

Loci & Constructions

"Construct" ?

Strictly speaking, it means to use *only* a straight edge and a compass. We can often soften this hard rule by allowing *measurement* (i.e. via a ruler along straight lines or via a *protractor* for angles).

What is a **scale drawing**?

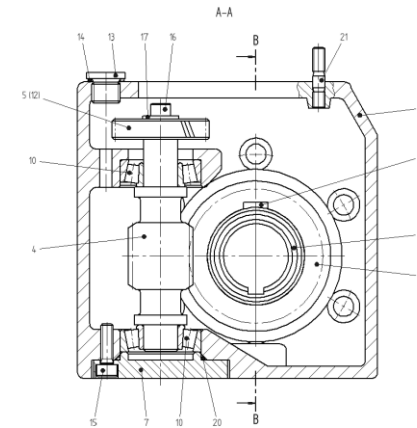
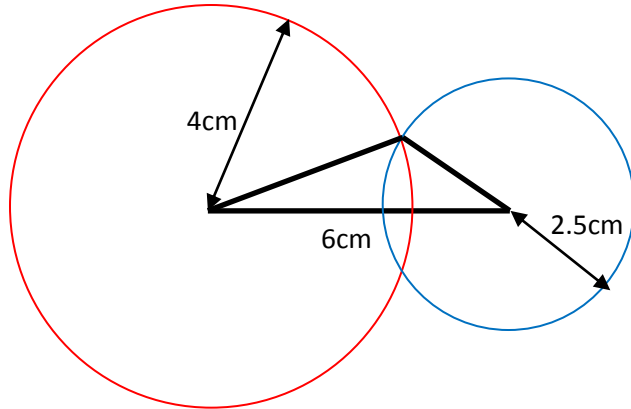
The construction of an *enlargement* of what the drawing represents in reality. All the proportions of shapes within the drawing are the same as in reality. Scale drawings are a fundamental component of design, from a transistor in a microprocessor to a set or architectural drawings, to a map of the world.

What does **loci** mean?

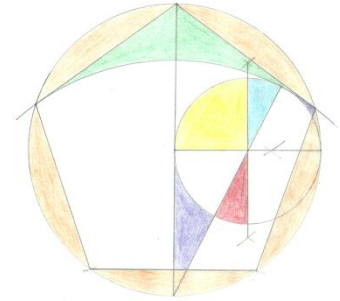
The *locus* of a point is "*the path it describes as it moves*", or "*the set the of all points that share a particular geometrical property*". So the locus of all points that are 10 units from (3,-2) describe a *circle* of centre (3,-2) and radius 10.

Constructing a triangle from three known sides

- To construct a triangle of sides 6cm, 2.5cm and 4cm, firstly use a ruler to measure out and draw a side. In the example below we have picked 6cm.
- Set a compass to one of the other side lengths using a ruler and draw an arc (or if needed an entire circle) centred on one of the ends of the line drawn in step 1.
- Repeat step 2 for the other side.
- Where the circular arcs join will be the third vertex of the triangle. Use a ruler to join this to the ends of the line drawn in step 1 to complete the scale drawing of the triangle.

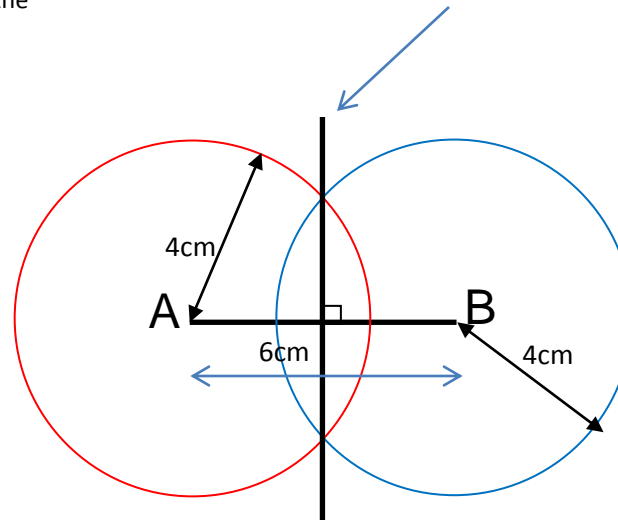


An engineering drawing of a piece of equipment constructed via Computer Aided Design (CAD)



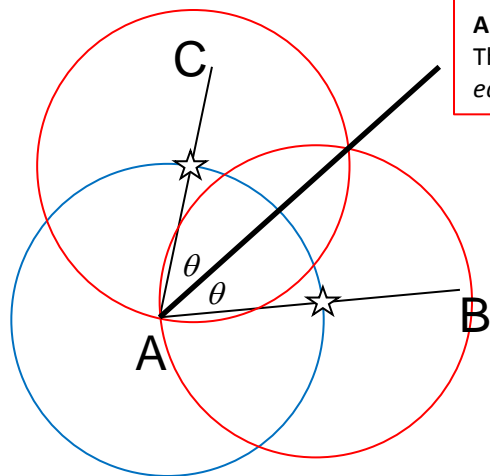
Construction of a regular pentagon via straight edge and compass only

Perpendicular bisector of line AB
This is the locus of points which are *equidistant* from points A and B



Bisect a straight line

- Draw the line to be bisected.
- Set your compass to have a radius that is *just beyond the mid-point* of the line.
- Draw two arcs (or full circles) from the ends of the line. Take care not to accidentally change the angle of your compass.
- Join the two circle intersections with a ruler to form a straight line. This is the *perpendicular bisector*.

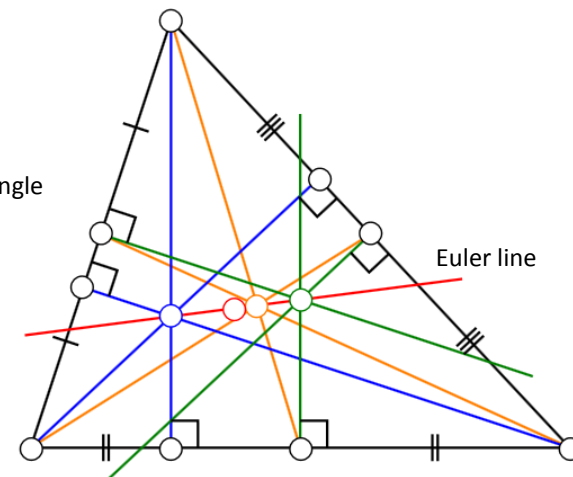
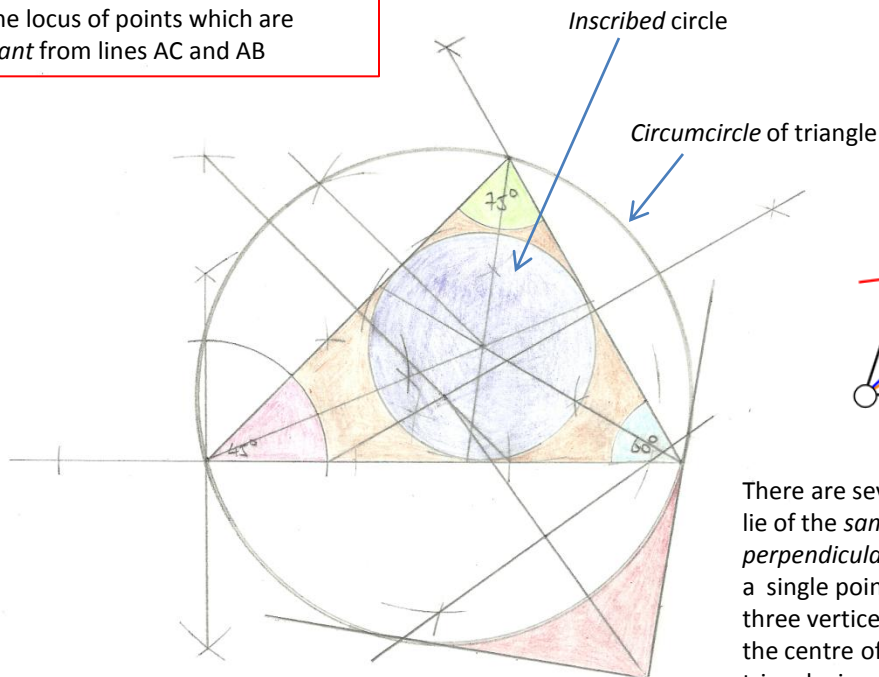
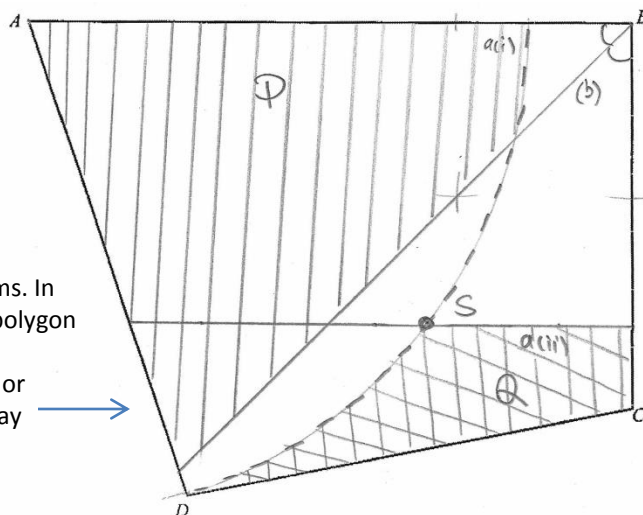


Angle bisector of angle \hat{CAB}
This is the locus of points which are equidistant from lines AC and AB

Bisect an angle

- Draw a set of lines with the desired angle that needs to be bisected.
- Centre a compass on the vertex of the two lines (A in this case) and draw a circle.
- Using the same opening angle for the compass, draw two further circles centred on the points (marked above with a star) where the circle drawn in step 2 intersects the lines drawn in step 1.
- Using a ruler, connect the point where these circles intersect with the vertex. The angle between the lines can now be bisected.

Construction skills can be applied to practical problems. In this example, region P is defined to be (i) within the polygon ABCD; (ii) Less than the distance AD from A; (iii) closer to line AB than line BC. Region Q is greater or equal to a fixed distance from line AB and further away from A than distance AD. Point S is on the horizontal boundary of Q and exactly AD away from A.



There are several 'mid-points' of a triangle, but they all lie of the *same* line (the *Euler line*). If one constructs *perpendicular bisectors* of each edge, they will meet at a single point. This point will be equidistant from all three vertices of the triangle. Hence this point will be the centre of a circle which will *circumscribe* the triangle, i.e. pass through all three vertices.

If one instead constructs *angle bisectors* of each angle, these lines will meet at a different point. If one constructs a perpendicular from this point to each edge, one shall discover this distance is the *same*. The radius of a circle with this distance, centred on the 'angle bisector intersection point' will be the largest circle which can fit entirely within the circle. It is said to be *inscribed* within the triangle.