$k = \frac{2\pi}{\lambda}$ $\omega = 2\pi f$ $\omega = \frac{c}{n}k$ $\text{wave speed} = \frac{c}{n}$ $\text{wave vector } \mathbf{k} = k\hat{\mathbf{z}}$

Electromagnetic waves

Electromagnetic waves (of a particular amplitude, and wavelength) comprise of sinusoidally varying vector components of electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. **Maxwell's Equations**, which describe the relationships between electric and magnetic fields (and charge) predict the following:

1. If an electromagnetic wave propagates in direction parallel to vector $\mathbf{k}$, the electric and magnetic field are both *perpendicular* to this direction. In other words $(\mathbf{E},\mathbf{B},\mathbf{k})$ forms a right handed set* in a Cartesian (x,y,z) sense. No vector component of $\mathbf{E}$ or $\mathbf{B}$ is parallel to the direction of propagation.

2. Electromagnetic waves travel at a *finite speed* through a medium. This is independent of any coordinate system, so you can never 'catch up' with an electromagnetic wave, no matter how fast you move. This idea is the main reason (in *Special Relativity*) behind the need to modify space and time as one approaches the speed of light. The speed of electromagnetic waves is c/n where $c = 2.998 \times 10^8\,\text{ms}^{-1}$ and n is the refractive index. For a vacuum, n is unity. Materials such as glass have a refractive index of about 1.5. Light still travels at c, but the charge-carrying atoms in the glass interact with the EM waves and effectively force them to take a more tortuous path. The net effect is that EM propagation appears to 'slow' in the medium.

3. At an interface between media of differing refractive index, vector components of $\mathbf{B}$ *perpendicular* to the interface surface must be *continuous* across the boundary. Also, components of the $\mathbf{E}$ and $\mathbf{H}$ fields which are *parallel* to the surface, must be continuous across the boundary.

4. For an electromagnetic wave: $|\mathbf{E}| = |\mathbf{B}|\frac{c}{n}$ $\mathbf{B} = \hat{\mathbf{k}} \times \frac{n\mathbf{E}}{c}$

For *isotropic* media:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.998 \times 10^8\,\text{ms}^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\text{Hm}^{-1}$$
$$\varepsilon_0 = 8.85 \times 10^{-12}\,\text{Fm}^{-1}$$

$$\mathbf{E} = \begin{pmatrix} E_{0x} \\ E_{0y} \\ 0 \end{pmatrix} e^{i(kz-\omega t)}$$

$$\mathbf{B} = \frac{n}{c}\begin{pmatrix} -E_{0y} \\ E_{0x} \\ 0 \end{pmatrix} e^{i(kz-\omega t)}$$

Note the actual $\mathbf{E}$ and $\mathbf{B}$ fields are the *real* parts of these *complex* quantities.

Note *De Moivre's Theorem*

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$\Rightarrow e^{i\frac{1}{2}\pi} = i \qquad e^{i\pi} + 1 = 0$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

Poynting vector i.e. Power / m²

$$\therefore |\mathbf{S}|_{\max} = \frac{1}{\mu_0}|\mathbf{E}||\mathbf{B}| = \frac{n}{\mu_0 c}|\mathbf{E}|^2$$

$$\langle S \rangle \approx \tfrac{1}{2}\frac{n}{377\Omega}|\mathbf{E}|^2$$

Power of an EM wave varies as the *square* of electric field strength

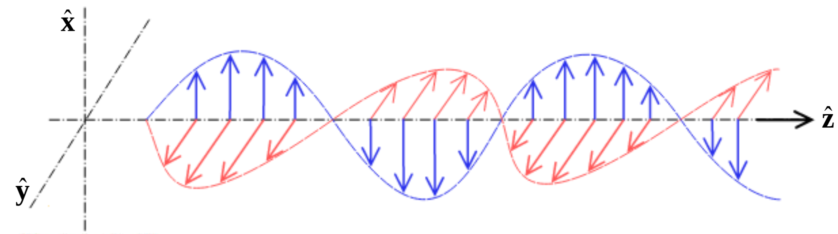
Energy per unit volume stored in $\mathbf{E}$ and $\mathbf{B}$ fields is:

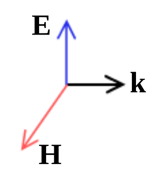
$$u = \tfrac{1}{2}\varepsilon_0|\mathbf{E}|^2 + \tfrac{1}{2}\frac{1}{\mu_0}|\mathbf{B}|^2$$

$$n = \sqrt{\mu\varepsilon}$$

μ **Relative permeability.** Unity for non magnetic materials. Magnetic materials such as iron have a relative permeability of about 5000. Ferrite is 640, Nickel is 100.

ε **Relative permittivity.** Unity for vacuum and approximately for air. Water is about 1.77, glass 3.7-10, diamond 5.5-10, sapphire 8.9-11.1

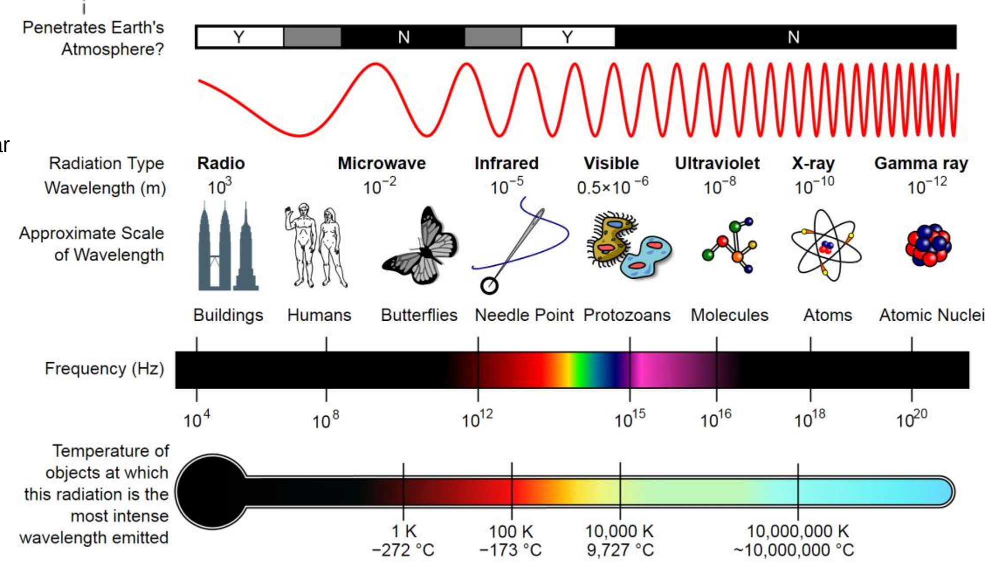




Electromagnetic waves have particular names depending on their frequency (or wavelength).

The complete set is called the **Electromagnetic Spectrum.**

Note the *energy* of an EM wave is proportional to *frequency*, so Gamma rays are produced from high energy nuclear processes, whereas one can generate radio ways by oscillating a few electrons rather gently in an antenna mast or radio receiver.



https://en.wikipedia.org/wiki/Electromagnetic_spectrum#/media/File:EM_Spectrum_Properties_edit.svg

*Actually, it is $(\mathbf{E},\mathbf{H},\mathbf{k})$. But for isotropic media, $\mathbf{B}$ is parallel to $\mathbf{H}$