Series & Parallel circuits

Electronic systems consist of networks of basic components which modify the rate of flow of charge (*current*) and energy per unit charge (*voltage*) between two points in the network. In order to model the behaviour of a system, it is useful to understand the basic mathematical features. We shall consider *resistive* components (i.e. those which obey *Ohm's Law*) and the difference between *series* and *parallel* arrangements of resistors.

Ohm's Law

$\boxed{V = IR}$

Voltage / volts
Energy per unit charge lost across a conductor of resistance R

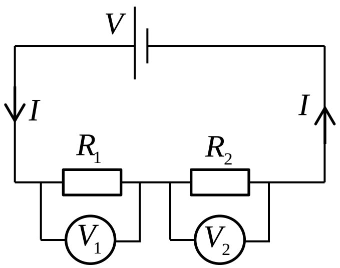
Resistance /ohms (Ω)

Current / amps
Rate of flow of charge through resistor R

Note due to a quirk of history, **conventional current is the 'flow of positive charge'.** Unfortunately **electrons** were discovered after many of the electrical conventions were fixed. Since it is the *electrons which are mobile in a conductor*, current can be thought of as the flow of '*holes*' of net positive charge left when an electron has been moved via an electric field.

Georg Ohm
1789-1854

Series circuits



The *same* current must flow through every component in the loop, otherwise charge would be created or lost.

$V_1 = IR_1$
$V_2 = IR_2$
Apply Ohm's law to each resistor in turn

$V = IR$ Apply Ohm's law to entire series loop. R is the total resistance

$V = V_1 + V_2$ The applied voltage V must be divided across the resistors

Hence:

$IR = IR_1 + IR_2$

$\therefore \boxed{R = R_1 + R_2}$ so **series** resistors ***add***

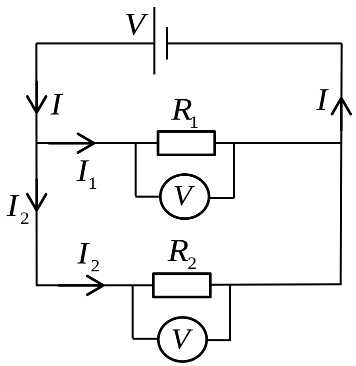
We can use this result to show that resistances in series can act as *potential dividers*

$$I = \frac{V_1}{R_1} = \frac{V}{R} \qquad \therefore \frac{V_1}{R_1} = \frac{V}{R_1 + R_2}$$

$$\boxed{V_1 = \frac{R_1}{R_1 + R_2} V}$$

i.e. the voltage across a resistor is the same fraction of the applied voltage as the ratio of resistance to the total resistance.

Parallel circuits



$V = IR$ Apply Ohm's law to entire circuit. R is the total resistance

$V = I_1 R_1$
$V = I_2 R_2$
Apply Ohm's law to each resistor in turn. Same *electric field* across each loop, so the ***same voltage is dropped*** across the resistors

Current is assumed to be contained within the circuit, hence:

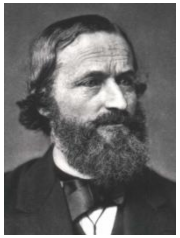
$I = I_1 + I_2$ ← This is *Kirchoff's law*

Therefore:

$V / R = V / R_1 + V / R_2$

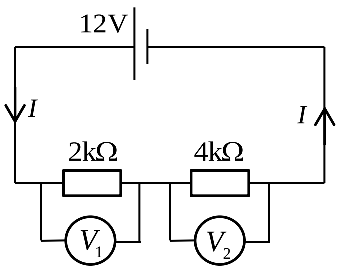
$$\therefore \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

so **parallel** resistor loop resistance ***reciprocals*** add



Gustav Kirchhoff
1824-1887

Series circuit example



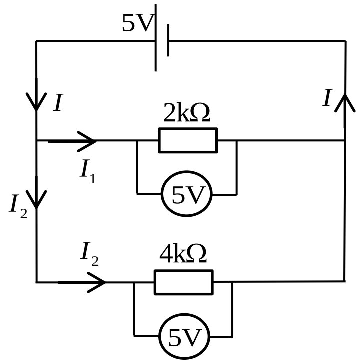
$$I = \frac{12}{2+4}\text{mA}$$

$$I = 2\,\text{mA}$$

$$V_1 = \frac{2}{2+4} \times 12 = \boxed{4\text{V}}$$

$$V_2 = \frac{4}{2+4} \times 12 = \boxed{8\text{V}}$$

Parallel circuit example



$I_1 = 5/2 = 2.5\text{mA}$
$I_2 = 5/4 = 1.25\text{mA}$
$I = I_1 + I_2 = \boxed{3.75\text{mA}}$

Ohm's Law then add currents

Or alternatively ...

$$R = \frac{1}{\frac{1}{2} + \frac{1}{4}} = \tfrac{4}{3}\text{k}\Omega$$

$$I = 5 / \tfrac{4}{3} = \tfrac{15}{4}\text{mA} = \boxed{3.75\text{mA}}$$

Use reciprocal resistance addition rule

$1\text{mA} = 10^{-3}\text{A} \qquad 1\text{k}\Omega = 10^{3}\Omega$