Electrical current I in Amps (A) is the rate of flow of **charge**. Charge Q is measured in ***coloumbs*** (C) and, in most practical scenarios regarding electrical circuits, ***electrons*** are flowing within a *conductor*. The charge associated with one a electron is $-e = -1.602 \times 10^{-19}\,\mathrm{C}$.

Note for various historical reasons (the electron was not discovered until 1897, by J.J. Thomson), ***current is the flow of positive charge***, which we may interpret as the 'holes' of charge left behind following the flow of negatively charged electrons. In an electrical circuit, current flows from the positive terminal of a power supply to the negative, whereas the associated electron flow is in the opposite direction.

Direct Current (DC) implies a uniform and constant rate of charge with time. ***Alternating Current*** (AC) is a sinusoidal charge flow with time. If current has amplitude I_0, frequency f and initial phase ϕ then $I(t) = I_0 \cos\left(2\pi ft - \phi\right)$.

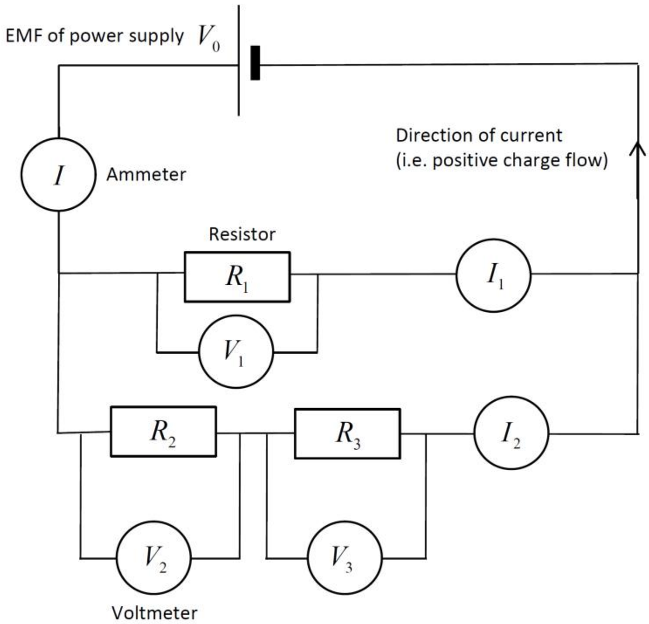
Kirchhoff's First Law applies the idea of *charge conservation* to a circuit. **For any junction of conductions, the total current flowing into a junction must equal the total current flowing out of a junction.**

A ***resistor*** is an element of an electrical circuit that results in energy dissipation when current flows through it. The resistance R (in ohms Ω) of a resistor is defined as the ratio between the energy per unit charge dissipated V (this is called the potential difference, or 'voltage') and the current. i.e. $R = V/I$. [1]

Since V is the **energy E dissipated per unit charge**, $E = QV$. The rate of energy dissipated, i.e. the power loss P, is the energy per unit charge $\times$ the charge flowing per second. Hence: $P = VI = I^2R = V^2/R$.

Note $R = V/I$ is often erroneously called 'Ohm's Law.' An ***ohmic conductor*** (which does obey 'Ohm's Law') is one which has a constant resistance, regardless of the current flowing through it. Common electrical components are *not ohmic*.

A *filament bulb* increases in resistance as current increases. This is due to increased metal lattice vibrations as it gets hotter, resulting in an impeded flow of electrons.



$V_1 = V_0$

$V_2 + V_3 = V_0$ Kirchoff II

$I = I_1 + I_2$ Kirchoff I

$V_1 = I_1R_1\ ;\ V_2 = I_2R_2\ ;\ V_3 = I_2R_3$

$R_{tot} = V_0/I = \left(\frac{1}{R_1} + \frac{1}{R_2+R_3}\right)^{-1}$

A *thermistor* (i.e. a *semiconductor*) will typically reduce in resistance as current increases. In this case, higher temperatures resulting from greater current promote more electrons to a 'conduction band'.

Electrical circuits comprise loops of power supplies, low resistance wires and resistive components. ***Kirchhoff's Second Law*** applies the idea of *conservation of energy* to a circuit. **For any closed loop, the sum of the potential differences must equate to the net electromotive force (EMF) applied to the loop.** The EMF is the voltage (i.e. the energy per unity charge) provided by a power supply such as a mains unit or battery. Note to evaluate Kirchhoff's Second Law, one must define a 'direction' round each loop (i.e. adopt a sign convention), particularly if a loop contains voltage sources that are opposed (and therefore count negatively).

[1] For 'reactive' circuits involving capacitors (conductive plates separated by an insulating 'dielectric') and inductors (coils of conductive wire) this equation generalizes to $V = IZ$ where Z is the *impedance*.

A ***voltmeter*** is a device for measuring the potential difference between two points in a circuit. An ideal voltmeter can be thought to have essentially infinite resistance, and therefore will draw very little current. Voltmeters must therefore be placed in parallel (i.e. 'across a component') rather than in series with a loop.

An ***ammeter*** is a device for measuring the rate of flow of charge and is placed within a circuit loop. (i.e. 'in series'). An ammeter will have very low resistance, and therefore the potential difference across it can be considered negligible.

Kirchhoff's laws can be used to determine how resistances add in *series* and in *parallel* circuits.

In *series* circuits, resistances add: $R_{tot} = R_1 + R_2 + ... + R_N$

For a series circuit, the addition of resistances, and $R_i = V_i / I$, means we can write $V_i = \frac{R_i}{R_1+R_2+...+R_N} V$, where V is the total potential difference $V = \sum_i V_i$. This is the ***potential divider*** idea, and allows series voltages to be computed directly without having to work out the current first. i.e. *"the fraction of total voltage dropped across a resistor is equal to the fraction of the resistor resistance to the total resistance in a series loop."*

In *parallel* circuits, resistances add in *reciprocals*: $1/R_{tot} = 1/R_1 + 1/R_2 + ... + 1/R_N$

These ideas explain how the *resistance* R *of a conductor* varies with cross sectional area A and length l .

$R = \rho l / A$ where ρ is the resistivity, a bulk material property. The resistivity of copper[2] is about $1.68 \times 10^{-8} \Omega\text{m}$. Doubling the length of a conductor of fixed cross-section is like adding two resistors in series. So $R \propto l$. Doubling the cross-section, for a given length, is like adding two resistors in parallel. Hence: $R \propto 1/A$.

The actual voltage V_0 across the terminals of a power supply is unlikely to equate exactly to the EMF once current is drawn through a circuit. There will typically be a non-zero *internal resistance* r such that $V_0 = \varepsilon - Ir$. In other words, we model a power supply as an ideal voltage source ε in series with a fixed resistor r .

Question 1

(i) A current of 1.00nA flows through an ammeter. How many electrons pass in 1.00ns? (Nano 'n' means 10^{-9}).

(ii) A 10 Ω resistor dissipates 2.0W. Calculate the current flowing through the resistor, and the potential difference across it. If the potential difference doubled and the resistance halved, what would happen to the power dissipated?

(iii) A 3k Ω resistor is wired in parallel with a 2k Ω resistor. This grouping is then wired in series with a 1k Ω resistor. What is the total resistance?

(iv) An electron is accelerated in a vacuum tube via a potential difference of 5,000V. Calculate the kinetic energy (in J) gained by the electron. Assume 100% efficiency![3] If the mass of an electron is $m_e = 9.109 \times 10^{-31}\text{kg}$, and the speed of light is $c = 2.998 \times 10^8 \text{ms}^{-1}$, calculate the speed of the electron when it exits the tube, as a fraction of the speed of light. Assume it started from rest, and ignore any relativistic effects.

(v) A copper wire of diameter 1mm has a resistance of 1 Ω . How long is the wire in m? If the wire length is trebled and the resistance is now 0.5 Ω , calculate the new wire diameter (in mm).

(vi) A voltmeter records a potential difference of V_0 volts across N resistors $R_1, R_2, ..., R_N$ wired in *series*. Why must the same current $I_i = I$ flow through each resistor? Use the definition of resistance $R_i = V_i / I_i$ and Kirchhoff's Second Law to show that *resistances add when wired in series.* V_i is the potential difference across the i^{th} resistor.

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[2] http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/rstiv.html

[3] Accelerating charges will *radiate*, but let us assume the loss of energy by the electron is negligible in this case.

(vii) A voltmeter records a potential difference of V_0 volts across N resistors $R_1, R_2, ..., R_N$ wired in *parallel*. Why must each loop (and therefore each resistor) have a potential difference of V_0? Use the definition of resistance $R_i = V_i / I_i$ and Kirchhoff's First Law to show that resistances add in reciprocals when wired in parallel.

(viii) A power supply provides an EMF ε and has internal resistance r. It is connected to a circuit with resistance R. Show that the voltage across the resistor is $V = \frac{\varepsilon R}{R+r}$. When $R = R_1 = 10.0\Omega$, $V = V_1 = 10.43\text{V}$. When $R = R_2 = 7.0\Omega$, $V = V_2 = 9.88\text{V}$. Use this information to find ε and r.

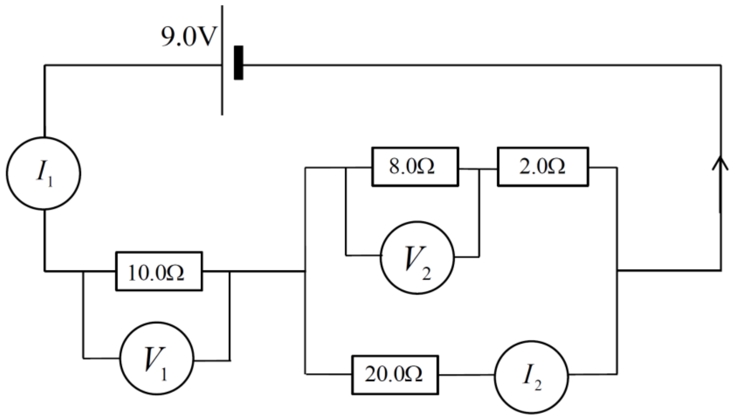
(ix) At room temperature, a tungsten filament lamp has a resistivity of $\rho = 5.4 \times 10^{-8}\Omega\text{m}$ and a resistance of $40\,\Omega$. If the filament has a diameter of $20\mu\text{m}$, calculate the length of the filament (in m). How are filaments presented in a typical bulb?

(x) A battery with EMF ε and internal resistance r is connected in a circuit to a variable resistor R via an ammeter, which measures the current flow I. A voltmeter is connected across the resistor and measures voltage V. As the variable resistor is changed, if one plots a V vs I graph, it will have a line-of-best fit $V = 12.0 - 0.2I$.

Draw a circuit diagram, and write down the EMF and internal resistance. What power is dissipated by the variable resistor (in W) when $R = 5.0\Omega$?

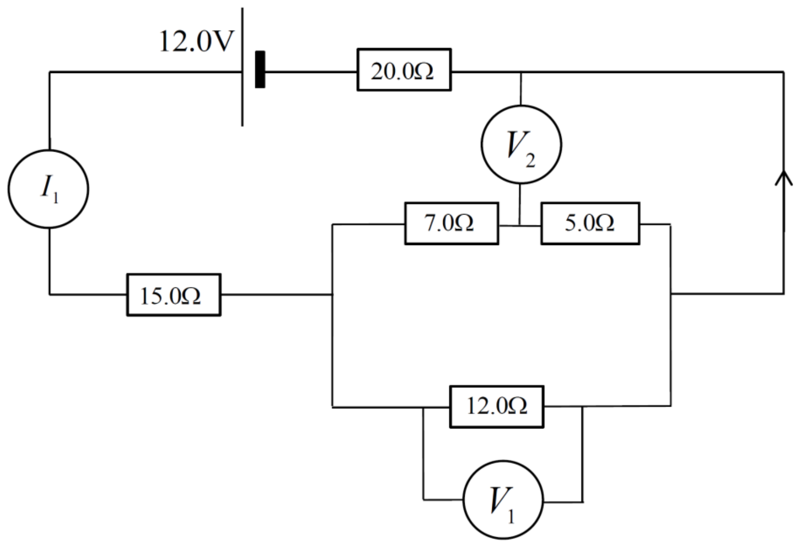
Question 2

Find all unknowns in the circuit below: (*Hint*: calculate the total resistance first...)

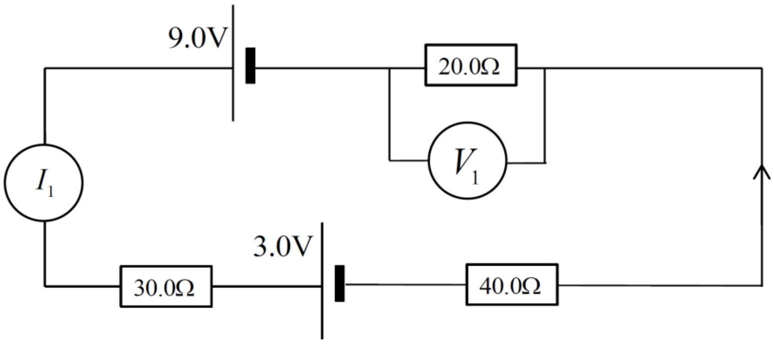


Question 3

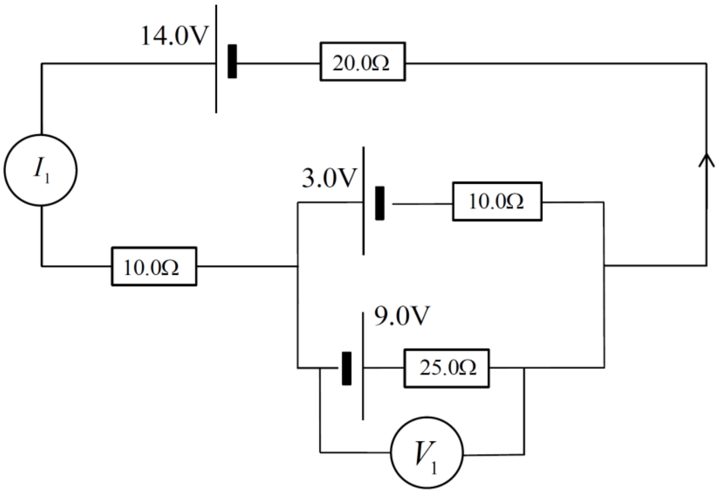
Find all unknowns in the circuit below: (*Hint*: add extra voltmeters across the resistors and work out their readings)



Question 4 Find all unknowns in the circuit below:



Question 5 Find all unknowns in the circuit below: (*Hint*: apply Kirchhoff II for possible loops).



Question 6

A uniform cylindrical cable made from aluminum of resistivity $\rho = 2.8\times10^{-8}\Omega\text{m}$ carries electricity generated from a hydroelectric power station. The maximum power that can be transported by the cable is 2,000MW, and it is transported at 750kV. If the power loss over 1,600km of cabling is 0.90%, calculate the radius of the cable in mm.[4]

Question 7

As in Q1 (x), a battery with EMF ε and internal resistance r is connected in a circuit to a variable resistor R via an ammeter, which measures the current flow I. A voltmeter is connected across the resistor and measures voltage V. Show that the power dissipated in the resistor R is given by $P = \varepsilon^2 R/(r+R)^2$ and sketch ε vs R. Show that the maximum power dissipated is $P_{\text{max}} = \varepsilon^2/4r$, i.e. when $R = r$. Use a spreadsheet to construct a model P vs R curve for given inputs ε, r. Plot curves for $\varepsilon = 5\text{V}, r = 4\Omega$ and $\varepsilon = 7\text{V}, r = 2\Omega$ on the same graph. Use an R scale of 0 ... 20Ω.

Question 8

A resistor in the shape of a solid truncated cone is constructed from a conductor of uniform resistivity ρ. The truncated cone has length l and the ends have cross sectional radii a and b respectively. By considering the cone to be assembled from cylinders of radius r and width dl wired in series, show (using integral calculus) that the total resistance is $R = \rho l/\pi ab$.

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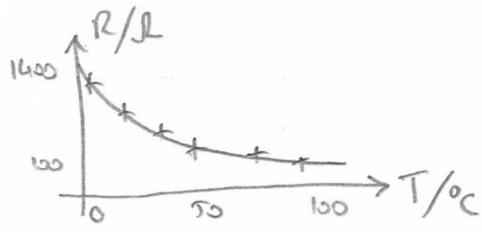
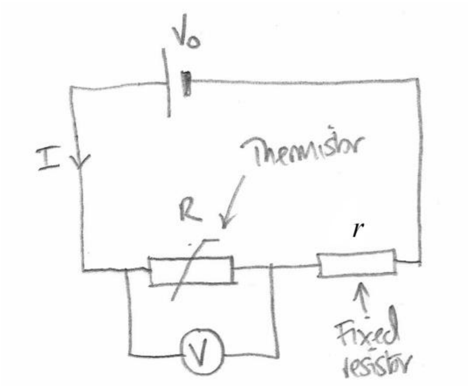
[4] Assume current travels uniformly throughout the conductor. This is true for direct current. For alternating current, conduction is mostly within a 'skin depth' of the surface. The skin depth varies with frequency of the AC. See Eclecticon.

Question 9 A thermistor is a semiconductor that decreases in resistance as temperature increases. A sensible model of this behaviour is $R = R_0 e^{-kT}$ where T is the temperature in ^{o}C. To construct a temperature sensor from a thermistor, it is wired in series with a fixed resistor r, and a power supply (of negligible internal resistance) provides a potential difference of V_0. The temperature is determined from the potential difference V measured across the thermistor. The thermistor operates over range $T_{\min} \leq T \leq T_{\max}$. What is the optimum value of r such that the variation of V over the operating temperature range is maximized? You are encouraged to construct a computer model of the system. Use $k = 0.0271$ and $R_0 = 1183\Omega$ as sensible values. If $0 \leq T \leq 90^{o}\text{C}$, you should discover the optimum r is about $349\,\Omega$.



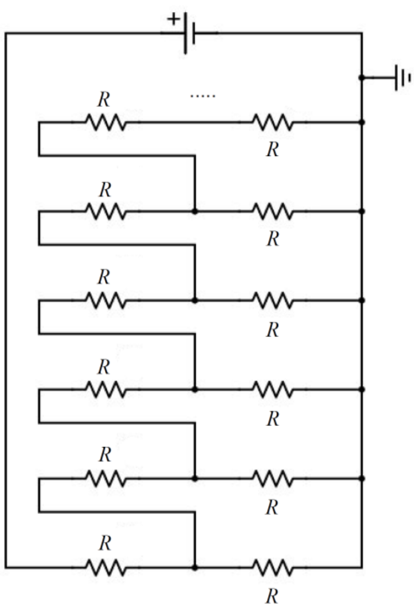


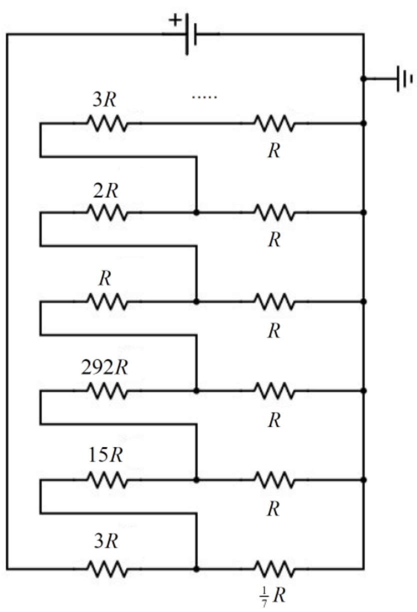
Hint: Find an expression for $V(T)$ and hence $\Delta V = V(T_{\min}) - V(T_{\max})$. Plot $\Delta V(r)$ and find r when $d\Delta V/dr = 0$.

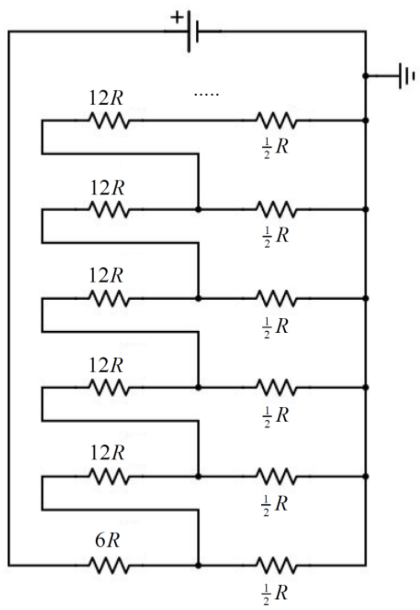
Question 10

If the pattern of the circuits below was continued *ad infinitum*, determine the total resistance in the form $k \times R$ for each circuit. k is a particularly special number for each!

Hint: The first circuit is "perfectly proportioned"..... The second circuit may be used for 'circular reasoning,' and the third is the *Circuit of Deep Thought* (!)

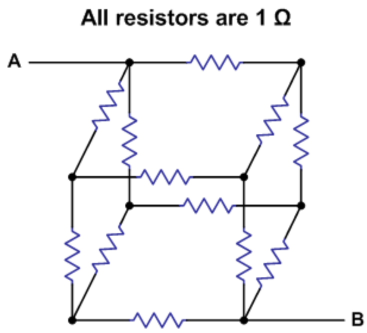






Question 11

Twelve identical resistors are wired to form the edges of a cube. If a power supply connects diagonally opposite corners, what is the resistance of the cube, if each resistor is 1 ohm?



Hint: Construct a (two dimensional!) equivalent circuit first.... Are any points the same potential? If so, can you re-draw the circuit?

**** Project **** Write a computer program to solve the *general case*, with all resistors being different! The idea is to write equations (using Kirchhoff's Laws) involving the current in each resistor, for every possible loop, and every junction.