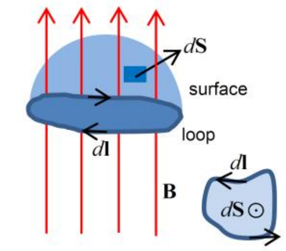
Electric $\mathbf{E}$ fields and *Magnetic* $\mathbf{B}$ fields act upon *charges*. For magnetic fields to have any effect, the charges must be *moving,* whereas electric fields act on charges, regardless of their motion. The source of electric fields are charges, and the source of magnetic fields are moving charges (i.e. currents). The force $\mathbf{f}$ on a charge q moving with velocity $\mathbf{v}$ is: $\mathbf{f} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. The cross-product term describes the mutually perpendicular *'Fleming's Left Hand Rule'* for *'Thrust, Field and Current.'* The latter is also known as the ***motor effect,*** as it is the basis for mechanical devices with an electricity-powered motive force. i.e. place a current carrying wire in a magnetic field (whose direction has a component B perpendicular to the current direction), and there will be a **force** on the wire of magnitude BIl in a *mutually perpendicular* direction to both the current and the magnetic field. l is the length of the wire and I the current.

As a converse to the motor effect, **Faraday's law of electromagnetic induction** describes how *electricity can be generated by moving a conductor in a magnetic field*. i.e. movement and magnetic field are now inputs, which *cause* current to flow. ***"The Electromotive Force (EMF - i.e. a voltage)*** ε ***induced in a closed loop of a conductor is proportional to the rate of change of magnetic flux*** Φ ***that the loop encloses."*** $\varepsilon = -d\Phi/dt$.

The negative sign above is a consequence of **Lenz's law.** Currents that are induced as a result of Faraday's law must result in a situation that *opposes, or resists, the production of further currents.* If this were not the case, then simply by moving a conductor in a magnetic field we could generate infinite amounts of energy as the system would 'self catalyze.' A typical example is the induction of currents in a *solenoid* when moving a bar magnet close to it. If the North pole of the magnet approaches the solenoid, the currents induced must result in an *opposing* North pole, which means it is *harder* to push the magnet further towards the coil. This 'dynamic drag force' is also the basis of *electromagnetic braking* of metal wheels using magnets. *Eddy currents* induced in the wheels resulting from the wheel rotating through a magnetic field result in a magnetic interaction that will slow down the rotation. This is without any physical contact between the wheels and the brakes, thus minimizing wear and frictional heating.



By convention, surface normals are defined as *outward* if the enclosing loop is *anti-clockwise*

Magnetic flux Φ is the integral ("summation" essentially) of the magnetic field projection on *vector area elements* of the surface enclosed by the conductor loop. $\Phi = \int_{surface} \mathbf{B} \cdot d\mathbf{S}$

A simplification of Faraday's law is **"Rate of change of magnetic flux linked is proportional to EMF induced."** The 'linked' word relates to the number of coils N that experience the same magnetic flux. In an ***AC generator,*** N loops of area A rotate at angular speed ω in a uniform magnetic field of strength B. The flux linked is therefore: $\Phi = NBA\cos\omega t$. When $t = 0$ the vector area of the loop is parallel to the magnetic field. Applying Faraday's law, the EMF induced is: $\varepsilon = -NBA\omega\sin\omega t$. i.e. the EMF induced is proportional to the rotation frequency as well as the maximum flux linked. It is also 90 degrees out of phase with the rotation of the loop vector area. i.e. the maximum EMF is induced when the plane of the loop is parallel to the magnetic field, and no flux is linked. *Flux linked* is in units of ***Webers*** (Wb). $1\text{Wb} = \text{m}^2\text{T}$

An **inductor** is a fundamental electrical component comprising of a (possibly miniaturized) solenoid. When current I flows through an inductor of inductance L, the *back-EMF induced* is: $\varepsilon = -L\,dI/dt$. If an inductor is a solenoid of N turns in total length l, has cross sectional area A and contains a core of relative permeability μ, the inductance is:

$$L = \frac{\mu\mu_0 N^2 A}{l}$$. The permeability of free space $\mu_0 = 4\pi \times 10^{-7}\,\text{NA}^{-2}$.

Combinations of inductors, capacitors and resistors form the basis of nearly all electrical devices. A series 'LCR' circuit, if driven by an AC source, can be used to boost signals at specific frequencies. i.e. it will become a 'resonant' circuit, and the basis of analogue radio receivers. The resonant frequency of an LCR circuit is $f_0 = \frac{1}{2\pi}\sqrt{1/LC}$ where C is the capacitance. If the components are wired in parallel the circuit has a minimum response at f_0. Inductance is measured in ***Henrys*** (H). $1\text{H} = 1\text{VsA}^{-1}$

Faraday's law also applies to a time varying magnetic field in the vicinity of a static conductor. This is the basis behind most **generators** (i.e. you spin a magnet rather than the coils to reduce friction in the *slip ring* electrical contacts) and also **transformers**, and also explains why most power distribution networks are **AC** *not* DC. This is *because voltages and currents can be varied with high efficiency* using electromagnetism in a transformer.

A transformer is a pair of windings round opposing sides of a loop made of iron or similarly ferromagnetic material. If the flux linked, respectively, by primary (P) and secondary (S) coils is: $\Phi_S = N_S BA$, $\Phi_P = N_P BA$; applying Faraday's law yields: $V_S/V_P = N_S/N_P$ which is the **Transformer Equation**. If the transformer is 100% efficient, this means power transfer from primary to secondary coils is without loss: i.e. $I_P V_P = I_S V_S$ so therefore $I_S/I_P = V_P/V_S = N_P/N_S$.

i.e. high voltage, low current from a power transmission line can be converted to low voltage, high current for light industrial or domestic use. The last two equations are approximations. In reality on must consider the frequency response of a transformer (which varies with primary and secondary coil inductances, and also the resistances in the secondary and primary circuits). One must also consider the *mutual inductance* between the coils in calculation of the induced EMF. The transformer equation is valid for high frequencies, with low secondary current and a high degree of coupling between primary and secondary coils.

Question 1

(i) The Airbus A380 double-decker airplane has a wingspan of $l = 79.8\text{m}$ and a cruising speed of $v = 1{,}050\text{km/h}$. The Earth's magnetic field has strength (in the polar regions) of about 65μT. Assume the A380 takes a high latitude route, where the Earth's magnetic field is perpendicular to the velocity of the aircraft. Calculate the magnetic flux linked by the aircraft wings per second, and hence determine the EMF induced between the wing tips. If the aluminum skin on the wings is on average about 8.0m wide and 0.7mm thick, estimate the current induced in the wings if the resistivity of aluminum is about $\rho = 2.7\times10^{-8}\,\Omega\text{m}$.

(ii) A solar flare consists of radio waves and a 'coronal mass ejection' (CME) of charged particles. Assume an average CME travels at 490km/s and consists of a beam of protons of mass $m = 1.67\times10^{-27}\,\text{kg}$ and charge $e = 1.60\times10^{-19}\,\text{C}$. When the particles reach Jupiter, they encounter a magnetic field perpendicular to their motion of strength B = 417μT. Determine the radius r of the circular paths the magnetic field causes the protons to execute.

(iii) A diesel-powered portable generator is taken on an adventure camp in the Lake District. It supplies mains AC at 50Hz, which means a sinusoidal voltage vs time output, with an amplitude of 325V. The generator employs a magnet of field strength $B = 0.05\text{T}$ and the internal windings enclose an area of $20\text{cm}\times15\text{cm}$. Calculate the number of windings N.

(iv) A 500kV aluminum power line connects a power station to a substation 100km away. The substation has a step-down transformer to produce appropriate voltages for industrial and domestic use. (a) Calculate the number of primary coil windings N_P if the secondary windings $N_S = 50$ and the desired output is 4kV. (b) Assume the power line carries 660MW of power, and the transformer is 95% efficient. What current is flowing through the primary coil, and what flows through the secondary coil? (c) If the power line suffers a loss of 1% over the 100km, calculate the resistance of the cable and hence its radius r, assuming that the current flows uniformly through the whole body of the cable. (d) Compare r to the 'skin depth' $(\delta/\text{mm}) \approx 5.03\times10^5\sqrt{\rho/f}$, which is the effective depth of AC currents of frequency f in a conductor of resistivity ρ and with a relative permeability of unity. Assume $f = 50\text{Hz}$.

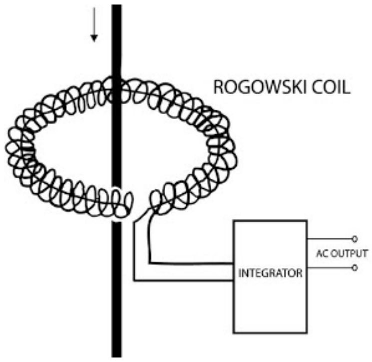
(v) In a school laboratory there is a cylindrical air-cored inductor of diameter 9.0cm and height 5.0cm which is labeled as having an inductance of exactly 1.0H. Calculate the number of turns N.

(vi) An analogue electrical circuit consisting of a resistor, capacitor and inductor wired in parallel are used to filter out a 'mains hum' at 50Hz from low-frequency radio signals. If a 0.2H inductor is used, calculate the capacitance required.

(vii) A bar magnet is rotated at 1000RPM directly above a solenoid of 2400 turns. The bar magnet is cuboid in shape, with the end being a square of sides 0.5cm. An EMF of 12.6mV is induced in the coil. Calculate the magnetic field strength (in T) experienced by the solenoid. How would the EMF change if the magnet was spun at 5000RPM, and twice as far away from the solenoid, and the solenoid had only 1200 turns? What was the original magnetic field strength (in mT) ?

(viii) A circuit consisting of a resistor of resistance R, a capacitor of capacitance C and inductor of inductance I is connected in series to a voltage source of EMF ε. If current I flows through the circuit, and voltage V is between the capacitor plates, explain why: $I\varepsilon = I^2R + IL\frac{dI}{dt} + CV\frac{dV}{dt}$ and hence why the energy stored in an inductor and capacitor (respectively) is: $\frac{1}{2}LI^2$ and $\frac{1}{2}CV^2$.

Question 2 A *Rogowski coil* is an electrical device for measuring AC currents in a wire *without* inserting an ammeter in series with the wire. It is a *toroidal* coil of wire of N turns, with loop cross section A, and the torus has radius r. A wire is coiled around in a 'nearly complete' ring and then looped back through the centre of the torus.



If the magnetic field at radius r from a current carrying wire is: $B = \mu_0 I/2\pi r$, show that an AC current of the form $I = I_0 \cos\omega t$ will result in an induced voltage output from the Rogowski coil of the form: $V = \frac{\omega N A \mu_0 I_0}{2\pi r}\sin\omega t$.

Question 3 A *Magnetron*, the key component in microwave ovens, and also in many radar transmitters, consists of a hot cathode which produces electrons, which are then accelerated in a radial sense to an anode. The cathode is a cylindrical wire of radius r and the anode is an outer cylinder of radius R. Assume a vacuum between these. A radial electric field exists between inner cathode and outer anode of strength $E = \lambda/2\pi\varepsilon_0 a$, where a is the radius from the centre of the cathode. Now consider the effect of a uniform magnetic field, perpendicular to the electric field and longitudinal to the cylinder. If voltage V is placed between anode and cathode, determine an expression for the magnetic field strength B which eventually results a circular path of electrons (mass $m = 9.11\times10^{-31}\,\text{kg}$) of radius R, and also calculate the frequency of these circular orbits. *Hint #1:* $V = \int_r^R E dr$. Use this to find λ. *Hint #2:* Think why the magnetic field does no work on the electron, and hence by conservation of energy: $\frac{1}{2}mv^2 = eV$ where v is the tangential velocity of circular orbits at radius R. *Note in a Magnetron, the circulating electrons 'pump' further circulations in* ***circular cavities*** *in the anode. The result of this acceleration of charge is an emission of high frequency electromagnetic radiation which is typically in the high MHz to low GHz range.*

Question 4 A small neodymium magnet is dropped from rest at the top of a vertical plastic tube. At about mid-way down the tube are a tight coil of wires, perhaps several thousand turns in a few cm. The coils are connected to a pair of identical light-emitting diodes (LEDs) wired in parallel, which are themselves wired in opposite directions. When the magnet reaches the coils the first solenoid lights up, peaking in brightness and then dropping to zero. At this point the second LED lights up, increasing in brightness to an even greater extent to the first LED, before also dropping to zero.

(a) Sketch a graph of the LED brightness vs time. Use a solid line for LED #1 and a dotted line for LED #2.

(b) Sketch a graph of the EMF induced in the coil by the movement of the magnet through it. The EMF is initially positive.

(c) Use a sequence of diagrams involving the magnetic field linking the coil to (qualitatively) explain the features of the EMF vs time graph. Assume the magnet North pole is the underside i.e. pointing downwards.

(d) The magnet is now dropped down a thick pipe made from aluminum. Explain qualitatively what you think will happen, and why. In your answer, explain the similarities and differences between this situation and the 'coil drop' in parts (a) to (c).

Question 5 A filament light bulb consists of a tungsten wire of $N = 50$ coils in length $l =$ 2.0cm. The wire diameter is 0.046mm and the uncoiled length is 580mm. The resistivity of tungsten is $\rho = 5.60\times10^{-8}\,\Omega\text{m}$.

(a) Calculate (a) the resistance of the filament and (b) the inductance of the filament.

(b) A voltage of $\varepsilon =$ 230V is applied to the filament. By modeling it at an inductor and fixed resistor in series, show from Ohm's Law that: $\varepsilon = L\frac{dI}{dt} + IR$. Hence show that if ε is constant, the current established in the bulb is: $I = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right)$. Sketch I vs t using appropriate scales.

(c) Assume $\varepsilon = 325\sin\left(2\pi \times 50t\right)$ where t is in seconds i.e. UK mains AC. Comment on the 'current establishment time' as a result of the inductance and resistance of the filament, compared to the period of main AC.

Question 6 A ring of aluminum of radius $a = 3.5\text{cm}$, thickness $b = 1.00\text{mm}$, height $h = 1.5\text{cm}$, density $\rho = 2700\text{kgm}^{-3}$ and conductivity (i.e. the reciprocal of resistivity) $\sigma = 1/\left(2.7\times10^{-8}\Omega\text{m}\right)$ is placed around the bottom of an acrylic tube of height $H = 0.2\text{m}$ containing iron. Assume $a \gg b$. The ring initially rests on the top of a solenoid of $n = 10{,}000$ turns per meter. When AC current $I = I_0 \sin\omega t$ is (briefly) passed through the solenoid, the ring is fired upwards into the air. For mains AC: $\omega = 2\pi\times 50\,\text{rads}^{-1}$. $I_0 = 0.2\text{A}$.

(a) In order for there to be an *upward* force on the ring, draw a diagram to indicate the direction of the ring currents and the magnetic field it experiences. Show that the upward force has magnitude $F = 2\pi a I_r B_r$ where I_r is the current induced in the ring and B_r is the magnetic field strength the ring experiences.

(b) Show that the resistance of the ring is $R = 2\pi a/\sigma bh \approx 4.0\times10^{-4}\Omega$

(c) The magnetic field strength (acting upwards) in the centre of the solenoid is $B = \mu\mu_0 nI$ where $\mu = 500$ is the relative permeability of the iron core. Show that the maximum EMF induced in the ring is (approximately) $\varepsilon = \pi a^2 \mu\mu_0 n I_0 \omega \approx 1.5\text{V}$ due to the solenoid and hence show the maximum current in the ring is about 3840A (!)

An EMF induced in the ring will cause a current to flow. However, since this is time varying, the inductance of the ring will mean a 'back EMF' results, which will reduce the answer to (c). Also, the ***mutual inductance*** *between the ring and the solenoid may not be as in (c), and indeed the direction an strength of the magnetic field experienced (upwards) by the ring will not be as much as* $B = \mu\mu_0 nI$. For more details see: Smith J.M.B. "The jumping ring and Lenz's law - an analysis." *Physc Educ* 2008; **43**: 265-269.

(d) If $B_r = kB$, where $k = 5\times10^{-4}$, calculate the upward force on the ring (in N). Hence determine how high it rises above the top of the acrylic/iron tube. Assume F is a constant for the whole tube height H.

Question 7 It can be shown that the (complex) *impedance* of an inductor is $Z_L = i\omega L$, the impedance of a capacitor is $Z_C = 1/i\omega C$ and the impedance of a resistor is $Z_R = R$. $f = \frac{1}{2\pi}\omega$ is the frequency of electrical signals flowing through these components. For a *reactive* circuit consisting of these components, Ohm's Law in the form $V = IZ$ applies in exactly the same way as it does for circuits of (passive) resistors i.e. $V = IR$ where I is current and V is potential difference. Consider an AC voltage source of the form $\varepsilon = V_0 e^{i\omega t}$, applied to a *series* circuit of a resistor, capacitor and inductor.

(a) Explain why $V_C = \frac{Z_C}{Z_C + Z_L + Z_R}\varepsilon$, and hence show that $V_C = V_0 A e^{i(\omega t - \phi)}$ where:

$$A = 1\Big/\sqrt{\left(1-\omega^2 LC\right)^2 + \omega^2 R^2 C^2} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right)$$

(b) Sketch the *amplitude response* $A = |V_C/V_0|$ vs frequency, and also *phase response* ϕ. Show that the peak of $A(\omega)$ is at: $\omega = \frac{1}{\sqrt{LC}}\left(1 - \frac{1}{2}\frac{R^2C^2}{LC}\right)^{\frac{1}{2}}$ and $A = \frac{\sqrt{LC}}{RC}\left(1 - \frac{1}{4}\frac{R^2C^2}{LC}\right)^{-\frac{1}{2}}$. How do the curves vary with L, C and R?

Question 8 Repeat the process in Q5 with the inductor and capacitor in *parallel*, and this pairing in *series* with the resistor. Also, instead of V_C, consider the voltage V_R across the resistor. What could this circuit be used for?