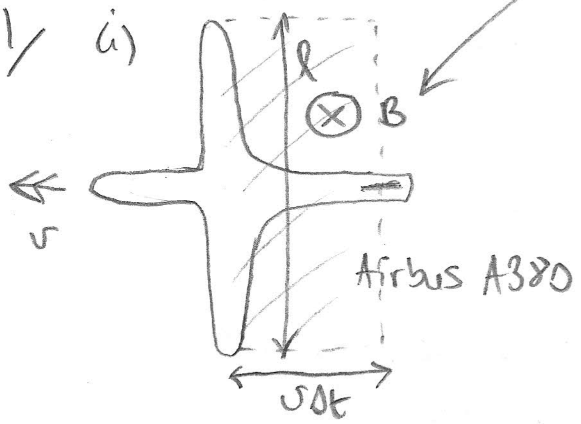
ELECTROMAGNETISM

1/ (i)



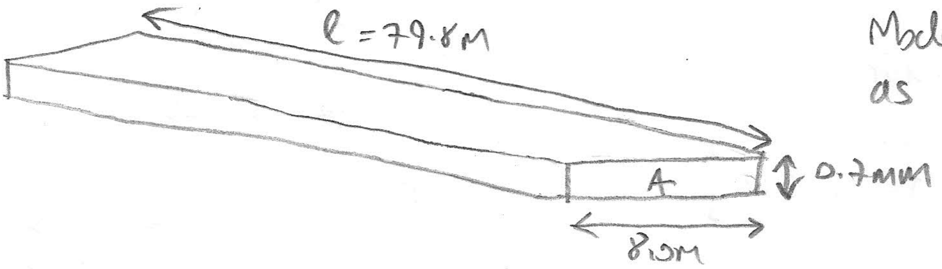
In δt seconds, the wings sweep out area $v\delta t l$.

The magnetic flux linked is $\therefore$ $Bv\delta t l$ and $\therefore$ the rate of change of magnetic flux linked is $\frac{d\Phi}{dt} = Bvl$.

By Faraday's law, this is the EMF induced across the wings.

$$\therefore \quad V = Bvl = 65 \times 10^{-6}\,\text{T} \times \frac{1050 \times 1000\,\text{m}}{3600\,\text{s}} \times 79.8\,\text{m}$$

$$= \boxed{1.51\ \text{V}}$$



Modelling the wings as cuboids of Aluminium

$$I = V/R \qquad R = \frac{\rho l}{A}$$

$$\therefore \quad I = \frac{VA}{\rho l} = \frac{1.51 \times 8.0 \times 0.7 \times 10^{-3}}{2.7 \times 10^{-8} \times 79.8}$$

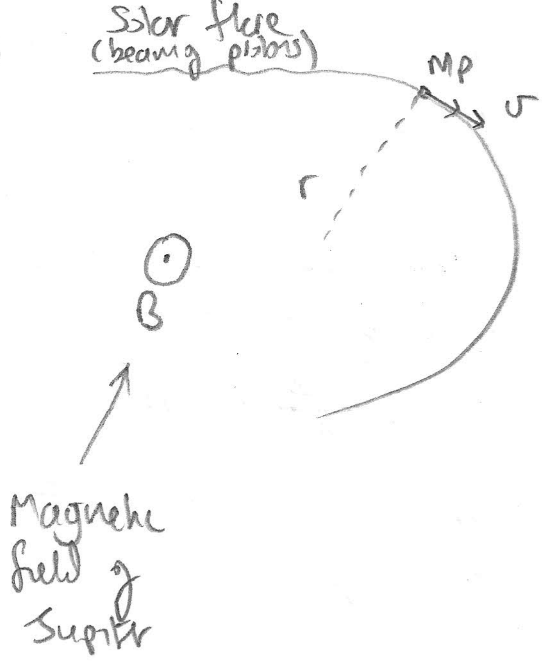
$$= \boxed{3920\ \text{A}}$$

This seems rather large, so one hopes the wings can either (i) harness this current or (ii) increase the resistance of the wings via insulating material.

However, the power of 5.9 kW (VI) is likely to be relatively minor in terms of the jet engine power.

.... Although, with a thrust of × 65.1 kW (Boeing 747 → the A380 will probably be larger) the induced currents and resulting EM braking effect might actually be significant.

(ii)



Newton II:

$$\frac{m_p v^2}{r} = Bev$$

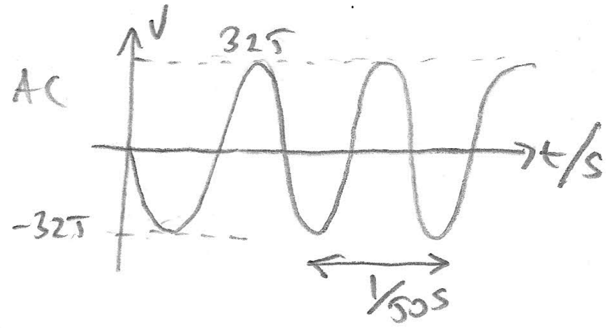
$$\therefore \frac{m_p v}{Be} = r$$

$$\therefore r = \frac{1.67\times10^{-27} \times 490\times10^{3}}{417\times10^{-6} \times 1.60\times10^{-19}}$$

$$\boxed{r = 12.3\,\text{m}}$$

which on the scale of Jupiter is tiny. So expect a visible aurora effect at the poles.

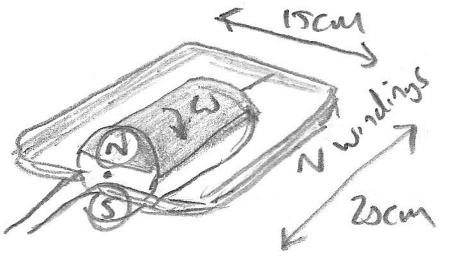
(iii) Diesel generator produces mains AC



Now $\boxed{V = -NBA\omega\sin\omega t}$

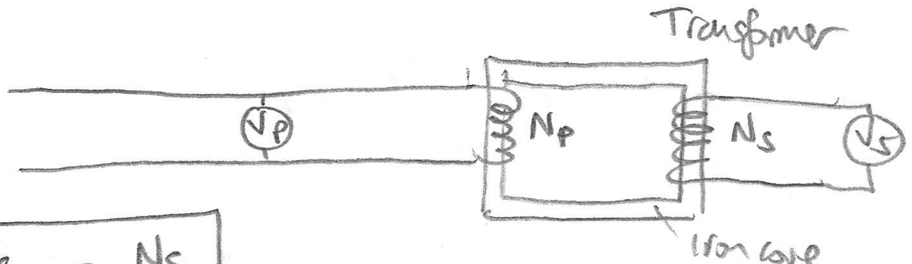
So $N = \dfrac{V_{max}}{BA\omega}$ # windings in generator

$$\therefore N = \frac{325}{0.05 \times (20\times10^{-2} \times 15\times10^{-2}) \times 2\pi \times 50} = \boxed{690}$$



AC generator.

(iv)



a) $\boxed{\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}}$ $\therefore N_p = \dfrac{V_p}{V_s} N_s = \dfrac{500\,\text{kV}}{4\,\text{kV}} \times 50$

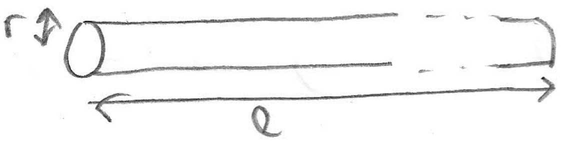
$= \boxed{6250}$

b) $I_s V_s = 0.95 \underbrace{I_p V_p}_{660\,MW}$ if transformer is 95% efficient.

So $I_p = \frac{660\times10^6}{500\times10^3} = \boxed{1320\ A}$

∴ $I_s = \frac{0.95\times660\times10^6}{4000} = \boxed{1.57\times10^5\ A}$

c) Assume power line suffers a 1% loss over 100km = l



$R = \rho \frac{l}{\pi r^2}$

∴ $\underbrace{0.01\times660\times10^6}_{P_{loss}} = I_p^2 R$

∴ $\frac{\rho l}{\pi r^2} = P_{loss}/I_p^2$

∴ $\frac{\pi r^2}{\rho l} = \frac{I_p^2}{P_{loss}}$

∴ $\boxed{r = \sqrt{\frac{\rho l I_p^2}{\pi P_{loss}}}}$

∴ $r = \sqrt{\frac{2.7\times10^{-8}\times100\times10^3\times1320^2}{\pi\times0.01\times660\times10^6}}$ (m)

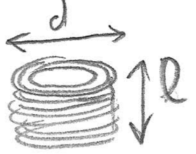
$= \boxed{15.1\ mm}$

d) Compare to skin depth $\left(\delta/mm\right) \simeq 5.03\times10^5\sqrt{\frac{\rho}{f}}$

↑ only for AC signals of frequency f.

$= 5.03\times10^5\sqrt{\frac{2.7\times10^{-8}}{50}} = \boxed{11.7\ mm}$

So since current is AC, might get away with slightly thinner cables (which will be lighter).

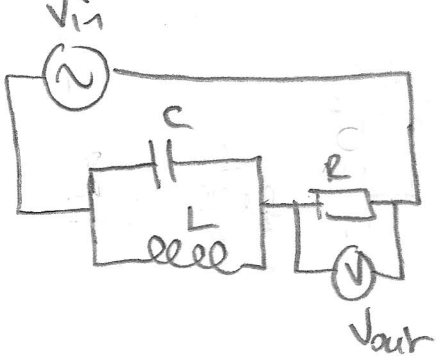
(v) 

Inductor of inductance $\boxed{L = \frac{\mu_0 N^2 \pi \left(\frac{d}{2}\right)^2}{l}}$

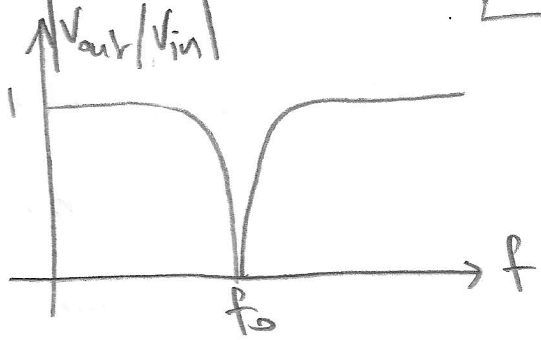
∴ # of turns $N = \sqrt{\frac{4lL}{\mu_0 \pi d^2}}$ $\left(L = \mu_0 \mu N^2 A / l\right)$

$$N = \sqrt{\frac{4\times 5.0\times 10^{-2}\times 1.0}{4\pi\times 10^{-7}\times\pi\times(9.0\times 10^{-2})^2}}$$

$\boxed{N = 2501}$

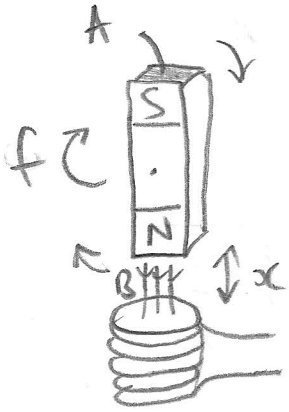
(vi) 

Resonance frequency $\boxed{f_0 \approx \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$



∴ $4\pi^2 f_0^2 = \frac{1}{LC}$ ∴ $\boxed{C = \frac{1}{4\pi^2 f_0^2 L}}$

∴ $C = \frac{1}{4\pi^2\times 50^2\times 0.2} = \boxed{50.7\ \mu F}$

(vii) 

Solenoid

[rot frequency f]

Rate of magnetic flux linked by solenoid is $\boxed{B_0 A N_0 f_0 = \mathcal{E}_0}$, the EMF induced.

If $x \to 2x$, expect $B_1 = B_0/2^3$

∴ $\mathcal{E}_1 = B_1 A N_1 f_1$ so $\boxed{\frac{\mathcal{E}_1}{\mathcal{E}_0} = \frac{B_1}{B_0}\frac{N_1}{N_0}\frac{f_1}{f_0}}$

$$\therefore \quad \mathcal{E}_1 = 12.6\text{ mV} \times \left(\frac{1}{8}\right)\left(\frac{1200}{2400}\right)\left(\frac{5000}{1000}\right)$$

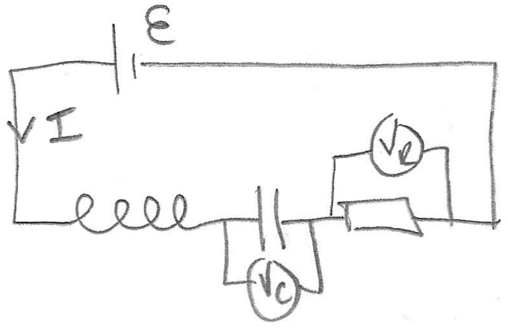
$$\boxed{\mathcal{E}_1 = 3.9\text{ mV}}$$

Now $A = (0.5 \times 10^{-2})^2 \text{ m}^2$

$$\therefore \quad B_0 = \frac{12.6 \times 10^{-3}}{(0.5\times 10^{-2})^2 \times 2400 \times 1000 \times 2\pi/60}$$

$$\boxed{B_0 = 2.01\text{ mT}}$$

(viii)



ohm's law:

$$\mathcal{E} - \underbrace{L\frac{dI}{dt}}_{\text{"Back EMF"}} = V_c + V_R$$

$$V_R = IR$$

$$CV_c = \int I\,dt \qquad \text{"}Q = CV\text{"}$$

$\therefore \quad I = C\dfrac{dV_c}{dt}$ let $V = V_c$ for brevity.

Now input power to capacitor is IV (ditto for other components)

From energy conservation $\Rightarrow \quad CV\dfrac{dV}{dt} = I\mathcal{E} - I^2R - LI\dfrac{dI}{dt}$

$$\therefore \quad \boxed{I\mathcal{E} = I^2R + LI\frac{dI}{dt} + CV\frac{dV}{dt}}$$

Now $LI\dfrac{dI}{dt} = \dfrac{d}{dt}\left(\tfrac{1}{2}LI^2\right) \qquad CV\dfrac{dV}{dt} = \dfrac{d}{dt}\left(\tfrac{1}{2}CV^2\right)$

So

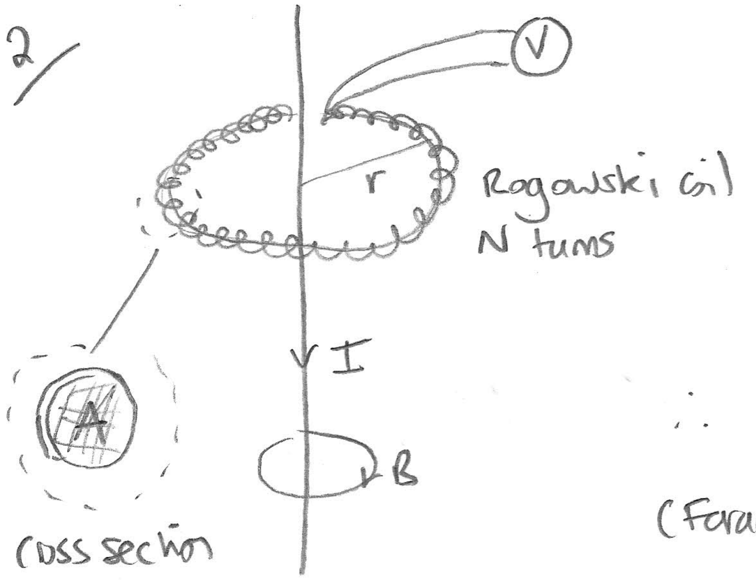
$$I\mathcal{E} = I^2R + \frac{d}{dt}\left(\tfrac{1}{2}CV^2 + \tfrac{1}{2}LI^2\right)$$

input power — power dissipated in resistor — Energy stored in C and L

∴ Stored energy in Capacitor is $\boxed{\frac{1}{2}CV^2}$

and Stored energy in Inductor is $\boxed{\frac{1}{2}LI^2}$

2/



Flux linked by coil is

$NBA = \Phi$

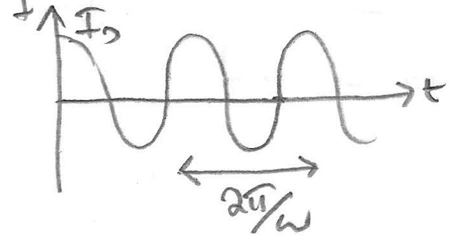
$\Phi = NA \times \mu_0 I / 2\pi r$

∴ EMF induced $V = -\frac{d\Phi}{dt}$

(Faraday's law)

$$\Rightarrow \quad V = -\frac{NA\mu_0}{2\pi r}\frac{dI}{dt}$$

If $I = I_0 \cos \omega t$



$\omega = 2\pi/T$

$T = 2\pi/\omega$

$$\therefore \quad V = -\frac{NA\mu_0}{2\pi r}\left(-I_0\omega \sin \omega t\right)$$

$$\therefore \quad \boxed{V = \frac{\omega N A \mu_0 I_0 \sin \omega t}{2\pi r}}$$

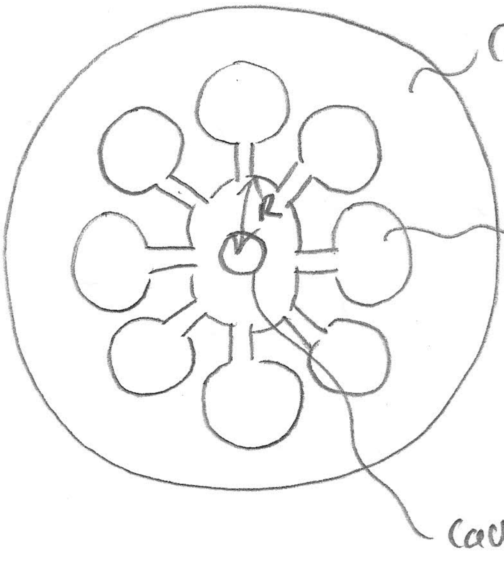
[let's try sensible numbers $N = 50$; $A = \pi \times (5\times10^{-3})^2\ m^2$

$\omega = 2\pi \times 50\,Hz$, $r = 2.0\times10^{-2}\ m$

$$\therefore \quad V = \left(\frac{f}{50\,Hz}\right)\left(\frac{I_0}{A}\right)\sin(2\pi f t)\left(\frac{N}{50}\right) \times \boxed{2.46\times10^{-7}\ V}$$

So ideally want quite high frequency AC at around 1-6 A, with a few 100 coils to get a reasonable CRO reading for V.

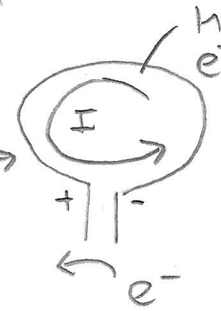
3 MAGNETRON



$\odot$ B field (uniform)

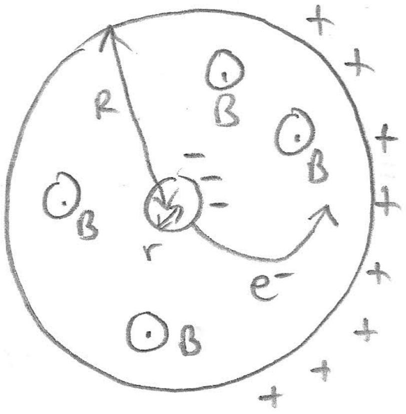
(Source of electrons)

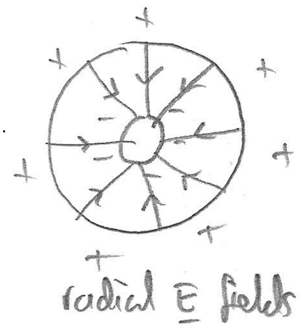
oscillating $\underline{E}$ and $\underline{B}$ fields result in EM radiation in microwave spectrum



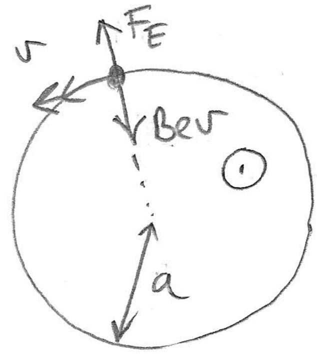
[Circulating e^- result in end of cavity acting like a capacitor. ⇒ current round cavity ⇒ inductor.]

↳ simplified model:





If stable circular orbits at radius a (of electrons)
↑
mass M



NII: $$\frac{Mv^2}{a} = Bev - F_E$$

Now $$F_E = \frac{e\lambda}{2\pi\varepsilon_0 a}$$ ← Electric field strength

where λ is the charge / unit length along the cathode

Voltage V between anode and cathode is $V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R}{r}\right)$

$\left(V = \int_r^R E\,dr\right)$

↑

$-\frac{dV}{dr} = E$

$$\therefore \lambda = \frac{2\pi\varepsilon_0 V}{\ln(R/r)}$$

$$\therefore \frac{Mv^2}{a} = Bev - \frac{e}{2\pi\varepsilon_0 a}\frac{2\pi\varepsilon_0 V}{\ln(R/r)}$$

$$\therefore \quad \boxed{\frac{mv^2}{a} = Bev - \frac{eV}{a \ln(R/r)}}$$

Now since the magnetic force always acts $\perp$ to the electron velocity, it does no work.

$\therefore$ By conservation of energy: $\quad eV = \frac{1}{2} m v^2$

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

is work done by electric field. This is when $a = R$.

So
$$\frac{2eV}{R} = Be\sqrt{\frac{2eV}{m}} - \frac{eV}{R \ln(R/r)}$$

$$\therefore \quad B = \frac{eV}{R}\left(1 + \frac{1}{\ln(R/r)}\right)\sqrt{\frac{m}{2eV}} \cdot \frac{1}{e}$$

$$B = \left(1 + \frac{1}{\ln(R/r)}\right)\sqrt{\frac{mV^2}{2eVR^2}}$$

$$\boxed{B = \left(1 + \frac{1}{\ln(R/r)}\right)\sqrt{\frac{mV}{2eR^2}}}$$

Let $r = 2\text{mm} \qquad R = 15\text{mm} \qquad V = 1\text{kV}$

$m = 9.109 \times 10^{-31}$ kg

$$\Rightarrow \quad B = \left(1 + \frac{1}{\ln\left(\frac{15}{2}\right)}\right)\sqrt{\frac{9.109 \times 10^{-31} \times 1000}{2 \times 1.602 \times 10^{-19} \times (15 \times 10^{-3})^2}}$$

$$= \boxed{0.011\ \text{T}}$$

which is quite achievable.

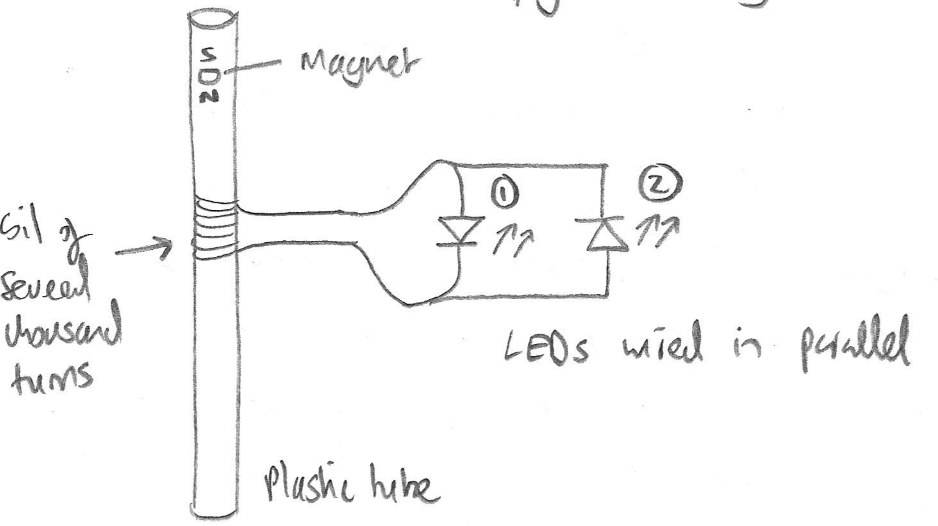
Frequency of circulating electrons is $f = \frac{\omega}{2\pi}$

$v = R\omega \qquad$ so $\quad f = \frac{v}{R}\frac{1}{2\pi} = \frac{1}{2\pi R}\sqrt{\frac{2eV}{m}}$

$$= \boxed{0.2\ \text{GHz}}$$

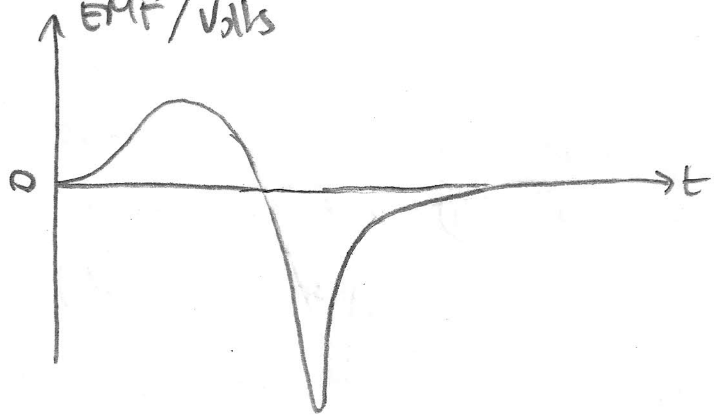
4/

$g = 9.81$ N/kg

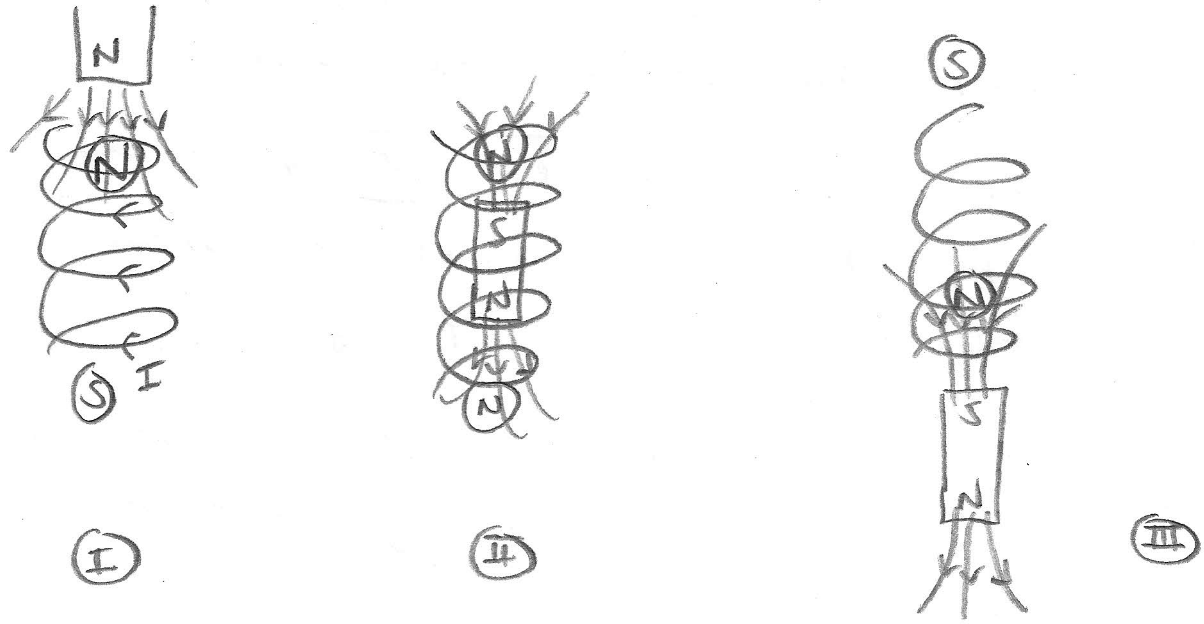


a)

b)



c)



(I) Magnet accelerates towards coil.
coil experiences a rate of change of magnetic flux linked
∴ EMF induced across the coil. LENZ's law means
an opposing (N) pole is induced in the top of the solenoid.
This means the magnet will experience a resistive (i.e. upwards acting) force. However, if the coil is short, we might not expect the effect of this force to slow the magnet very much.

Since magnet is accelerating, expect EMF ↑

(II) As magnet passes through the coil it will induce an EMF of opposing sign to the top half. when the magnet gets half way these will cancel out
⇒ zero EMF.

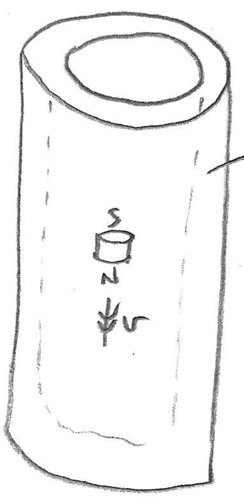
(III) As the magnet leaves the coil, the −ve EMF will dominate, as a (N) pole is induced in the lower half of the coil. (opposite to (I)).

Now the magnet is still accelerating, so we would expect the maximum EMF to be larger (and more −ve) than before.

It will also take $\boxed{\text{less time}}$ to rise and fall since the time to traverse the bottom half of the coil is < the top half.

once the magnet leaves the coil, the flux linkage drops by $\simeq \frac{1}{r^3}$ since the magnet $\simeq$ dipole.

d)



In a sense, one could imagine a thick conductive pipe as a very long solenoid comprising very thick coils.

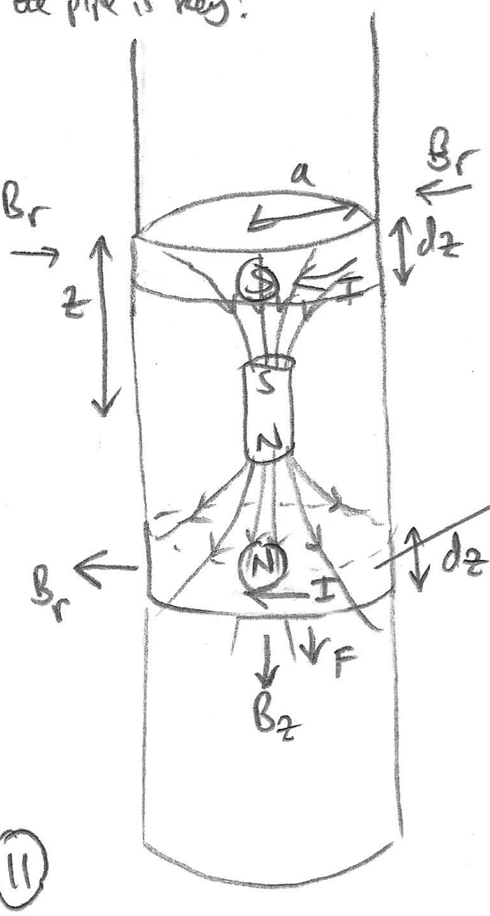
The analysis is more complex though as currents can swirl in eddies throughout the volume of the pipe – they are not confined to tangentially coiled wires.

However, by Fleming's rules, expect tangential currents

Experimentally one observes a significant drag force on the falling magnet, and models predict this is $\propto$ speed.

$\therefore$ expect the magnet to attain a $\boxed{\text{terminal velocity}}$ where the weight of the magnet is balanced by the force of interaction between the falling magnet and the magnetic field set up by the eddy currents induced in the cylinder. {Ignore air resistance ↪ use a heavy magnet}.

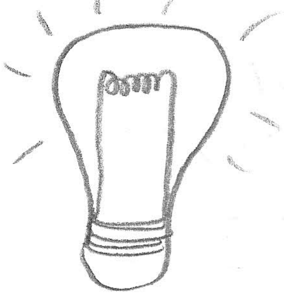
It looks like the effects might cancel each other out mid pipe... So perhaps the $\boxed{\text{START}}$ of the pipe is key?



One way of doing this is to work out the force on a ring element of the pipe:

$$F = B_r \times I \times 2\pi a \qquad \text{By Newton III}$$

this is also the upward force on the magnet. But you have to consider the effect of the South pole too!



Filament bulb

$N = 50$ coils in length

$l = 2.0\,cm$. Wire diameter is

$d = 0.046\,mm$.

uncoiled length of wire is

$l_0 = 580\,mm$.

Wire is made from Tungsten

with resistivity $\rho = 5.60 \times 10^{-8}\,\Omega m$.

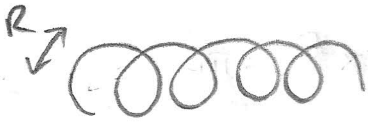
5/

a) (i) $\boxed{R = \dfrac{\rho L}{\pi \left(\frac{d}{2}\right)^2}}$ $= \dfrac{5.60 \times 10^{-8} \times 580 \times 10^{-3}}{\pi \left(0.046 \times 10^{-3}/2\right)^2}$

$= \boxed{19.5\,\Omega}$

b (ii) Treating the filament as a solenoid (air cored)

inductance $\boxed{L = \dfrac{\mu_0 N^2 A}{l}}$

Now  N loops of coil of length l_0

$\therefore$ radius of loop R is given by $\boxed{2\pi R N = l_0}$

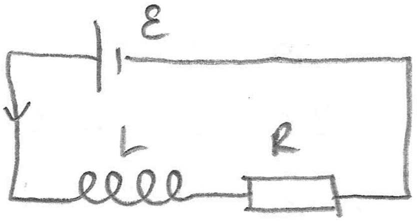
$\therefore \quad R = \dfrac{l_0}{2\pi N}$

$\therefore \quad A = \pi R^2 = \dfrac{\pi\, l_0^2}{4\pi^2 N^2} = \dfrac{l_0^2}{4\pi N^2}$

$\therefore \quad L = \dfrac{\mu_0 N^2}{l} \dfrac{l_0^2}{4\pi N^2} \Rightarrow \boxed{L = \dfrac{\mu_0\, l_0^2}{4\pi l}}$

$\therefore \quad L = \dfrac{4\pi \times 10^{-7} \times \left(580 \times 10^{-3}\right)^2}{4\pi \times 2.0 \times 10^{-2}} = \boxed{1.68 \times 10^{-6}\,H}$

b)



$\varepsilon = 230V$

$L = 1.68 \times 10^{-6} H$

$R = 19.5\,\Omega$

$$\underset{\substack{\uparrow \\ \text{Applied} \\ \text{EMF}}}{\varepsilon} - \underbrace{L\frac{dI}{dt}}_{\substack{\text{Back EMF} \\ \text{due to inductor}}} = \underbrace{IR}_{\text{ohm's law}} \quad \Rightarrow \quad \boxed{\varepsilon = L\frac{dI}{dt} + IR}$$

let $\boxed{I = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right)}$

$$\frac{dI}{dt} = \frac{\varepsilon}{R}\frac{R}{L}e^{-tR/L} = \frac{\varepsilon}{L}e^{-tR/L}$$

$$\therefore \quad L\frac{dI}{dt} + IR = \cancel{\varepsilon e^{-tR/L}} + \varepsilon - \cancel{\varepsilon e^{-tR/L}}$$

$$= \varepsilon \checkmark$$

[let $I = \frac{\varepsilon}{R} - z$ $\quad \therefore \frac{dI}{dt} = -\frac{dz}{dt}$

$$\therefore \quad \varepsilon = -L\frac{dz}{dt} + \left(\frac{\varepsilon}{R} - z\right)R$$

$$\cancel{\varepsilon} = -L\frac{dz}{dt} + \cancel{\varepsilon} - Rz$$

$$L\frac{dz}{dt} = -Rz$$

$$\int_{z_0}^{z} \frac{1}{z'}\,dz' = -\frac{R}{L}t$$

when $t = 0$, $I = 0$

so $z_0 = \varepsilon/R$

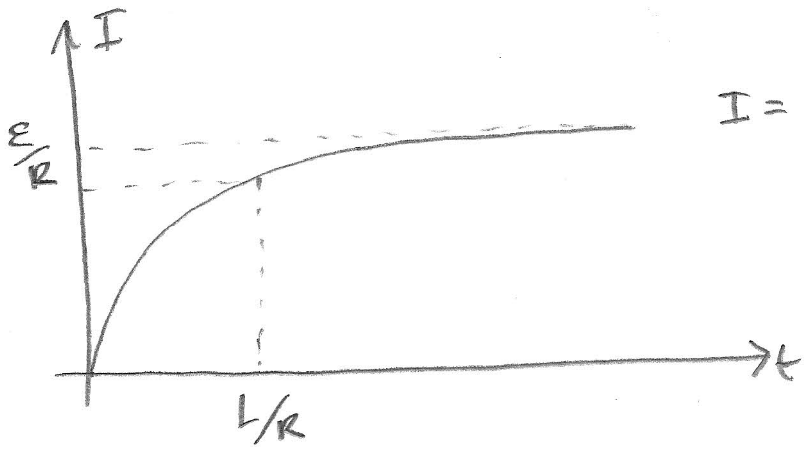
$$\therefore \int_{\varepsilon/R}^{z} \frac{1}{z}\,dz = -\frac{R}{L}t$$

$$\therefore \left[\ln z\right]_{\varepsilon/R}^{z} = -\frac{Rt}{L}$$

$$\ln\left(\frac{z}{\varepsilon/R}\right) = -\frac{Rt}{L}$$

$$z = \frac{\varepsilon}{R}e^{-Rt/L}$$

$\therefore$ since $I = \frac{\varepsilon}{R} - z \Rightarrow \boxed{I = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right)}$



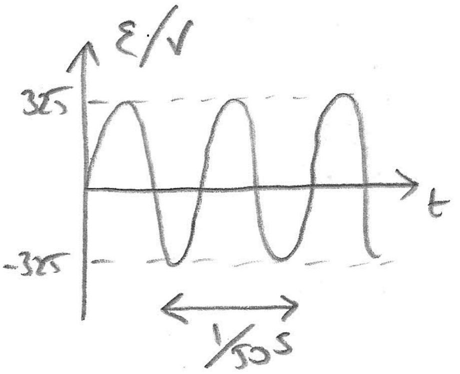
$I = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right)$

For our filament bulb : $L/R \simeq \boxed{8.6 \times 10^{-8} s}$

when $t = L/R$: $I = \frac{\varepsilon}{R}\left(1 - e^{-1}\right) \simeq 0.632\,\varepsilon/R$

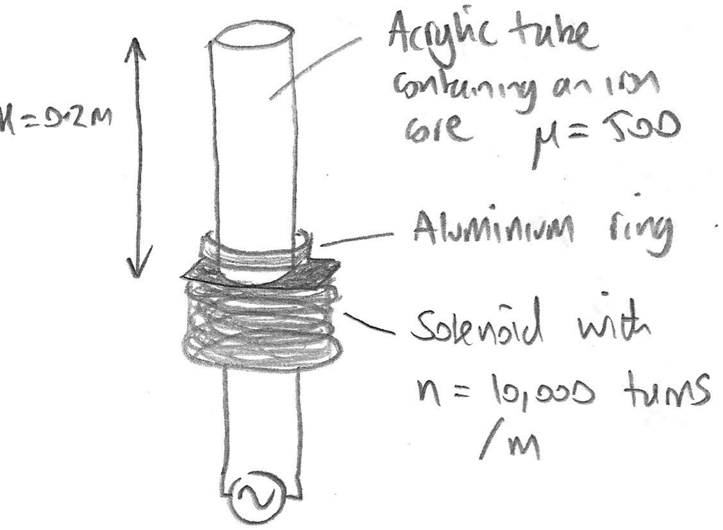
$\varepsilon/R = \frac{230}{19.5} = \boxed{11.8 A}$

c) Mains AC: $\varepsilon = 325 \sin\left(2\pi \times 50 t\right)$



The current establishment time $\simeq 8.6 \times 10^{-8} s$
is << than the period of the AC $(2.0 \times 10^{-2} s)$
(a factor of about 4.3×10^{-6}). So ∴ we can largely
ignore the induction effects on the timescales of mains AC.
Conversely, the assumption of constant ε in $I = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right)$
is valid. over a $10^{-7} s$ time period, the mains voltage
$\simeq$ constant.

6/

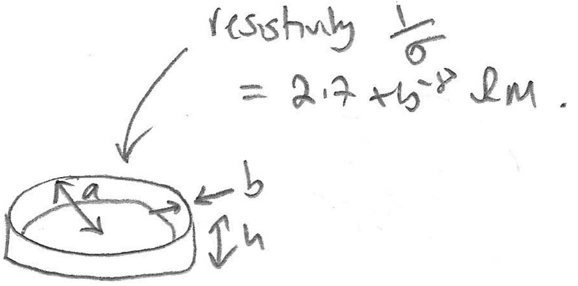


Mains AC

$I = I_0 \sin \omega t$

$I_0 = 0.2$ A

$\omega = 2\pi \times 50$ Hz

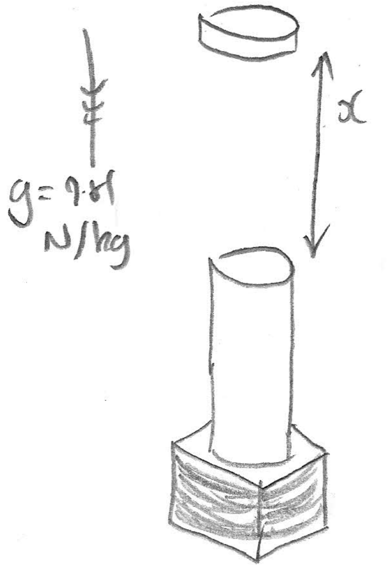


$a = 3.5$ cm

$b = 1.00$ mm

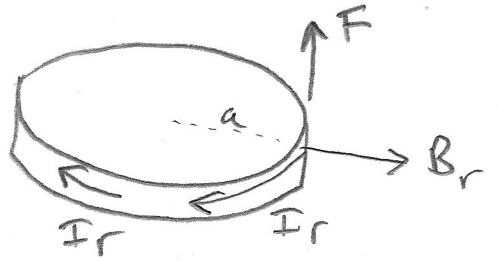
$h = 1.5$ cm

$\rho = 2700$ kg/m^3



Apogee of jumping ring.

a)

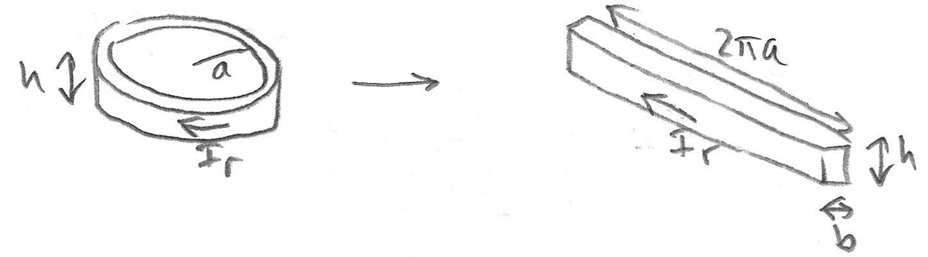


By Fleming's left hand rule, for an upward force on the ring, magnetic field must be radial and induced current tangential as shown.

∴ using "$F = BIl$"

$$\boxed{F = 2\pi a B_r I_r}$$

b) Since $b \ll a$, ring is $2\pi a$ long and with a cross section of bh.



∴ resistance $R = \frac{1}{\sigma}\frac{2\pi a}{bh}$

$$\boxed{R = \frac{2\pi a}{\sigma b h}}$$

$$\therefore\ R = \frac{2\pi \times 3.5\times10^{-2}\,m}{\frac{1}{2.7\times10^{-8}\,\Omega m}\times 1.00\times10^{-3}\,m \times 1.5\times10^{-2}\,m} = \boxed{4.0\times10^{-4}\,\Omega}$$

c) Flux linked by ring is $\Phi = \mu\mu_0 n I \times \pi a^2$

So EMF induced (if ignore "Back EMF" due to inductance of ring) is: $\mathcal{E} = -\frac{d\Phi}{dt} = \pi a^2 \mu\mu_0 n \frac{dI}{dt}$

If $I = I_0 \sin\omega t$

∴ $\frac{dI}{dt} = \omega I_0 \cos\omega t$.

∴ Max EMF induced is: $\mathcal{E} = \pi a^2 \mu\mu_0 n \omega I_0$

$\simeq \pi\times(3.5\times10^{-2})^2 \times 500 \times 4\pi\times10^{-7}\times 10^4 \times 2\pi\times 50 \times 0.2$ (V)

$\simeq \boxed{1.5\,V}$

Hence max current in the ring is $\simeq \frac{1.5\,V}{4.0\times10^{-4}\,\Omega}$

$= \boxed{3840\,A}$ (if don't use rounded results) i.e. I_r is really large! If ring is 'held down' expect significant heating.

d) let $B_r = 5\times10^{-4}\ B$

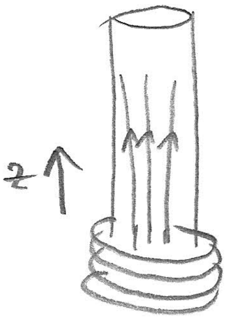
$\uparrow$

$B = \mu\mu_0 n I$

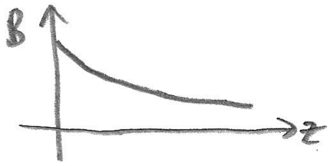
i.e. don't expect much radial deviation of magnetic field in iron side

$\therefore F = 2\pi a I_r B_r$

so $F = 2\pi\times3.5\times10^{-2}\times3840$
$\times 500\times4\pi\times10^{-7}\times10^{4}\times0.2$
$\times 5\times10^{-4}$
$= \boxed{0.53\ N}$



Might expect B (and $\therefore B_r$) $\downarrow$ above solenoid though also.



[Compare to Mg

$M = \rho\times2\pi a\times b\times h = 2700\times2\pi\times3.5\times10^{-2}\times1\times10^{-3}\times1.5\times10^{-2}$
$= 8.91\times10^{-3}$ kg

$\therefore Mg = 0.087\ N$ i.e. 16% of 'magnetic upthrust']

$\uparrow$

so probably can't ignore.

So net work done in rising x above top of iron column is $FH - MgH$

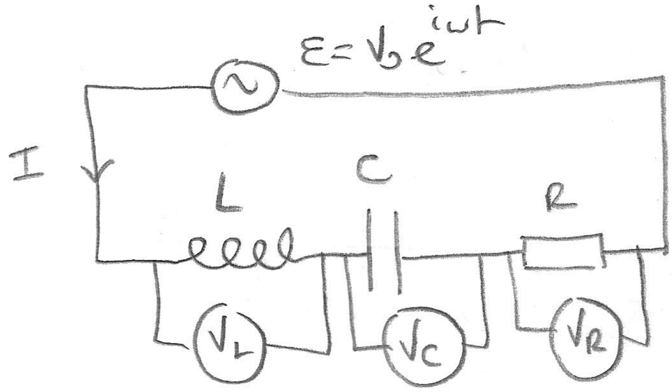
Let this turn into GPE Mgx, i.e. total energy at apogee, if GPE zero taken at top of column.

$$\therefore x = \frac{FH - MgH}{Mg}$$

$$x = \left(\frac{F}{Mg} - 1\right)H$$

$$x = \left(\frac{0.53}{0.087} - 1\right)\times0.2 \simeq \boxed{1.01\ m}$$

7/ a)



ohm's law generalized to 'complex impedances'

$$V_C = I Z_C$$

$$\mathcal{E} = I(Z_L + Z_C + Z_R)$$

$$\therefore \quad V_C / \mathcal{E} = \frac{Z_C}{Z_L + Z_C + Z_R}$$

$$Z_C = \frac{1}{i\omega C}$$

$$Z_L = i\omega L$$

$$Z_R = R$$

$$\therefore \quad V_C = \frac{V_0 e^{i\omega t} \frac{1}{i\omega C}}{i\omega L + \frac{1}{i\omega C} + R}$$

[kirchoff II: $\mathcal{E} = V_L + V_C + V_R$

$$\mathcal{E} = +L\frac{dI}{dt} + \frac{1}{C}\underbrace{\int I\,dt}_{CV_C = \int I dt} + IR$$

let $I = I_0 e^{i\omega t - i\phi}$ $\quad \therefore$

$$\mathcal{E} = +Li\omega I + \frac{1}{i\omega C} I + IR$$

$$\mathcal{E} = I\underbrace{\left(i\omega L + \frac{1}{i\omega C} + R\right)}_{Z}$$

$$\mathcal{E} = IZ$$]

$$\therefore \quad V_C = \frac{V_0 e^{i\omega t}}{-\omega^2 LC + 1 + i\omega RC}$$

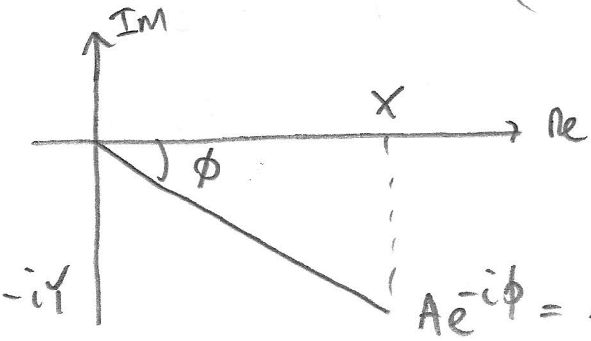
$$\boxed{V_C = \frac{V_0 e^{i\omega t}\left(1 - \omega^2 LC - i\omega RC\right)}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

$$\left[\frac{1}{a+ib} = \frac{a-ib}{(a+ib)(a-ib)} = \frac{a-ib}{a^2 - i^2 b} = \frac{a-ib}{a^2+b^2} \right]$$

(18)

let $V_c = A e^{i(\omega t - \phi)} V_0$

$$\therefore \quad A e^{-i\phi} = \frac{(1-\omega^2 LC - i\omega RC)}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$



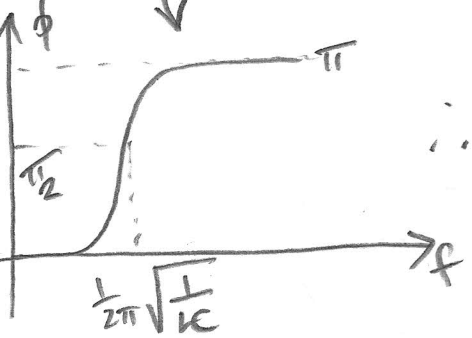
$A e^{-i\phi} = X - iY$

$$X = \frac{1-\omega^2 LC}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$

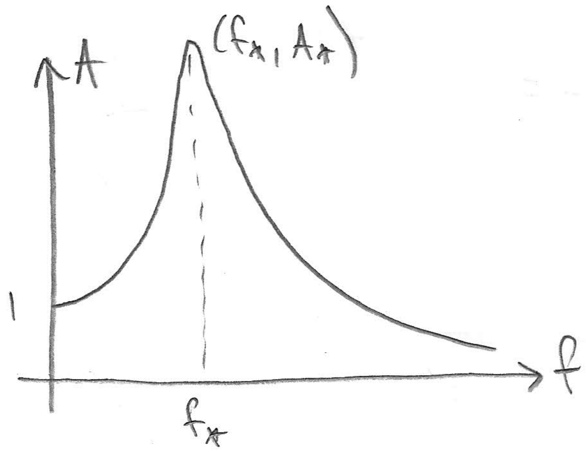
$$Y = \frac{\omega RC}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right)}$$

$$A = \sqrt{X^2 + Y^2} = \sqrt{\frac{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}{[(1-\omega^2 LC)^2 + \omega^2 R^2 C^2]^2}}$$



$$\therefore \quad \boxed{A = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}}$$



$$\frac{dA}{d\omega} = -\frac{1}{2}\left((1-\omega^2 LC)^2 + \omega^2 R^2 C^2\right)^{-3/2}$$

$$\times \left[2(1-\omega^2 LC)(-2\omega LC) + 2\omega R^2 C^2\right]$$

so $\frac{dA}{d\omega} = 0$ when

$\omega = 2\pi f$

$$2\cancel{\omega} LC(1-\omega^2 LC) = \cancel{\omega} R^2 C^2$$

$$1-\omega^2 LC = \tfrac{1}{2} R^2 C^2 / LC$$

$$\frac{1 - \frac{1}{2} R^2 C^2 / LC}{LC} = \omega^2$$

$$\therefore \quad \omega_* = \sqrt{\frac{1}{LC} - \frac{R^2C^2}{2L^2C^2}}$$

$$\omega_* = \sqrt{\frac{1}{LC} - \frac{1}{LC}\left(\frac{R^2C^2}{2LC}\right)}$$

$$\omega_* = \sqrt{\frac{1}{LC}}\left(1 - \frac{R^2C^2}{2LC}\right)^{1/2}$$

$$\therefore \quad \boxed{f_* = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}\left(1 - \frac{R^2C^2}{2LC}\right)^{1/2}}$$

Now: $\quad 1 - \omega_*^2 LC = \frac{1}{2} R^2C^2/LC$

$$\omega_*^2 R^2C^2 = \left(\frac{1}{LC} - \frac{R^2C^2}{2L^2C^2}\right)R^2C^2$$

$$= \frac{R^2C^2}{LC} - \frac{R^4C^4}{2L^2C^2}$$

$$= \frac{R^2C^2}{LC}\left(1 - \frac{R^2C^2}{2LC}\right)$$

$$\therefore \quad \left(1 - \omega_*^2 LC\right)^2 = \frac{1}{4}\frac{R^2C^2}{LC}\frac{R^2C^2}{LC}$$

$$\therefore \quad \left(1 - \omega_*^2 LC\right)^2 + \omega_*^2 RC = \frac{R^2C^2}{LC}\left(\frac{1}{4}\frac{R^2C^2}{LC} + 1 - \frac{R^2C^2}{2LC}\right)$$

$$= \frac{R^2C^2}{LC}\left(1 - \frac{1}{4}\frac{R^2C^2}{LC}\right)$$

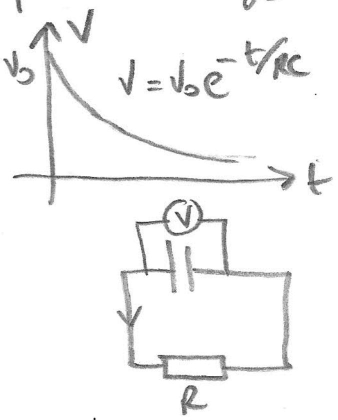
$$\therefore \quad \boxed{A_* = \frac{\sqrt{LC}}{RC}\left(1 - \frac{1}{4}\frac{R^2C^2}{LC}\right)^{-1/2}}$$

Now clearly a natural frequency of the system is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

so period $T = \frac{1}{f_0} = \boxed{2\pi\sqrt{LC}}$

RC is a characteristic decay time for capacitor charge and discharge.



Define : $\tau_c = RC$

$\tau_L = \sqrt{LC}$

$\therefore$ $$\boxed{f_* = f_0\left(1 - \frac{1}{2}\left(\frac{\tau_c}{\tau_L}\right)^2\right)^{\frac{1}{2}}}$$

and $$\boxed{A_* = \frac{\tau_L}{\tau_c}\left(1 - \frac{1}{4}\left(\frac{\tau_c}{\tau_L}\right)^2\right)^{-\frac{1}{2}}}$$

if $\tau_L \gg \tau_c$ $\quad \boxed{\begin{array}{l} f_* \approx f_0 \\ A_* \approx \frac{\sqrt{L}}{R\sqrt{C}} \end{array}}$ $\Rightarrow f_* \approx \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

So $L\uparrow \Rightarrow$ $f_*\downarrow$ as $f_* \propto \frac{1}{\sqrt{L}}$

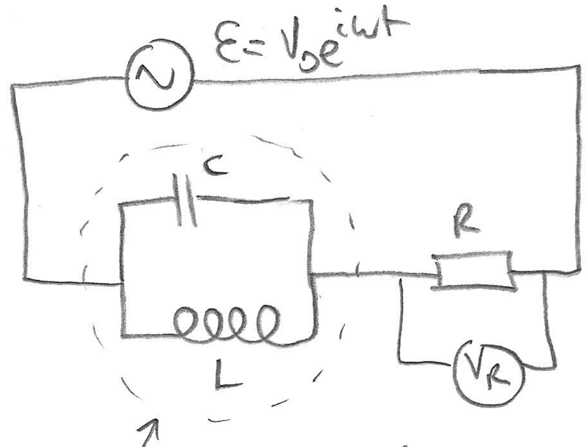
$A_*\uparrow$ as $A_* \propto \sqrt{L}$

$C\uparrow \Rightarrow$ $f_*\downarrow$ as $f_* \propto \frac{1}{\sqrt{C}}$

$A_*\downarrow$ as $A_* \propto \frac{1}{\sqrt{C}}$

$R\uparrow \Rightarrow$ $A_*\downarrow$ as $A_* \propto \frac{1}{R}$

8/



$$V_R = \frac{\mathcal{E} R}{Z + R}$$

$$V_R = \frac{\mathcal{E} R}{\frac{i\omega L}{1-\omega^2 LC} + R}$$

impedance $Z = \dfrac{1}{\frac{1}{Z_C} + \frac{1}{Z_L}}$

$$= \frac{1}{i\omega C + \frac{1}{i\omega L}}$$

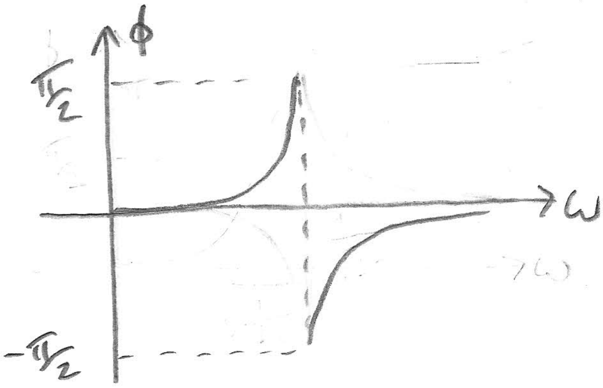
$$= \frac{i\omega L}{-\omega^2 LC + 1}$$

$$V_R = \frac{\mathcal{E} R (1-\omega^2 LC)}{i\omega L + R(1-\omega^2 LC)}$$

$$V_R = \frac{\mathcal{E} R (1-\omega^2 LC)(R(1-\omega^2 LC) - i\omega L)}{\omega^2 L^2 + R^2(1-\omega^2 LC)^2}$$

$$V_R = A V_0 e^{-i\phi} e^{i\omega t}$$

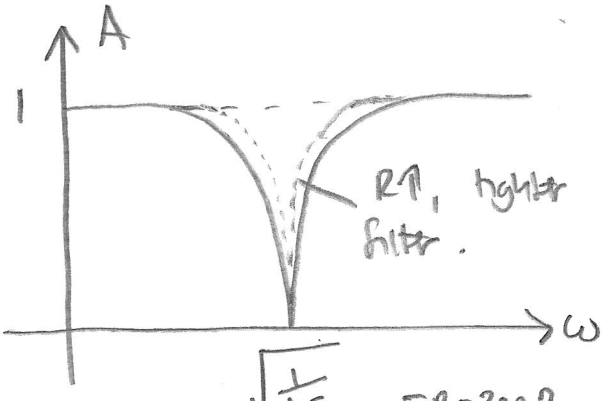
$$\therefore \boxed{\phi = \tan^{-1}\left(\frac{\omega L/R}{1-\omega^2 LC}\right)}$$



$(1-\omega^2 LC < 0$
$\Rightarrow \omega^2 > \frac{1}{LC} \Rightarrow \omega > \frac{1}{\sqrt{LC}})$

$$A = \sqrt{\frac{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}{(\omega^2 L^2 + R^2(1-\omega^2 LC)^2)^2}} \times |R(1-\omega^2 LC)|$$ ← is $\sqrt{(R(1-\omega^2 LC))^2}$

$$\boxed{A = \frac{|R(1-\omega^2 LC)|}{\sqrt{\omega^2 L^2 + R^2(1-\omega^2 LC)^2}}}$$



This could be used as a $\boxed{\text{filter}}$
eg remove 'mains hum' if $\frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 50\text{Hz}$.

[$R = 200\Omega$
$C = 203\,\mu F$
$L = 0.05\,H$
$\Rightarrow f_0 = 50\text{Hz}$]