1^3	8 2^3	<u>27</u> 3^3	$8^2=2^6$ 64 4^3	<u>125</u> 5^3	216 6^3	<u>343</u> 7^3	512 8^3	$27^2=3^6$ <u>729</u> 9^3

multiply grid for 1 : 20. Rule = cube.
Number of elements obeying rule = 12 (3%)

20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

multiply grid for 1 : 8000. Rule = cube highlighted
Number of elements obeying rule = 3154 (0.0049281%)



More mathematical powers!

N¹	2	3	4	5	6	7	8
N²	4	9	16	25	36	49	64
N³	8	27	64	125	216	343	512
N⁴	16	81	256	625	1,296	2,401	4,096
N⁵	32	243	1,024	3,125	7,776	16,807	32,768
N⁶	64	729	4,096	15,625	46,656	117,649	262,144
N⁷	128	2,187	16,384	78,125	279,936	823,543	2,097,152
N⁸	256	6,561	65,536	390,625	1,679,616	5,764,801	16,777,216

N¹	9	11	12	13	14	15	16
N²	81	121	144	169	196	225	256
N³	729	1,331	1,728	2,197	2,744	3,375	4,096
N⁴	6,561	14,641	20,736	28,561	38,416	50,625	65,536
N⁵	59,049	161,051	248,832	371,293	537,824	759,375	1,048,576
N⁶	531,441	1,771,561	2,985,984	4,826,809	7,529,536	11,390,625	16,777,216
N⁷	4,782,969	19,487,171	35,831,808	62,748,517	105,413,504	170,859,375	268,435,456
N⁸	43,046,721	214,358,881	429,981,696	815,730,721	1,475,789,056	2,562,890,625	4,294,967,296