The force $\mathbf{f}$ on a (stationary) charge of q coulombs in an ***electric field*** $\mathbf{E}$ is $\mathbf{f} = q\mathbf{E}$. An isolated charge Q will *produce* an electric field according to Coulomb's law $\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$. r is the radial displacement from the charge, $\hat{\mathbf{r}}$ is a radial unit vector and permittivity of free space $\varepsilon_0 = 8.85\times10^{-12}\,\mathrm{C^2m^{-2}N^{-1}}$. i.e. field lines emanate radially from positive charges. Charges can be positive (e.g. protons) or negative (electrons). The magnitude of charge on a proton or electron is $e = 1.602\times10^{-19}\,\mathrm{C}$. The potential V is the work done per unit charge, to bring a charge from 'infinity' to radius r from a source of an electric field. $V = \int \mathbf{E}\cdot d\mathbf{r} = \frac{Q}{4\pi\varepsilon_0 r}$.

Similarly to gravitational fields, the ***electric field strength = - the gradient of potential***: $\mathbf{E} = -\frac{dV}{dr}\hat{\mathbf{r}}$.

Pairs of positive and negative charges form **dipoles**, and the field between them looks like that of a bar magnet. The electric field strength on axis to a dipole diminishes as $1/r^3$.

The field between two **parallel plates** can be approximately ***uniform*** (if the plates are large enough). In this case $E = \frac{V}{d}$ where V is the potential difference ('voltage') between the plates and d is the distance between them. If an insulating dielectric is placed between the plates, charge will build up on the plates and can be stored. This is a ***capacitor***.

Capacitance C is the charge stored per unit volt. $Q = CV$. Capacitance of a parallel plate capacitor is: $C = \frac{\varepsilon\varepsilon_0 A}{d}$ where A is the plate area and ε is the relative permittivity or 'dielectric constant'. For vacuum $\varepsilon = 1$ and for air $\varepsilon \approx 1.00$. For water $\varepsilon \approx 78$. Energy stored in a capacitor is $\frac{1}{2}QV = \frac{1}{2}CV^2$. *Parallel* capacitors add like *series resistors*, and *series* capacitors *add in a reciprocal sense like parallel resistors.* The capacitance of a charged sphere of radius a is $C = 4\pi\varepsilon_0 a$.

If we charge up a capacitor via a circuit of resistance R with applied voltage V_0 : then the potential difference across the capacitor is: $V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$. RC is the time constant for the circuit.

If we discharge the capacitor : $V = V_0 e^{-\frac{t}{RC}}$ i.e. an *exponential decay.*

Gauss's law states that $\int_S \mathbf{E}\cdot d\mathbf{S} = \frac{Q}{\varepsilon_0}$ where charge Q (the source of electric field $\mathbf{E}$) is enclosed by surface S with surface normal $d\mathbf{S}$.

Question 1

(i) A spark will travel between the charged cap of a *Van der Graaf* generator and an earthed metal probe when the probe is less than about 15cm away. If air ceases to be an effective insulator when electric field strengths exceed $3.0\times10^6\,\mathrm{V/m}$, calculate the potential difference between the charged cap and ground.

(ii) If the Van der Graaf cap in (i) has a radius of 0.20m, calculate its capacitance, and hence the amount of charge Q stored prior to discharge. How much electrical energy is discharged with each spark?

(iii) Calculate the electric field strength (in V/m) experienced by an electron at range of $r = 5.3\times10^{-11}\,\mathrm{m}$ from a proton, which constitutes the nucleus of a Hydrogen atom. What is the electrical force (in N) between the electron and proton?

(iv) Determine the ratio of electrical and gravitational forces between two electrons separated by distance r. The mass of the electron is: $m_e = 9.109\times10^{-31}\,\mathrm{kg}$ and $G = 6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$. Comment on your answer.

(v) An air-gap parallel plate capacitor made from discs of radius r and plate separation d has a capacitance of 0.42 μF. If the plate separation is halved and the radius tripled, calculate the new capacitance.

(vi) (a) A 10nF, 20nF and 30nF capacitor are connected in series. What is the equivalent capacitance? (b) Now compute the equivalent capacitance if they are all connected in parallel.

(vii) A $3.0\mu F$ capacitor is connected in parallel with a $4.0\mu F$ capacitor. This assembly is wired in series with a $5.0\mu F$ capacitor. What is the total capacitance?

(viii) A $123\mu F$ capacitor is charged up to 12.0V and then discharged through a resistance R ohms. A voltmeter placed across the resistor reads 4.41V after 3.21s. Calculate R. How long will it take to discharge such that the voltmeter reads 1.0V? What is the current in the resistor after 2.0s?

(ix) A capacitor charges to a voltage of 6.0V in 2.0s via a charging circuit of resistance $300k\Omega$. The circuit has a constant applied voltage of 8.0V. Calculate the capacitance C (in F) and sketch a graph of the *charge* on the capacitor vs time. Calculate suitable scales. What is the maximum energy (in J) the capacitor can store?

(x) Show that *Gauss' law* yields Coulomb's inverse square law for the electric field at radius r from a spherical conductor of radius $R \le r$. If the spherical conductor is hollow, use Gauss' law to explain why the field strength inside must be zero. Does the shape of a hollow conductor matter? Explain with examples how such a *Faraday Cage* can be useful in high voltage engineering applications.

(xi) Prove the addition rules for capacitors in (a) series and (b) parallel.

(xii) Use Gauss' law + the idea of electric field = - potential gradient to prove that the capacitance of a parallel plate capacitor of plate area A and plate separation d is $C = \varepsilon_0 A/d$.

Question 2 In an experiment to demonstrate *electron diffraction*, electrons are produced from *thermionic emission* from a hot wire (at essentially zero velocity) and accelerated between two charged plates. The plate has a small hole which enables a beam of high speed electrons to collide with a graphite target. (For what happens next - see the Quantum Mechanics problem sheets!)

(a) Using *Classical Mechanics*, determine the voltage between the plates to accelerate the electron to 80% of the speed of light $c = 2.998 \times 10^8 \text{ms}^{-1}$. (b) Now repeat the calculation using the *Relativistic* expression for kinetic energy: $E = (\gamma - 1)mc^2$ where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$.

Question 3 Two spherical conductors are suspended from insulated wires. One has a radius R_1 and charged to voltage V_1. The other has radius R_2 and is charged to voltage V_2. (a) Determine an expression for (i) the charge and (ii) the stored energy in each sphere. (b) Without loss of charge, they are now connected by a copper wire. Explain why, after a very short time, both spheres must have the same voltage. Calculate an expression for this voltage, and hence determine the charge, and stored energy on each sphere. (c) Determine an expression for the energy lost due to heating of the wire, and evaluate if $V_1 = 300\text{kV}$, $V_2 = 400\text{kV}$, $R_1 = 10.0\text{cm}$ and $R_2 = 7.0\text{cm}$.

Question 4 A ping-pong ball of mass 2.8g is painted with a conductive coating. (The 2.8g includes the mass of the coating). It is suspended via a light string of length $l = 15\text{cm}$ from a retort stand and given a negative charge $-kQ$. The ball is then placed equidistant between two isolated charged spheres of separation $d =$ 20.0cm. The left sphere has charge Q and the right sphere has charge $-Q$. The ball is attracted to the left plate and hovers at an angle of 30° from the vertical. (a) Draw a diagram to represent the situation, and carefully sketch the electric field lines between the ball and the spheres. Explain why electric fields must only be perpendicular to a conductor. (b) Assuming $g = 9.81\text{N/kg}$, and $k = \frac{1}{2}$ calculate Q (in C). (c) If there spheres have radius 10.0cm, what is their electric potential in V?

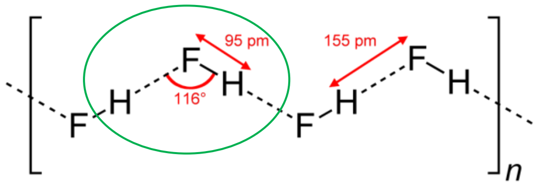
Question 5 (a) Use *Gauss' law* to show that the electric field strength E at radius r from a long wire containing charge λ per unit length is $E = \lambda/2\pi\varepsilon_0 r$. (b) Hence determine the potential difference $V = V_a - V_b$ between a wire of radius a and a cylindrical conductor of radius b (such that $b > a$) that encloses the wire. (This is what constitutes a *coaxial cable*, used in many electrical communications systems such as a connection to an oscilloscope or the wired link between an antenna and a satellite TV receiver). Hence show that the capacitance per unit length is $C = 2\pi\varepsilon_0/\ln\frac{b}{a}$.

(c) What is the capacitance of a 10.0m cable with $a = 0.5\text{mm}$ and $b = 3.0\text{mm}$, if a dielectric (polythene) with $\varepsilon = 2.25$ is placed between the inner and outer conductors?

Question 6 Hydrogen Fluoride (HF) is a small molecule comprising Hydrogen and Fluorine. The molecule is naturally polarized to form an *electric dipole*. The hydrogen end has a positive charge, and the fluorine end has an equal and opposite negative charge. i.e. the net charge of the molecule is zero. Assume for the purposes of this question that the charge magnitude of H or F equals the charge on the electron *e*.

(a) The H-F bond length d is about 95pm $(1\text{pm} = 10^{-12}\text{m})$. Calculate the electric field strength at the centre of the HF molecule.

(b) HF forms zig-zag structures of about five or six molecules in the liquid phase, due to the strong hydrogen bonding between the negatively polarized F of one molecule and the positively polarized H of an adjacent molecule. A HF hydrogen bond is about 155pm long.



Show that the electric field $\mathbf{E} = 1.0 \times 10^{10}(1.99, 2.71)\,\text{Vm}^{-1}$ is felt by the H atom of one molecule *just* due to the H and F atoms of a nearby molecule (the trio is circled). Take (x, y) coordinates to be centered on the H atom.

(c) A beta particle (an electron in this case) is r pm from the H atom in HF. No other charged molecules are in the vicinity and the electron and the HF bond are all on the same line. Determine an expression for the electric field strength $E(r)$ and use a suitable first-order binomial expansion approximation to show that $E \propto 1/r^3$ when $r \gg d$. Sketch this variation, with an appropriate distance and electric field scale.

Question 7 By considering rings of charge $dQ = \dfrac{2\pi r dr}{\pi R^2} Q$, determine the electrical field strength $E(z)$ on axis from a uniformly charged disc of radius R, containing total charge Q. z is the perpendicular distance from the centre of the disc. Plot $E(z)$ for increasing R values, assuming a charge per unit area σ. Show that in the limit of $R \to \infty$, $E = \sigma/2\varepsilon_0$.

Question 8 In the *Bohr* model of the Hydrogen atom, electrons still in a sense 'orbit' a proton nucleus in an approximately circular fashion at radius r.

(a) Use Coulomb's law + your prior knowledge of uniform circular motion to show that the centripetal acceleration $a = \dfrac{e^2}{4\pi\varepsilon_0 r^2 m_e}$, the orbital period is: $\tau = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 m_e r^3}{e^2}}$ and the energy of the electron is: $E = -\dfrac{e^2}{8\pi\varepsilon_0 r}$.

(b) Accelerating charges *radiate* electromagnetic waves, and hence lose energy. It can be shown that the rate of change of energy is given by: $\dfrac{dE}{dt} = -\dfrac{e^2}{6\pi\varepsilon_0 c^3}a^2$ where a is the acceleration and c is the speed of light. This is called the *Larmor* formula. Show that $\dfrac{dE}{dt}\tau \Big/ E = \frac{1}{3\sqrt{\pi}}\left(\dfrac{e^2}{\varepsilon_0 m_e c^2 r}\right)^{\frac{3}{2}}$ and hence show that the orbital energy changes little per rotational period if $r \gg \frac{1}{\sqrt[3]{9\pi}}\dfrac{e^2}{\varepsilon_0 m_e c^2}$. Use this approximation of 'quasi-stable circular orbits' to combine the Larmor formula with the results in (a) and hence determine the trajectory of an 'orbiting' electron, starting with $\theta = 0$ (i.e. 'horizontal') and $r(0) = 50\text{pm}$. Determine equations for the trajectory in polar coordinates $r(t)$, $\theta(t)$ and sketch the trajectory. (Even better: write a short computer program to plot it accurately). How long does it take before the electron in-spirals into the nucleus?

(Why electrons ***don't*** *do this, and indeed why we exist as a result, is one of the main motivators for a Quantum theory of the electron. In the Bohr model, electrons can be thought of as 'standing waves' at radius r and therefore don't lose energy. However, to calculate the energy levels in the Hydrogen atom, we still use the mechanics of the 'orbit' model.)*