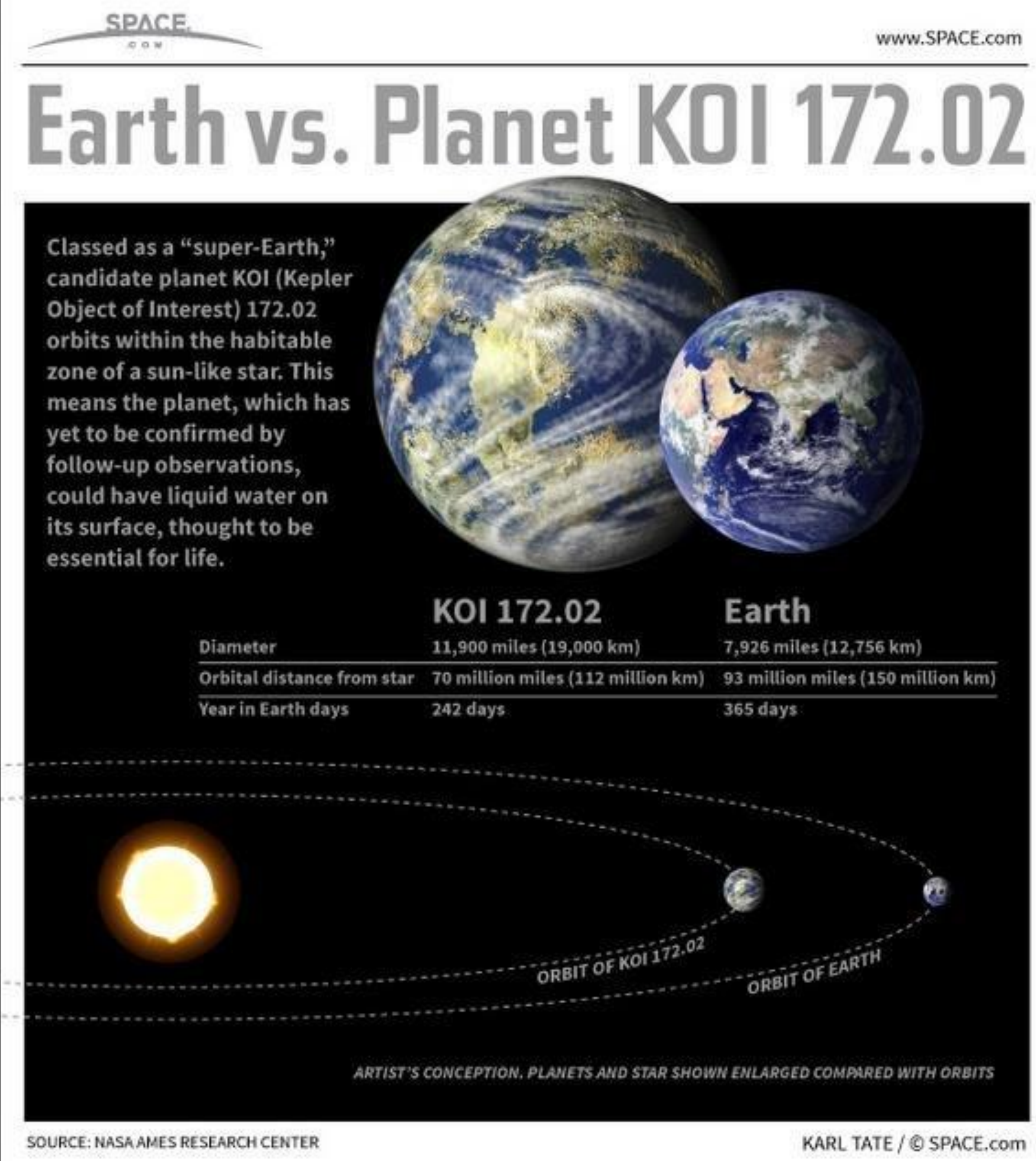
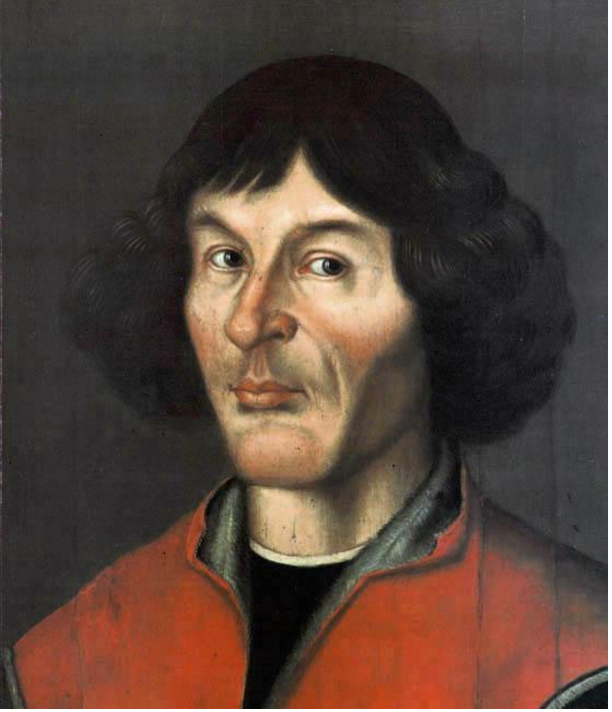
Extrasolar planets and Kepler's Third Law

$$T^2 = \frac{4\pi^2}{G(M+m)} a^3$$

AF. May 2015



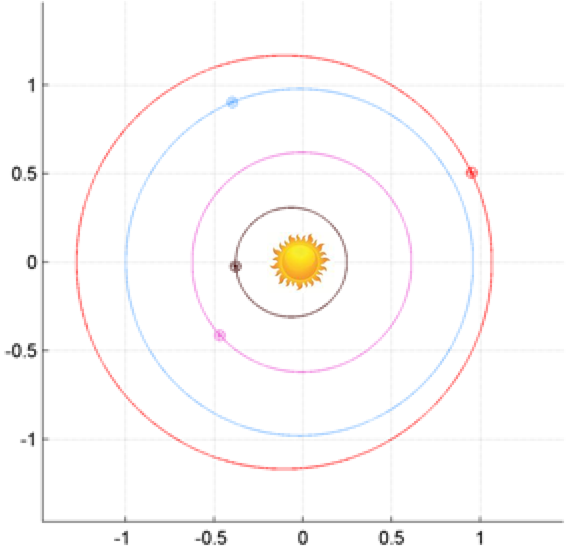


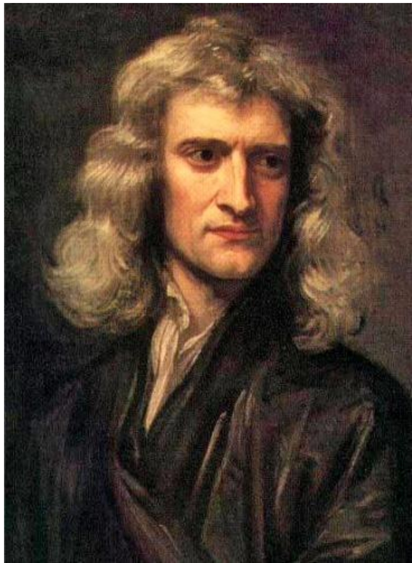
Nicolaus Copernicus
1473-1543

Tycho Brahe
1546-1601

Nose lost in 1566 following a sword duel with third cousin Manderup Parsberg over the legitimacy of a mathematical formula!

Johannes
Kepler
1571-1630





Isaac
Newton
1642-
1727

Kepler's three laws are:

1. *The orbit of every planet in the solar system is an ellipse with the Sun at one of the two foci.*
2. *A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.*
3. *The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.*

   The wording of Kepler's laws implies a specific application to the solar system. However, the laws are more generally applicable to any system of two masses whose mutual attraction is an inverse-square law.

$$r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\theta}$$ *Polar* equation of ellipse

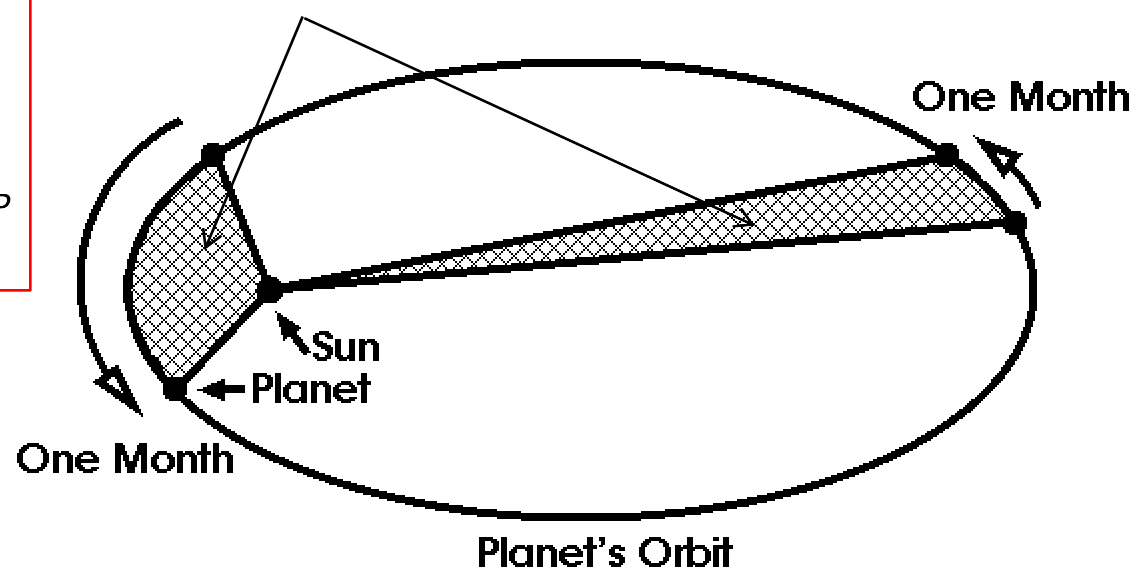
$$\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$$ *Eccentricity* of ellipse

$$P^2 = \frac{4\pi^2}{G(M+M_\odot)}a^3$$ Orbital period *P*

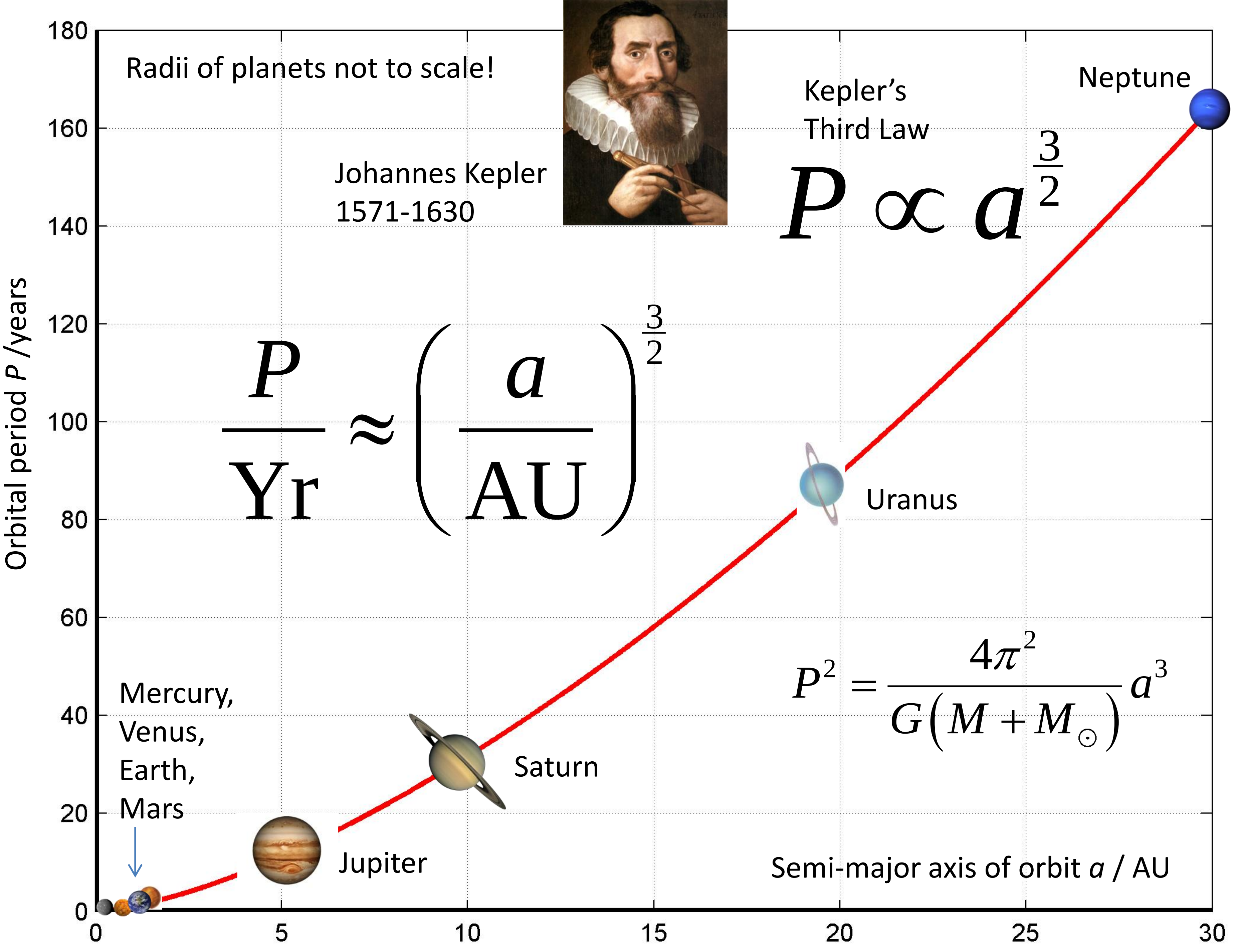
$$\frac{dA}{dt} = \tfrac{1}{2}\sqrt{G(M+M_\odot)\left(1-\varepsilon^2\right)a}$$

This is a constant

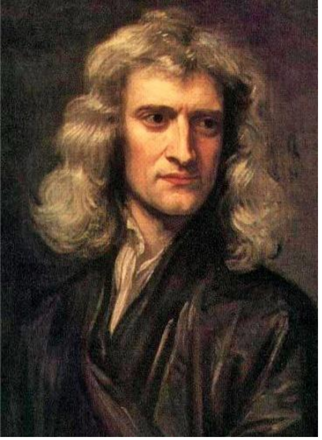
Equal areas swept out in equal times



Johannes Kepler
1571-1630



Radii of planets not to scale!
Johannes Kepler
1571-1630
Kepler's
Third Law
$P \propto a^{\frac{3}{2}}$
Neptune
$\frac{P}{\mathrm{Yr}} \approx \left(\frac{a}{\mathrm{AU}}\right)^{\frac{3}{2}}$
Uranus
$P^2 = \frac{4\pi^2}{G\left(M + M_{\odot}\right)} a^3$
Mercury,
Venus,
Earth,
Mars
Saturn
Jupiter
Semi-major axis of orbit a / AU
Orbital period P /years
0
20
40
60
80
100
120
140
160
180
0
5
10
15
20
25
30



Isaac Newton (1642-1727) developed a mathematical model of Gravity which predicted the elliptical orbits proposed by Kepler

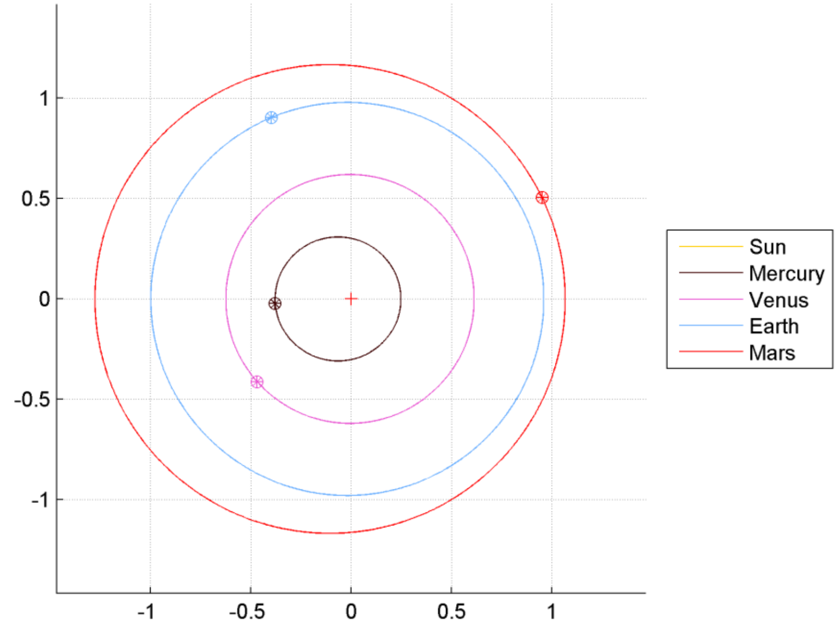
Force of gravity →
$$F = \frac{GMM_\odot}{r^2}$$
← Planet and Solar masses

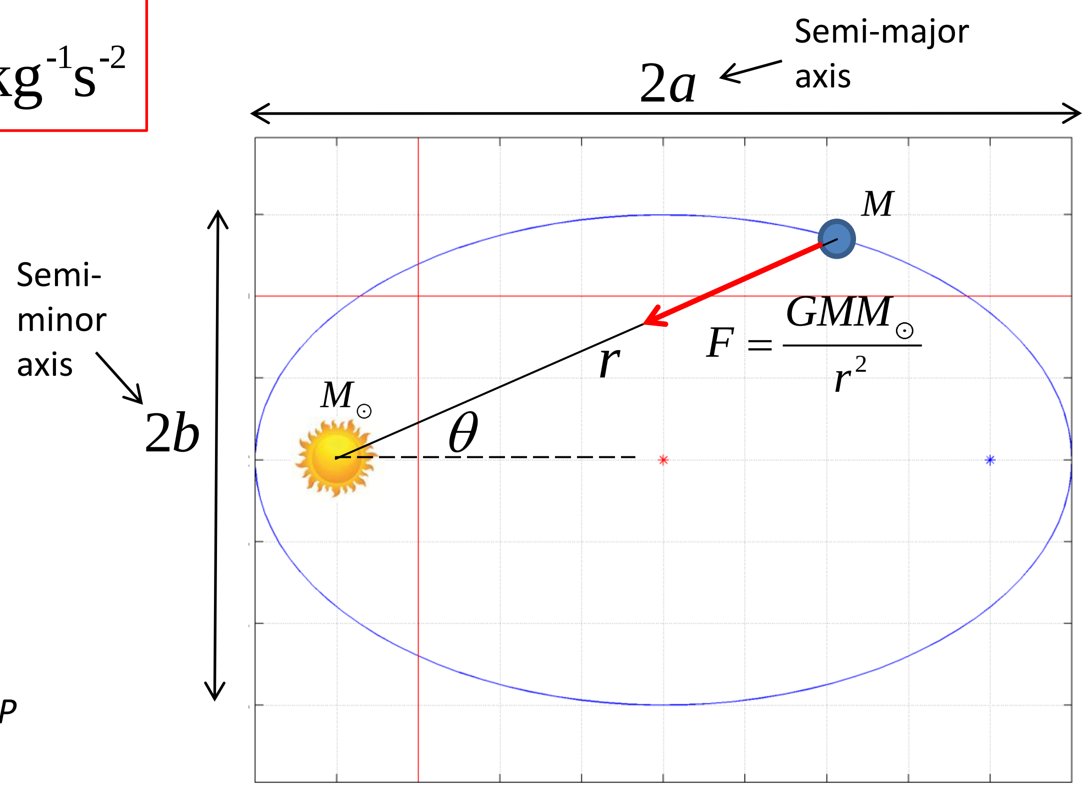
$$G = 6.67 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$$

$$r = \frac{a\left(1-\varepsilon^2\right)}{1-\varepsilon\cos\theta}$$ *Polar* equation of ellipse

$$\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$$ *Eccentricity* of ellipse

$$P^2 = \frac{4\pi^2}{G(M+M_\odot)}a^3$$ Orbital period *P*





Kepler's Third Law

$$M_\odot = 1.99\times10^{30}\,\mathrm{kg}$$

$$G = 6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$$

$$\mathrm{AU} = 1.49597871\times10^{11}\,\mathrm{m}$$

$$24\times3600\mathrm{s} = 1\mathrm{day}$$

$$T^2 = \frac{4\pi^2}{G(M+m)}a^3$$

$$\frac{T}{\mathrm{days}}\times3600\times24 = \frac{2\pi}{\sqrt{G(M+m)}}\times\mathrm{AU}^{\frac{3}{2}}\left(\frac{a}{\mathrm{AU}}\right)^{\frac{3}{2}}$$

$$\frac{T}{\mathrm{days}} = \frac{2\pi}{3600\times24\sqrt{G(M+m)}}\times\mathrm{AU}^{\frac{3}{2}}\left(\frac{a}{\mathrm{AU}}\right)^{\frac{3}{2}}$$

$$\frac{T}{\mathrm{days}} = \frac{2\pi\times\mathrm{AU}^{\frac{3}{2}}}{3600\times24\sqrt{GM_\odot}}\times\frac{1}{\sqrt{\frac{M}{M_\odot}+\frac{m}{M_\odot}}}\times\left(\frac{a}{\mathrm{AU}}\right)^{\frac{3}{2}}$$

$$\frac{T}{\mathrm{days}}f(M,m) = k\times\left(\frac{a}{\mathrm{AU}}\right)^{\frac{3}{2}}$$

$$k = \frac{2\pi\times\mathrm{AU}^{\frac{3}{2}}}{3600\times24\sqrt{GM_\odot}} \approx 365$$

$$f(M,m) = \sqrt{\frac{M}{M_\odot}+\frac{m}{M_\odot}}$$

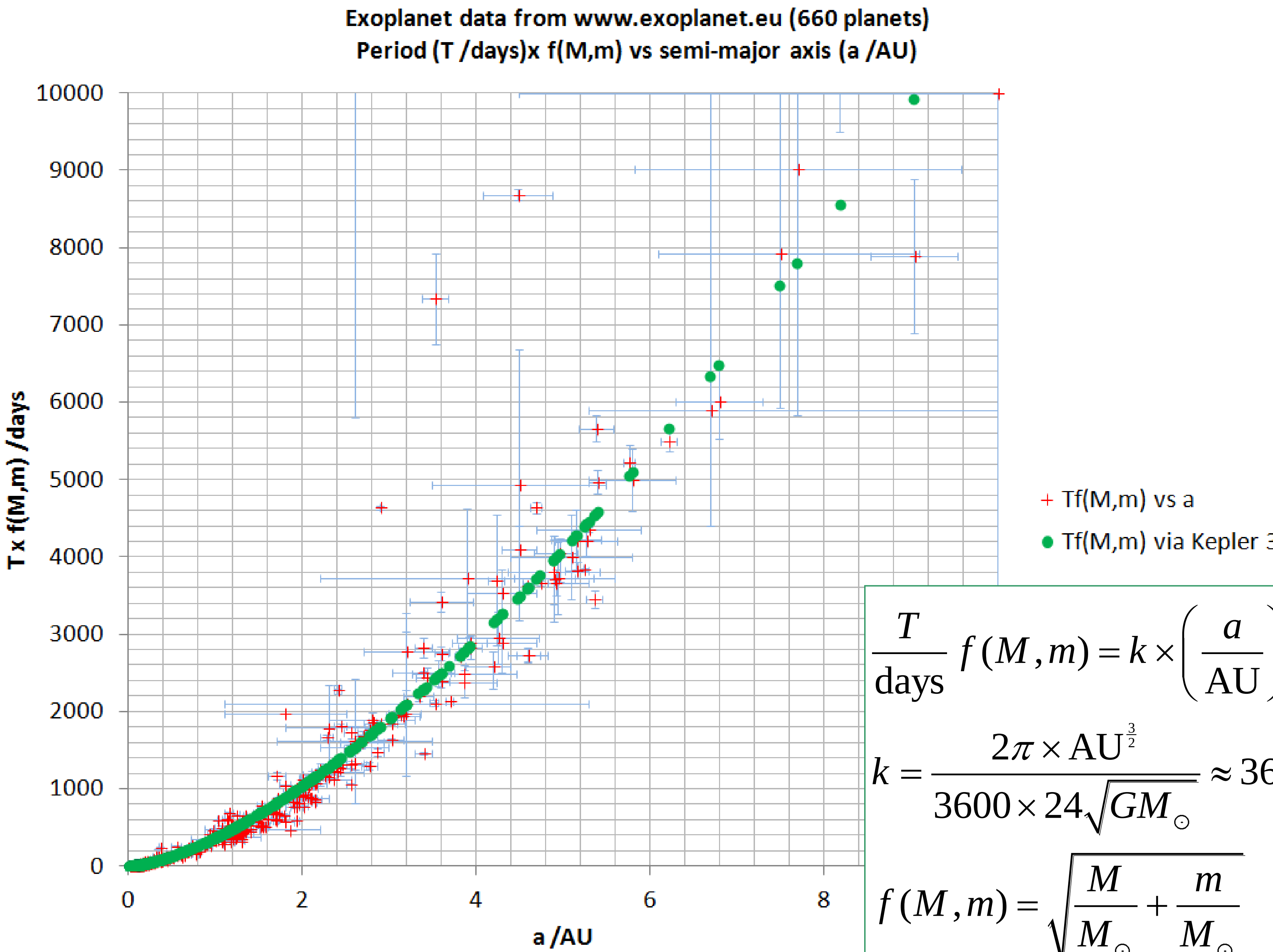
M star mass

m planet mass

T orbital period

a orbital 'radius'

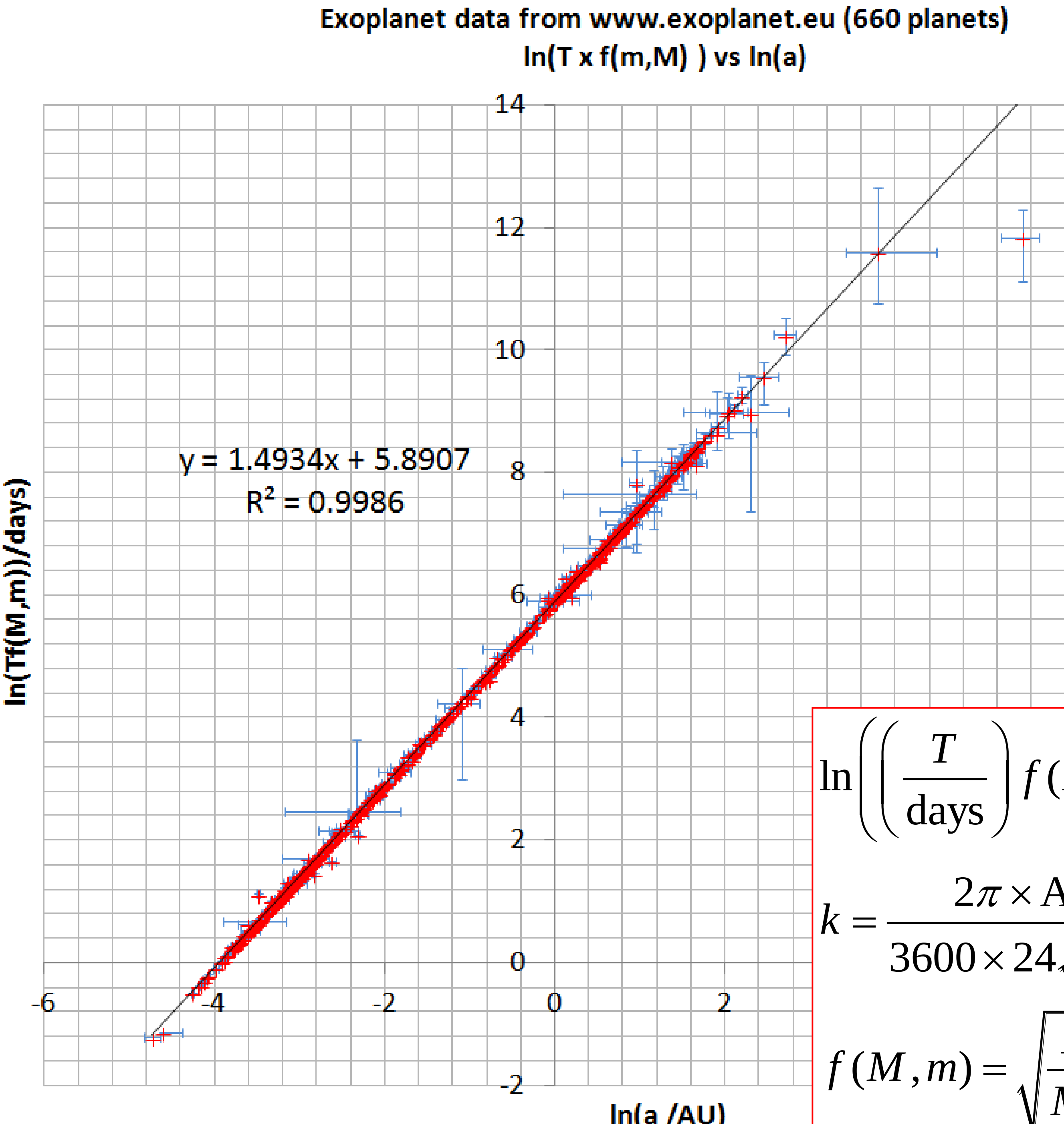
semi-major axis of elliptical orbit



$$\frac{T}{\text{days}} f(M,m) = k \times \left(\frac{a}{\text{AU}}\right)^{\frac{3}{2}}$$

$$k = \frac{2\pi \times \text{AU}^{\frac{3}{2}}}{3600 \times 24\sqrt{GM_{\odot}}} \approx 365$$

$$f(M,m) = \sqrt{\frac{M}{M_{\odot}} + \frac{m}{M_{\odot}}}$$



Vertical intercept is very close to ln(365) = 5.90

Gradient is 1.49, not far off the Kepler #3 prediction of 3/2

$$\ln\left(\left(\frac{T}{\text{days}}\right)f(M,m)\right) = \ln k + \tfrac{3}{2}\ln\left(\frac{a}{\text{AU}}\right)$$

$$k = \frac{2\pi \times \text{AU}^{\frac{3}{2}}}{3600 \times 24\sqrt{GM_\odot}} \approx 365$$

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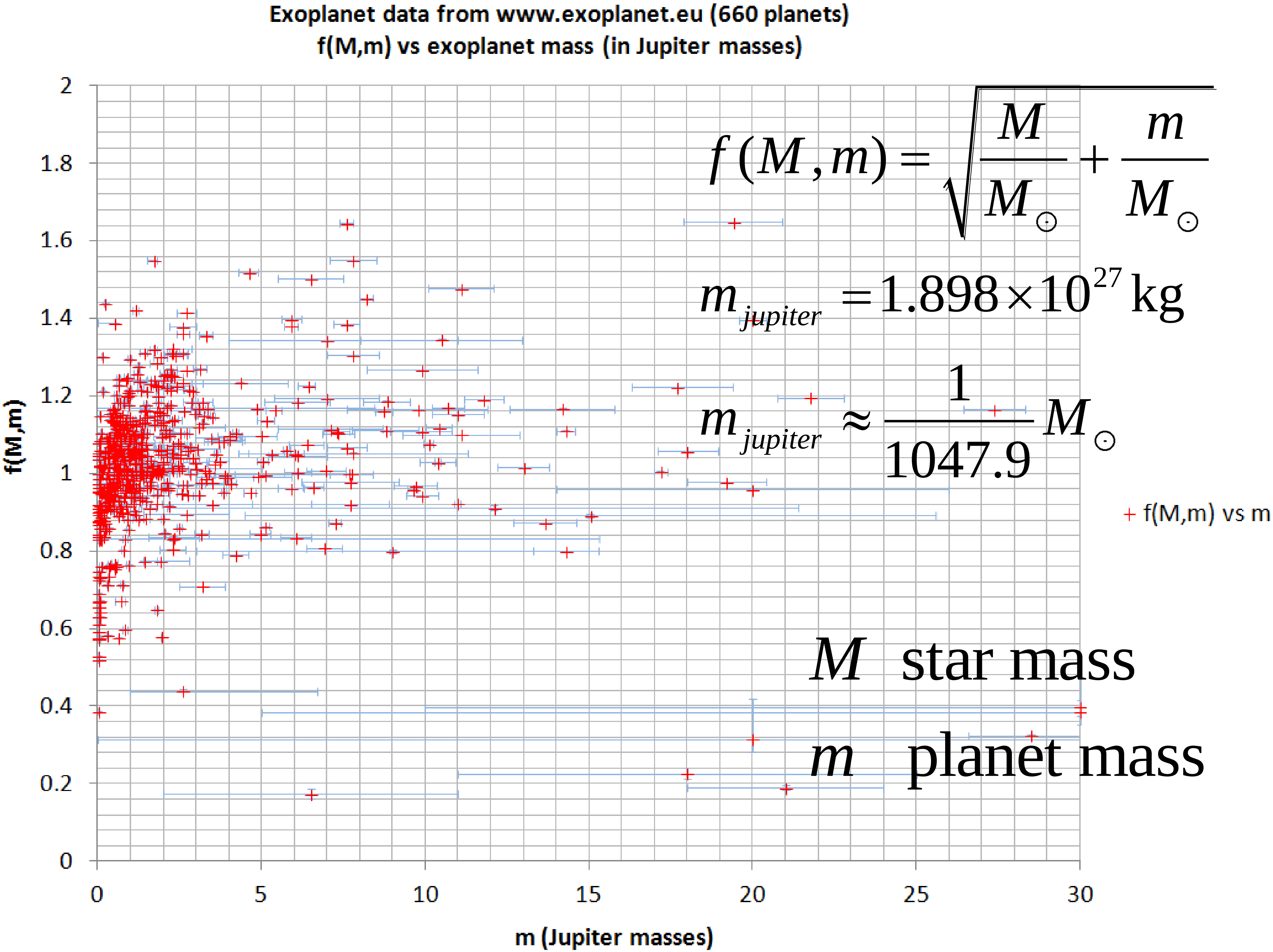
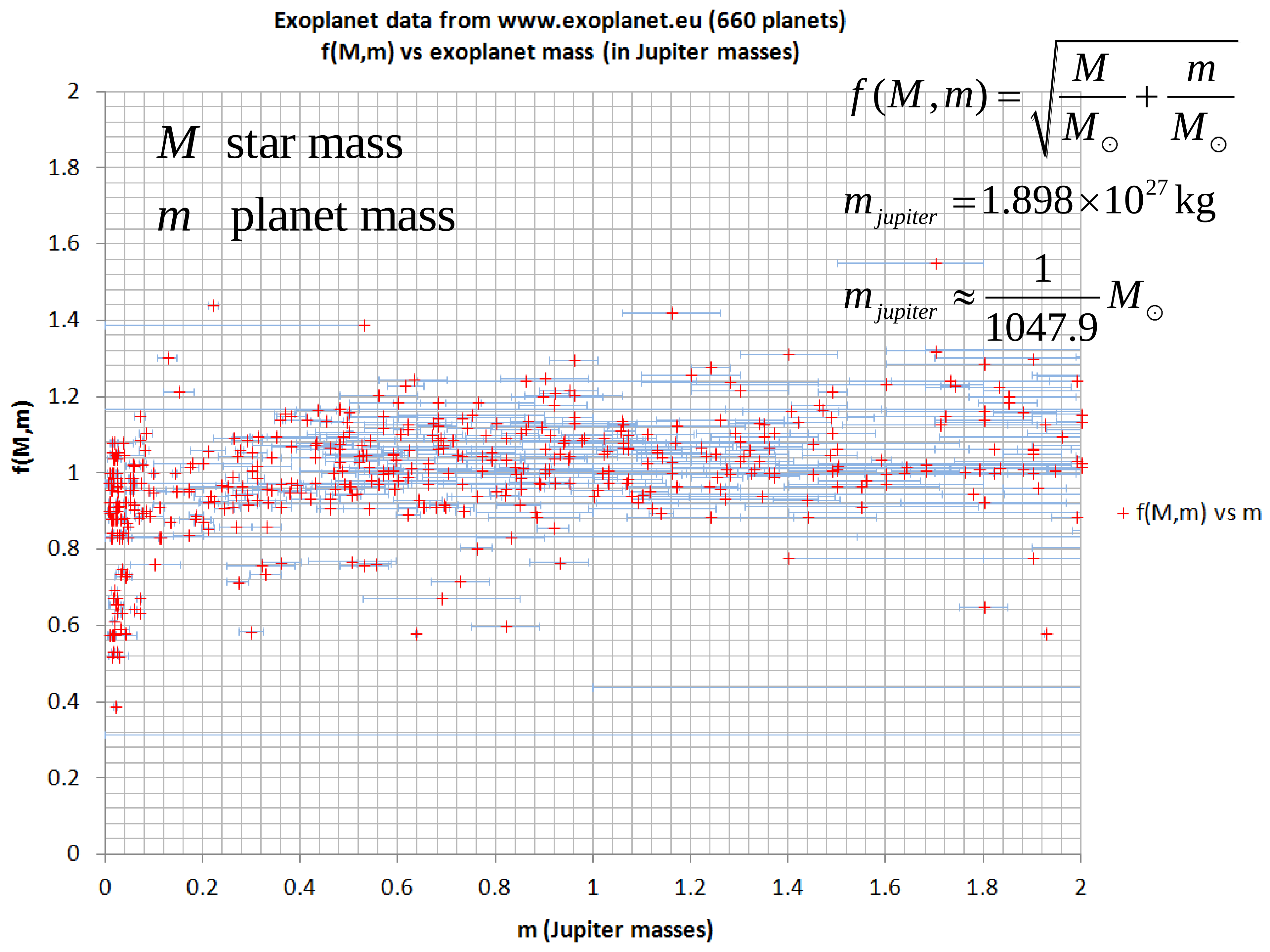

Exoplanet data from www.exoplanet.eu (660 planets)
f(M,m) vs exoplanet mass (in Jupiter masses)
$f(M,m)=\sqrt{\frac{M}{M_{\odot}}+\frac{m}{M_{\odot}}}$
$m_{jupiter}=1.898\times10^{27}\,\mathrm{kg}$
$m_{jupiter}\approx\frac{1}{1047.9}M_{\odot}$
M star mass
m planet mass
f(M,m) vs m
f(M,m)
m (Jupiter masses)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15
20
25
30



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M star mass
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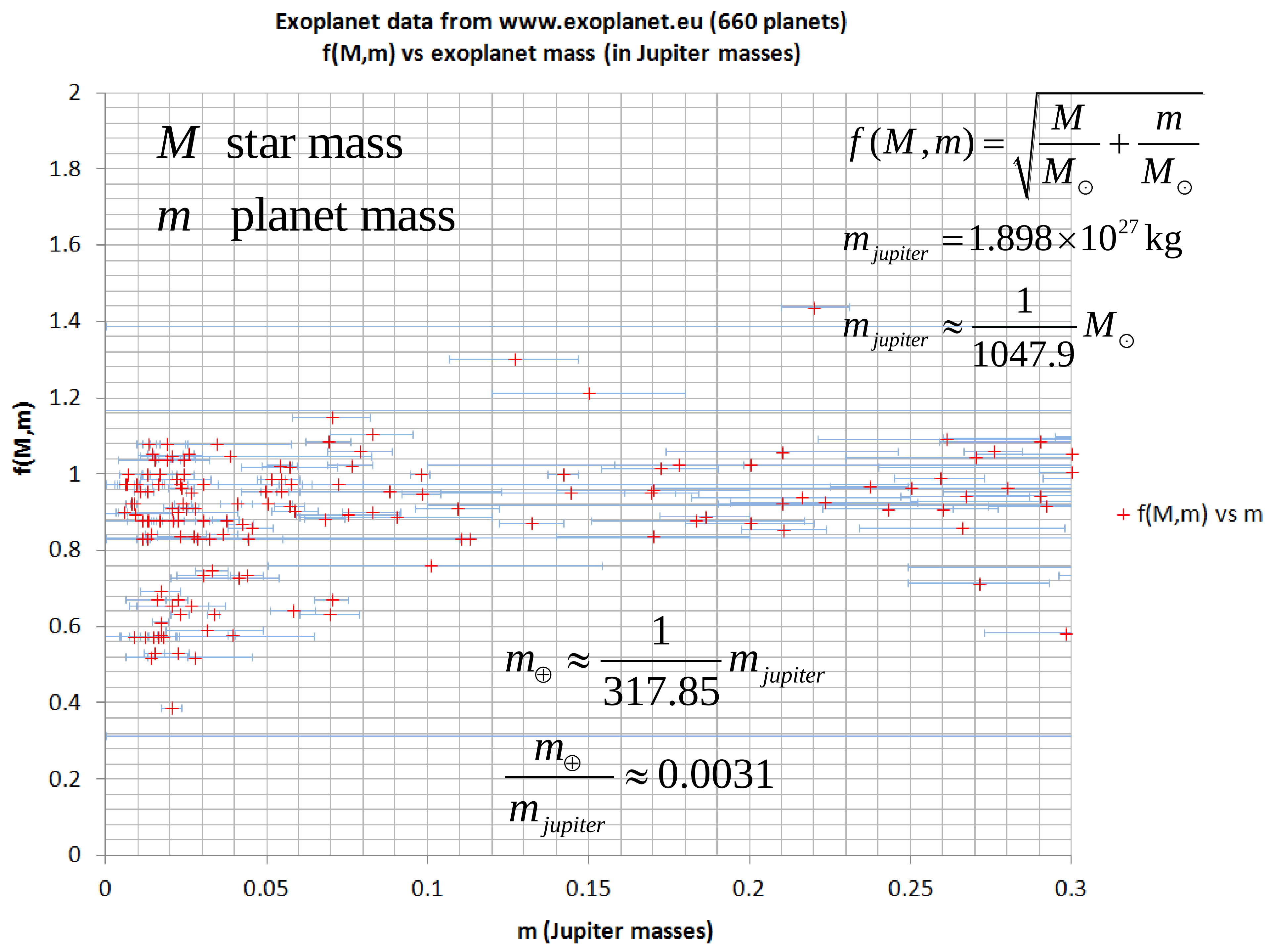
f(M,m)
m (Jupiter masses)
f(M,m) vs m
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2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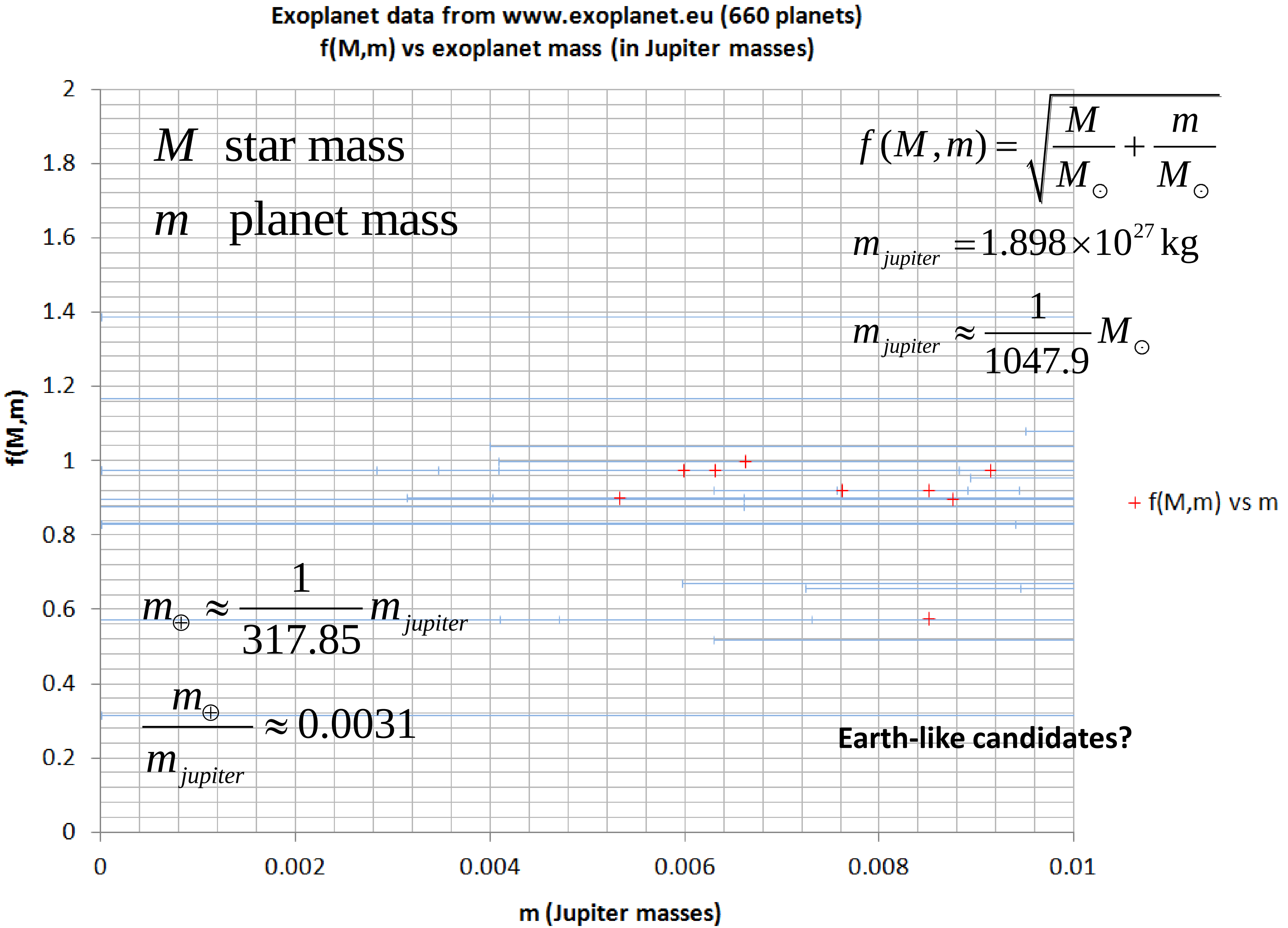
$m_\oplus\approx\frac{1}{317.85}m_{jupiter}$
$\frac{m_\oplus}{m_{jupiter}}\approx0.0031$
f(M,m)
m (Jupiter masses)
f(M,m) vs m
0
0.05
0.1
0.15
0.2
0.25
0.3
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2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$m_\oplus \approx \frac{1}{317.85} m_{jupiter}$
$\frac{m_\oplus}{m_{jupiter}} \approx 0.0031$
Earth-like candidates?
f(M,m) vs m
f(M,m)
m (Jupiter masses)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.002
0.004
0.006
0.008
0.01