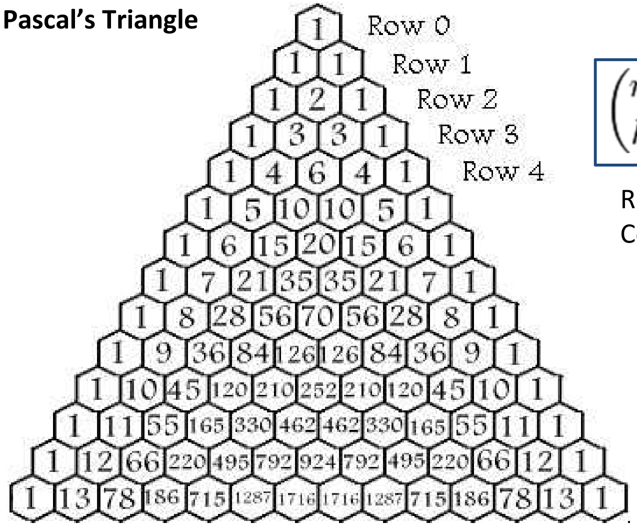
Pascal's Triangle

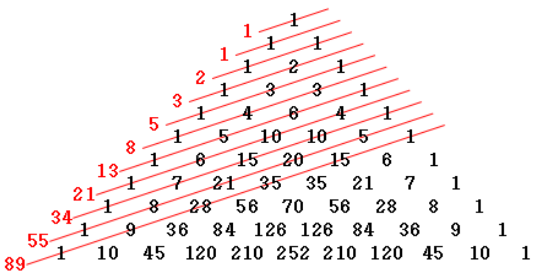


$$\boxed{\binom{n}{k} = \frac{n!}{k!(n-k)!}}$$

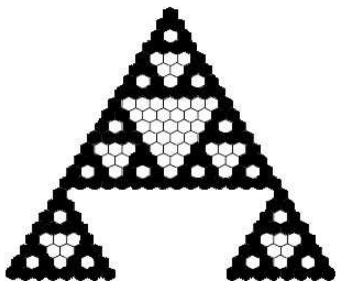
Row n
Column k

Blaise Pascal 1623-1662

- Concatenate row digits. Answer is 11^n. e.g. $1331 = 11^3$ $14641 = 11^4$
- If first (not 1) number in row n is prime, all other numbers are divisible by it
- Sum of numbers in rows = 2^n
- Generate the square numbers: $1+3 = 2^2$, $3 + 6 = 3^2$, $6 + 10 = 4^2$ ....
- Sum of 'shallow diagonals' gives the Fibonacci numbers $F_n = F_{n-1} + F_{n-2}$



- Odd numbers for the Sierpinski Gasket



Binomial expansion

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

Pascal's Triangle yields **Binomial Coefficients**

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k}$$

$(3x-4)^3 = (3x)^3 + 3(3x)^2(-4) + 3(3x)(-4)^2 + (-4)^3$

$= 27x^3 - 12(9)x^2 + 9(16)x - 64$

$= \boxed{27x^3 - 108x^2 + 144x - 64}$

$(2x+1)^4 = (2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x) + 1$

$= \boxed{16x^4 + 32x^3 + 24x^2 + 8x + 1}$

$\left(2x - \frac{3}{x}\right)^6$ has constant term

$= 20(2x)^3\left(-\frac{3}{x}\right)^3$

$= -8x^3(27)(20)\frac{1}{x^3} = \boxed{-4320}$

$(1+x)^{20} = \binom{20}{0}(1)^{20}x^0 + \binom{20}{1}(1)^{19}x + \binom{20}{2}(1)^{18}x^2 + \binom{20}{3}(1)^{17}x^3 + \ldots$

$\binom{20}{0} = 1$

$\binom{20}{1} = \frac{20!}{1!19!} = 20$

$\binom{20}{2} = \frac{20!}{2!18!} = \frac{20 \times 19}{2} = 190$

$\binom{20}{3} = \frac{20!}{3!17!} = \frac{20 \times 19 \times 18}{3 \times 2} = 10 \times 19 \times 6 = 60 \times 19 = 1140$

$\therefore \quad (1+x)^{20} = \boxed{1 + 20x + 190x^2 + 1140x^3 + ..}$

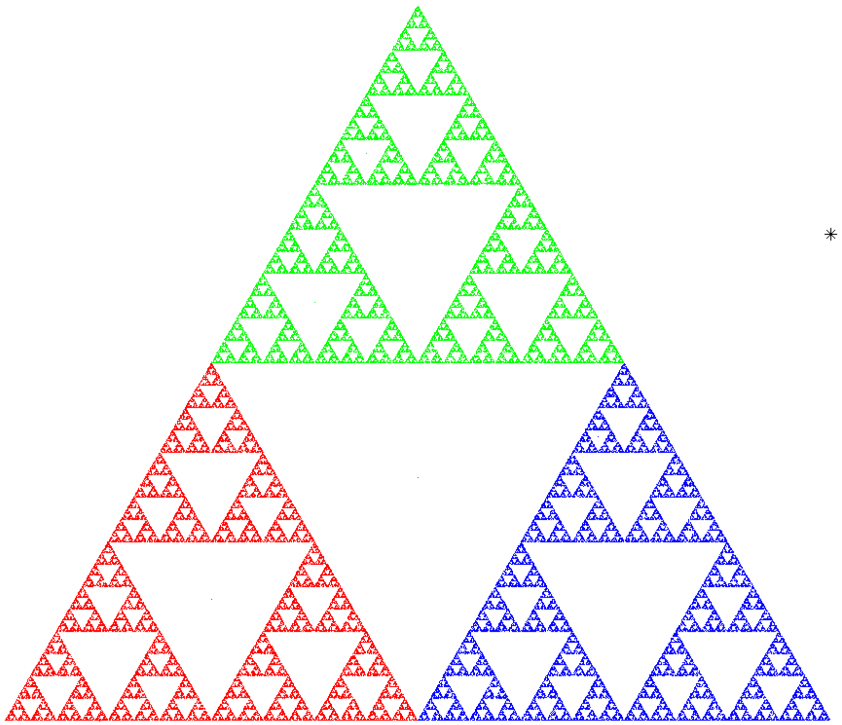
$\left(x^2 - \frac{2}{\sqrt{x}}\right)^4 = \binom{4}{0}(x^2)^4\left(-2x^{-\frac{1}{2}}\right)^0 + \binom{4}{1}(x^2)^3\left(-2x^{-\frac{1}{2}}\right)^1 + \ldots$

$+\ldots\binom{4}{2}(x^2)^2\left(-2x^{-\frac{1}{2}}\right)^2 + \binom{4}{3}(x^2)^1\left(-2x^{-\frac{1}{2}}\right)^3 + \binom{4}{4}(x^2)^0\left(-2x^{-\frac{1}{2}}\right)^4$

$= x^8 + 4x^6(-2x^{-\frac{1}{2}}) + 6x^4(4x^{-1}) + 4x^2(-8x^{-\frac{3}{2}}) + 16x^{-2}$

$= \boxed{x^8 - 8x^{\frac{11}{2}} + 24x^3 - 32x^{\frac{1}{2}} + 16x^{-2}}$

Sierpinski triangle via chaos game.
N = 100000, p = [0.33333 0.33333 0.33333].



Generalized Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k$$

$$\binom{n}{k}=\frac{n!}{k!(n-k)!}$$

$$(1+x)^n = \binom{n}{0}(1)^n x^0 + \binom{n}{1}(1)^{n-1}x^1 + \binom{n}{2}(1)^{n-2}x^2 + \dots$$

$$(1+x)^n = 1+nx+\frac{n!}{(n-2)!2!}x^2+\frac{n!}{(n-3)!3!}x^3+\dots$$

$$(1+x)^n = 1+nx+\frac{n(n-1)(n-2)!}{(n-2)!2!}x^2+\frac{n(n-1)(n-2)(n-3)!}{(n-3)!3!}x^3+\dots$$

$$\boxed{(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\dots}$$

This is *convergent* if the higher terms diminish in size i.e. $\boxed{|x|<1}$

This *Generalized Binomial Theorem* can be extended to values of *n* other than positive integers.
It is therefore a very useful tool in *approximating* complex expressions.

$$\left(1-\frac{1}{\sqrt{x}}\right)^{-\frac{1}{3}} = 1+\left(-\tfrac{1}{3}\right)\left(-x^{-\frac{1}{2}}\right)+\left(-\tfrac{1}{3}\right)\left(-\tfrac{4}{3}\right)\tfrac{1}{2!}\left(-x^{-\frac{1}{2}}\right)^2+\dots$$

$$+\left(-\tfrac{1}{3}\right)\left(-\tfrac{4}{3}\right)\left(-\tfrac{7}{3}\right)\tfrac{1}{3!}\left(-x^{-\frac{1}{2}}\right)^3+\dots$$

$$\boxed{\left(1-\frac{1}{\sqrt{x}}\right)^{-\frac{1}{3}} = 1+\tfrac{1}{3}x^{-\frac{1}{2}}+\tfrac{2}{9}x^{-1}+\tfrac{14}{81}x^{-\frac{3}{2}}+\dots}$$

where $\left|\frac{1}{\sqrt{x}}\right|<1 \Rightarrow \boxed{x>1}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1 19 171 969 3876 11628 27132 50388 75582 92378 92378 75582 50388 27132 11628 3876 969 171 19 1
1 20 190 1140 4845 15504 38760 77520 125970 167960 184756 167960 125970 77520 38760 15504 4845 1140 190 20 1
1 21 210 1330 5985 20349 54264 116280 203490 293930 352716 352716 293930 203490 116280 54264 20349 5985 1330 210 21 1
1 22 231 1540 7315 26334 74613 170544 319770 497420 646646 705432 646646 497420 319770 170544 74613 26334 7315 1540 231 22 1
1 23 253 1771 8855 33649 100947 245157 490314 817190 1144066 1352078 1352078 1144066 817190 490314 245157 100947 33649 8855 1771 253 23 1
1 24 276 2024 10626 42504 134596 346104 735471 1307504 1961256 2496144 2704156 2496144 1961256 1307504 735471 346104 134596 42504 10626 2024 276 24 1
1 25 300 2300 12650 53130 177100 480700 1081575 2042975 3268760 4457400 5200300 5200300 4457400 3268760 2042975 1081575 480700 177100 53130 12650 2300 300 25 1
1 26 325 2600 14950 65780 230230 657800 1562275 3124550 5311735 7726160 9657700 10400600 9657700 7726160 5311735 3124550 1562275 657800 230230 65780 14950 2600 325 26 1
1 27 351 2925 17550 80730 296010 888030 2220075 4686825 8436285 13037895 17383860 20058300 20058300 17383860 13037895 8436285 4686825 2220075 888030 296010 80730 17550 2925 351 27 1
1 28 378 3276 20475 98280 376740 1184040 3108105 6906900 13123110 21474180 30421755 37442160 40116600 37442160 30421755 21474180 13123110 6906900 3108105 1184040 376740 98280 20475 3276 378 28 1
1 29 406 3654 23751 118755 475020 1560780 4292145 10015005 20030010 34597290 51895935 67863915 77558760 77558760 67863915 51895935 34597290 20030010 10015005 4292145 1560780 475020 118755 23751 3654 406 29 1

$$\sqrt{1+x} = 1+\tfrac{1}{2}x+\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\tfrac{1}{2!}x^2+\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\tfrac{1}{3!}x^3+\dots$$

$$\boxed{\sqrt{1+x} = 1+\tfrac{1}{2}x-\tfrac{1}{8}x^2+\tfrac{1}{16}x^3+\dots}$$

$$\sqrt{1.1}\approx 1+0.05-\tfrac{0.01}{8}+\tfrac{0.001}{16}=\boxed{1.0488125}$$

$$\sqrt{1.1}=1.04880884817$$