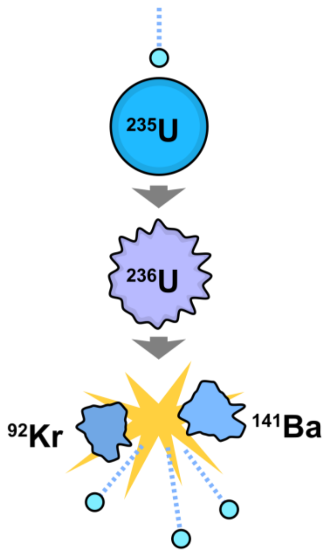
Nuclear Fission, Fusion and Binding Energy

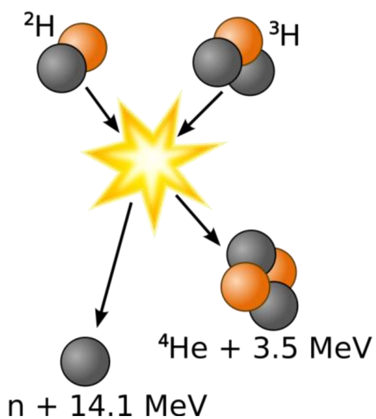
Einstein's mass-energy equivalence predicts that relatively small amounts of mass can produce very large quantities of energy, certainly vastly more than chemical changes. This conversion occurs naturally during nuclear processes of **Fission** and **Fusion**. *Fission* is where a typically 'heavy' nucleus (i.e. with a large number of nucleons such as Uranium) spontaneously fragments into smaller '*daughter*' nuclei, whereas *Fusion* is when two light nuclei (e.g. Deuterium and Tritium) fuse together to form a heavier nucleus (such as Helium).

Fission can be stimulated by exposing atoms to neutrons. This process occurs in a controlled fashion in nuclear power station reactors, and in an uncontrolled 'chain reaction' during the explosion of a nuclear weapon. Nuclear fusion is harder to achieve, as extremely high temperatures are required, but is the process which powers stars such as our Sun, and hence is the ultimate engine of all life on Earth.



$${}^{235}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\mathrm{n}$$

Example of Nuclear Fission



$${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + \mathrm{n}$$

Example of Nuclear Fusion

Example calculation: Nuclear Fission of Uranium 235

$${}^{235}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\mathrm{n}$$

$$\Delta m = \left(m_{\mathrm{U}} + m_{\mathrm{n}}\right) - \left(m_{\mathrm{Ba}} + m_{\mathrm{Kr}} + 3m_{\mathrm{n}}\right)$$

$$\Delta E = \Delta m \times c^2$$

$$\Delta m = 235.04393u - 140.91441u - 91.92516u - 2 \times 1.00867u$$

$$\Delta m = 0.18702u$$

$$\Delta E = 0.18702 \times 1.66054 \times 10^{-27} \times 299{,}792{,}458^2$$

$$\Delta E = 2.79112 \times 10^{-11}\,\mathrm{J} = \boxed{174.21\,\mathrm{MeV}}$$

Atomic mass unit u
i.e. one twelfth of the mass of Carbon 12

$$\boxed{\begin{aligned} m_e &= 9.10938356 \times 10^{-31}\,\mathrm{kg} \\ m_{\mathrm{p}} &= 1.672621777 \times 10^{-27}\,\mathrm{kg} \\ m_{\mathrm{n}} &= 1.674927471 \times 10^{-27}\,\mathrm{kg} \\ m_{\mathrm{u}} &= 1.660539040 \times 10^{-27}\,\mathrm{kg} \\ c &= 299{,}792{,}458\,\mathrm{ms}^{-1} \\ e &= 1.6021766209 \times 10^{-19}\,\mathrm{C} \end{aligned}}$$

$$1\,\mathrm{eV} = 1.6021766209 \times 10^{-19}\,\mathrm{J}$$

Electron-volts

$$E = \frac{2.79112 \times 10^{-11}\,\mathrm{J}}{\left(235.04393 + 1.00867\right) \times 1.66054 \times 10^{-27}\,\mathrm{kg}}$$

'Specific energy' of Fission fuel

$$E = \boxed{7.12066 \times 10^{13}\,\mathrm{Jkg}^{-1}}$$

$$M_{UK} \approx \frac{9.981 \times 10^{18}\,J}{7.12066 \times 10^{13}\,\mathrm{Jkg}^{-1}} = \boxed{140.2\ \mathrm{tonnes}}$$

Equivalent fuel mass to power UK for one year

$$E_{TNT} = 4.7 \times 10^{6}\,\mathrm{Jkg}^{-1}$$

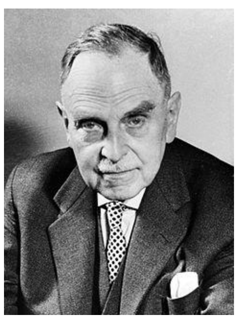
$$E_{dynamite} = 7.5 \times 10^{6}\,\mathrm{Jkg}^{-1}$$

$$E_{coal} < 35{,}000\,\mathrm{kJkg}^{-1} = 3.5 \times 10^{7}\,\mathrm{Jkg}^{-1}$$

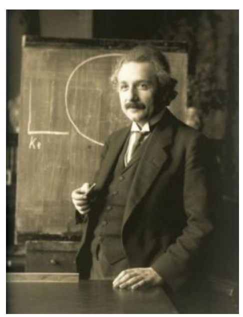
$$E_{gas} \approx 49\,\mathrm{MJkg}^{-1} = 4.9 \times 10^{7}\,\mathrm{Jkg}^{-1}$$

Per kg, there is over a **million times more energy** in fissile nuclear fuel than in coal or natural gas.

Hans Bethe 1906-2005
Stellar fusion mechanism
Nucleosynthesis (i.e. heavier elements are formed via Fusion during *Supernovae* events)



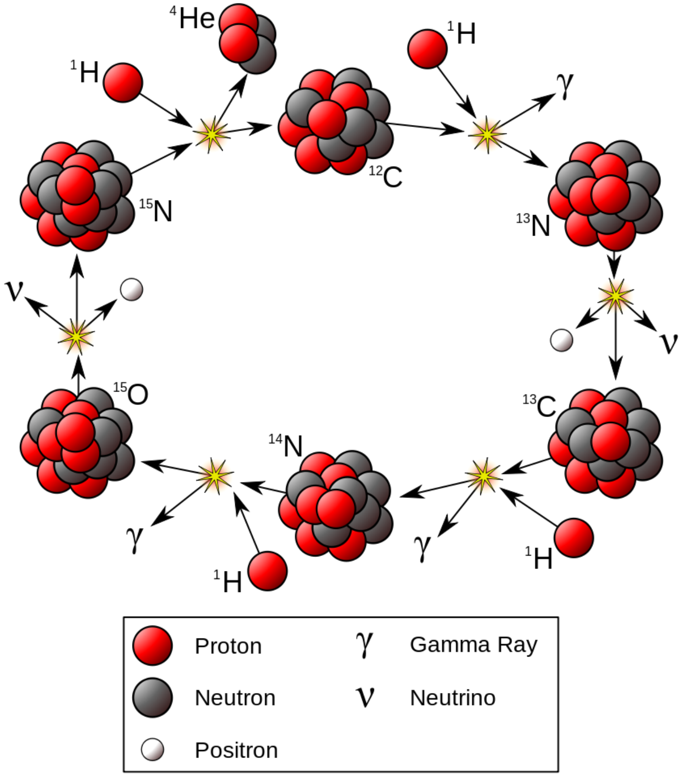
Otto Hahn
1879-1968
Discovery of Nuclear Fission



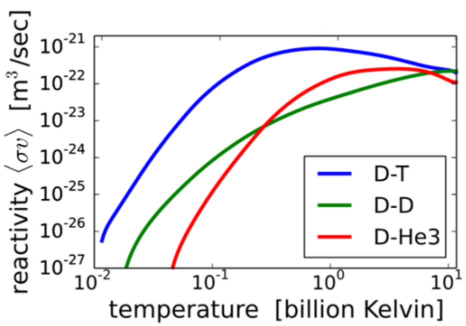
Albert Einstein
1879-1955
Quantum Mechanics
relativity

George Gamow
1904-1968
Applications of Quantum Mechanics to nuclear processes

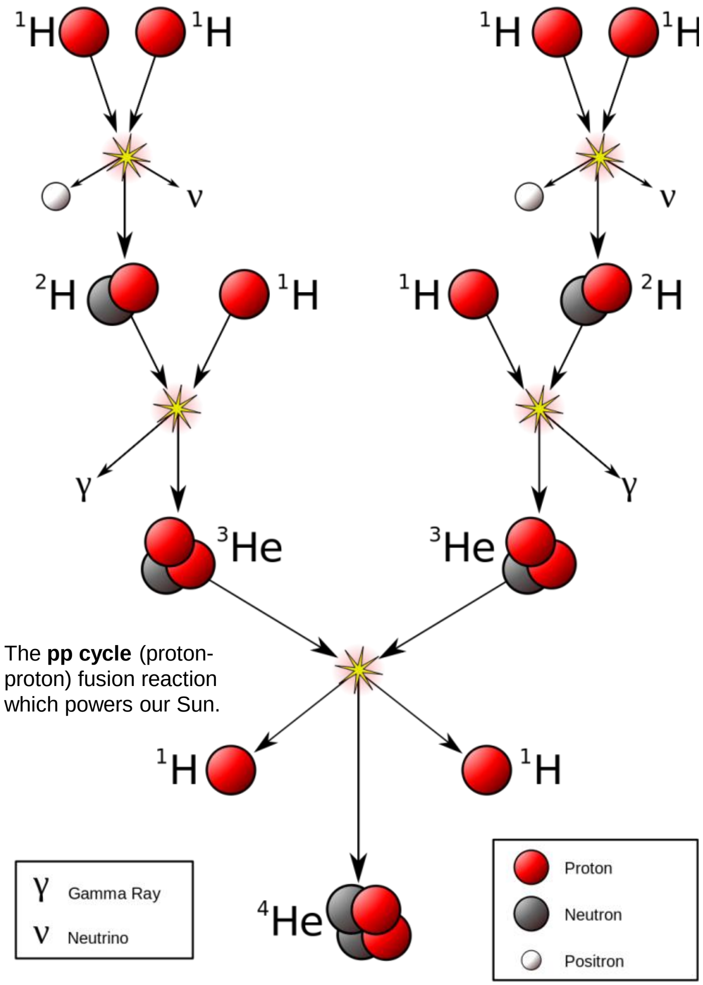
Carl Friedrich von Weizsäcker
1912-2007
Liquid Drop Model of Binding Energy



"CNO cycle (for carbon–nitrogen–oxygen) is one of the two sets of fusion reactions by which stars convert hydrogen to helium, the other being the proton–proton chain reaction. Unlike the latter, the CNO cycle is a *catalytic* cycle. Theoretical models suggest that the CNO cycle is the dominant source of energy in stars more massive than about 1.3 times the mass of the Sun." https://en.wikipedia.org/wiki/CNO_cycle



Very high temperatures are required to 'ignite' a fusion process



The **pp cycle** (proton-proton) fusion reaction which powers our Sun.

The **temperature** required for fusion can be estimated by equating thermal energy with the potential energy of two protons separated by one femtometre (10^{-15} m) i.e. the size of a small nucleus.

$$\frac{e^2}{4\pi\varepsilon_0 r} = k_B T \qquad \therefore T = \frac{e^2}{4\pi\varepsilon_0 k_B r}$$

$$T = \frac{\left(1.6\times10^{-19}\right)^2}{4\pi\times8.85\times10^{-12}\times1.38\times10^{-23}\times10^{-15}}$$

$$\boxed{T = 16.7\times10^{9}\text{ K}}$$

It turns out this is a large overestimate, as *Quantum Mechanical tunnelling* is more probable at these energies. About **100 million K** is typically required. The core temperature of the Sun is about 15 million K, so the process is less efficient. But the Sun is very large!

Example calculation: Nuclear Fusion of Deuterium (Hydrogen 2) and Tritium (Hydrogen 3) to form Helium 4

$$\underset{m_\mathrm{D}}{{}^2_1\mathrm{H}} + \underset{m_\mathrm{T}}{{}^3_1\mathrm{H}} \rightarrow \underset{m_\mathrm{He}}{{}^4_2\mathrm{He}} + \underset{m_n}{\mathrm{n}}$$

$$\Delta m = \left(m_\mathrm{D} + m_\mathrm{T}\right) - \left(m_\mathrm{He} + m_n\right)$$

$$\Delta E = \Delta m \times c^2$$

$$\Delta m = 2.01410u + 3.01605u - 4.00260u - 1.00867u$$

$$\Delta m = 0.01888u$$

$$u = 1.66054\times10^{-27}\text{ kg}$$

$$c = 299{,}792{,}458\text{ms}^{-1}$$

$$\Delta E = 0.01888\times1.66054\times10^{-27}\times299{,}792{,}458^2$$

$$\Delta E = 2.81769\times10^{-12}\text{ J} = \boxed{17.587\text{MeV}}$$

$$E = \frac{2.81769\times10^{-12}\text{ J}}{\left(2.01410+3.01605\right)\times1.66054\times10^{-27}\text{ kg}}$$

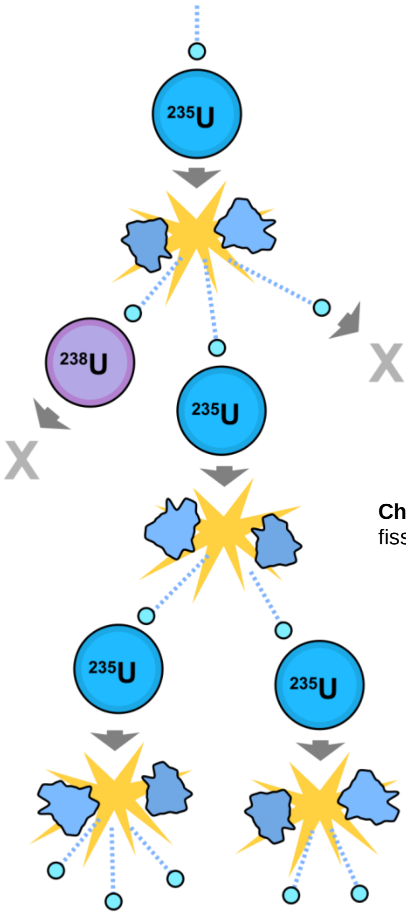
$$E = \boxed{3.37\times10^{14}\text{ Jkg}^{-1}}$$

$$M_{UK} \approx \frac{9.981\times10^{18}\,J}{3.37\times10^{14}\text{ Jkg}^{-1}} = \boxed{29.6\text{ tonnes}}$$

Fusion offers *even higher* 'specific energy' than Uranium fission, and without the radioactive waste that fission produces. It is perhaps the ultimate power source.....

However, the *very high temperatures* required mean it is a tough challenge to build a commercial fusion reactor. So far 'man-made-fusion' has been experimental, or apallingly demonstrated via a *thermonuclear weapon* such as a Hydrogen-bomb.

$$\boxed{1\text{J} = \frac{1}{10^6\times1.60217662\times10^{-19}}\text{MeV}}$$



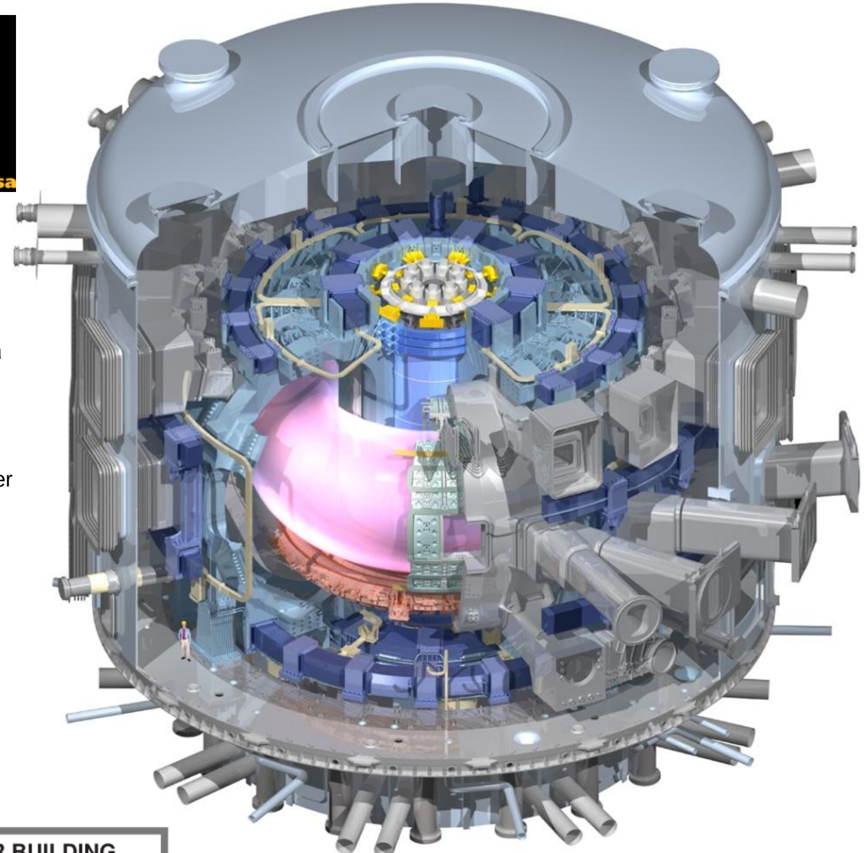
Chain reaction during nuclear fission of Uranium 235

iter

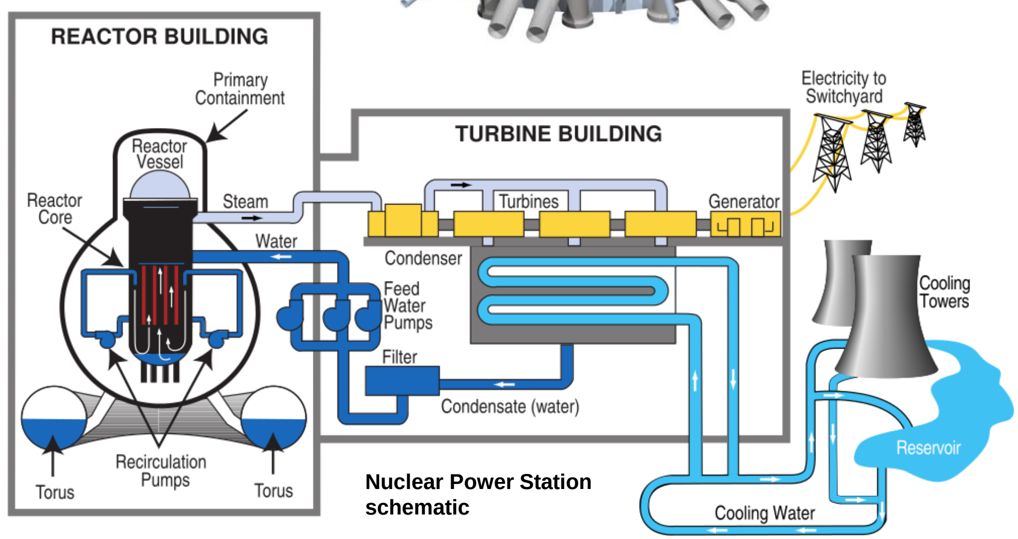
china eu india japan korea russia usa

International Thermonuclear Experimental Reactor (ITER)

- 500 MW of fusion power sustained for up to 1,000 seconds (compared to JET's * peak of 16 MW for less than a second)
- Fusion of about 0.5 g of deuterium/tritium mixture in its approximately 840 m^3 reactor chamber
- 10 times more energy than the amount consumed to heat up the plasma to fusion temperatures



Although radioactive waste disposal is a serious environmental hazard, the **factor of a million specific energy ratio** between nuclear and coal is a powerful argument. *You need a million times less Uranium for the same energy!* Although you can't make a weapon of mass destruction from coal, the climate change potential of significant Greenhouse gas emissions could make it much more dangerous to humanity in the long run.



Nuclear Power Station schematic

* https://en.wikipedia.org/wiki/Joint_European_Torus

The '**Liquid Drop Model**' was proposed by Weizsäcke, Bethe and others to explain the difference in mass (and hence possibility for energy to be released) when a nucleus is fragmented into its component proton and neutron parts. The mass difference, expressed as energy, is called the Binding Energy. The name derives from an idea of Neils Bohr and George Gammow that a nucleus could be thought of being composed of a spherical drop of incompressible 'nuclear fluid.'

Carl Friedrich von Weizsäcker 1912-2007 Liquid Drop Model of Binding Energy

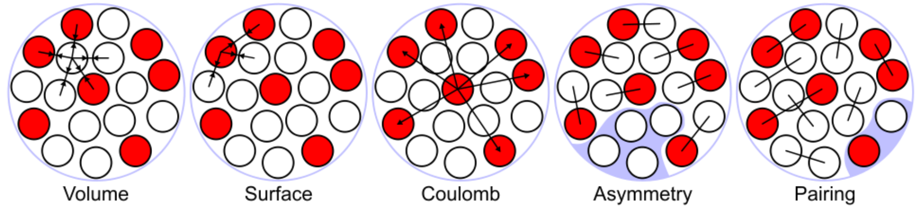
The Liquid Drop binding energy formula can be written in terms of **mass number A** (i.e. total number of nucleons i.e. protons + neutrons) and **Atomic number Z** (number of protons). Recall it is Z which *defines* the element, and A which specifies the *isotope*. (e.g. Carbon 12 and Carbon 14 have 6 protons each, but 6 and 8 neutrons respectively).

$${}^{A}_{Z}M \xrightarrow{B} Z\times \mathrm{p} + (A-Z)\times \mathrm{n} \qquad Mc^2 + B = Zm_p c^2 + (A-Z)m_n c^2$$

$$B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

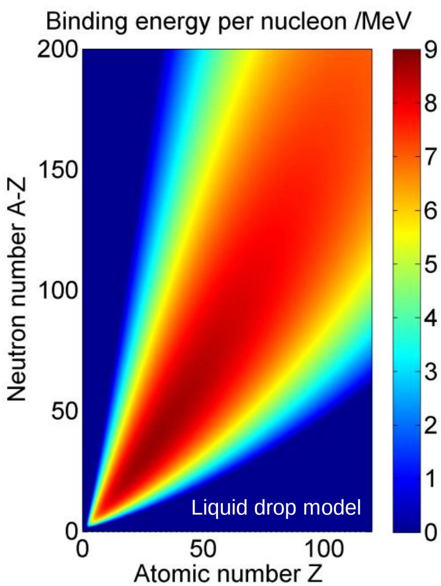
$$\delta(A,Z) = v\begin{cases} a_P A^{-3/4} & Z, A-Z \text{ even} \\ -a_P A^{-3/4} & Z, A-Z \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

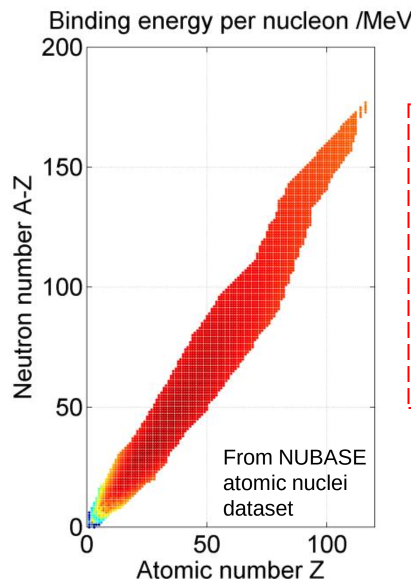
$$a_V = 15.76\,\mathrm{MeV}$$
$$a_S = 17.81\,\mathrm{MeV}$$
$$a_C = 0.711\,\mathrm{MeV}$$
$$a_A = 23.702\,\mathrm{MeV}$$
$$a_P = 34.0\,\mathrm{MeV}$$



Each term of the Liquid Drop Model (or the '**Semi-Empirical Mass Formula**') corresponds to a different nuclear effect which will result in either repulsion or attraction of nucleons, and thus contribute to the magnitude of the binding energy.

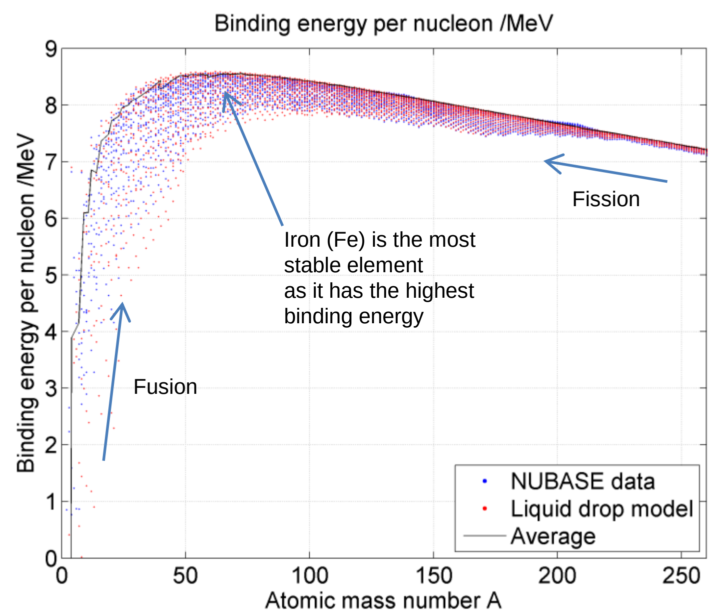
The Liquid Drop Model helps to explain why light nuclei can undergo fusion, whereas only heavy nuclei typically undergo fission. When light elements fuse, the binding energy *increases*. The difference is given off as gamma rays in a fusion reaction. When a heavy element (such as Uranium) undergoes fission, the binding energy of the daughter products is also a net increase, so the process can also generate energy.





The Liquid Drop model is not such a good fit for light nuclei, since it does not consider the 'shell structure' of the nucleus.

Also, certain combinations of protons and neutrons (the 'magic numbers') result in extra stability and an increase in binding energy. The Liquid Drop model does not predict this.



Revisit of Fission and Fusion reaction calculations using Binding Energy

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**** Nuclear Fission demo: U-235 + n -> Ba-141 + Kr-92 +3n ****

 Uranium-235
 NUBASE: B = 1736.8463MeV, M = 235.044u
 Liquid drop: B = 1740.8459MeV, M = 235.0396u

 Barium-141
 NUBASE: B = 1145.3585MeV, M = 140.9144u
 Liquid drop: B = 1141.7215MeV, M = 140.9183u

 Krypton-92
 NUBASE: B = 764.7682MeV, M = 91.9262u
 Liquid drop: B = 762.5381MeV, M = 91.9286u

 Energy released during fission (NUBASE) = 173.2804MeV
 Energy released during fission (Liquid Drop) = 163.4137MeV
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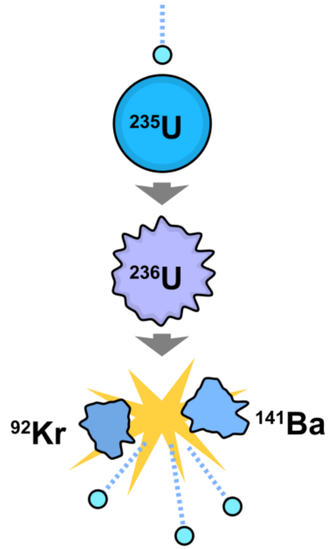

$$ {}^{235}_{92}\mathrm{U} + \mathrm{n} \to {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\mathrm{n} $$

$$ \Delta m = \left(m_{\mathrm{U}} + m_{\mathrm{n}}\right) - \left(m_{\mathrm{Ba}} + m_{\mathrm{Kr}} + 3m_{\mathrm{n}}\right) $$

$$ \Delta E = \Delta m \times c^2 $$

$$ m_{\mathrm{U}}c^2 + B_{\mathrm{U}} + m_{\mathrm{n}}c^2 = m_{\mathrm{Ba}}c^2 + B_{\mathrm{Ba}} + m_{\mathrm{Kr}}c^2 + B_{\mathrm{Kr}} + 3m_{\mathrm{n}}c^2 $$

$$ \therefore \Delta E = \Delta m \times c^2 = B_{\mathrm{Ba}} + B_{\mathrm{Kr}} - B_{\mathrm{U}} $$

$$ \therefore \Delta E = 1145.3585\mathrm{MeV} + 764.7682\mathrm{MeV} - 1736.8463\mathrm{MeV} $$

$$ \therefore \boxed{\Delta E = 173.2804\mathrm{MeV}} $$

$$ {}^{235}_{92}\mathrm{U} + \mathrm{n} \to {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\mathrm{n} $$

Example of Nuclear Fission

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**** Nuclear Fusion demo: H-2 + H-3 -> He-4 + n ****

 Deuterium
 NUBASE: B = 1.7135MeV, M = 2.0141u
 Liquid drop: B = -17.9325MeV, M = 2.0352u

 Tritium
 NUBASE: B = 7.9707MeV, M = 3.016u
 Liquid drop: B = 1.2401MeV, M = 3.0233u

 Helium-4
 NUBASE: B = 27.2736MeV, M = 4.0026u
 Liquid drop: B = 27.5908MeV, M = 4.0023u

 Energy released during fusion (NUBASE) = 17.5893MeV
 Energy released during fusion (Liquid Drop) = 44.2832MeV
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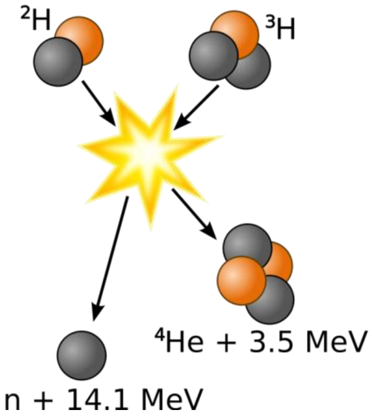

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \to {}^{4}_{2}\mathrm{He} + \mathrm{n} $$

Example of Nuclear Fusion

$$ \underset{m_{\mathrm{D}}}{{}^{2}_{1}\mathrm{H}} + \underset{m_{\mathrm{T}}}{{}^{3}_{1}\mathrm{H}} \to \underset{m_{\mathrm{He}}}{{}^{4}_{2}\mathrm{He}} + \underset{m_n}{\mathrm{n}} $$

$$ \Delta m = \left(m_{\mathrm{D}} + m_{\mathrm{T}}\right) - \left(m_{\mathrm{He}} + m_n\right) $$

$$ \Delta E = \Delta m \times c^2 $$

$$ m_{\mathrm{D}}c^2 + B_{\mathrm{D}} + m_{\mathrm{T}}c^2 + B_{\mathrm{T}} = m_{\mathrm{He}}c^2 + B_{\mathrm{He}} + m_{\mathrm{n}}c^2 $$

$$ \therefore \Delta E = \Delta m \times c^2 = B_{\mathrm{He}} - B_{\mathrm{D}} - B_{\mathrm{T}} $$

$$ \therefore \Delta E = 27.2736\mathrm{MeV} - 1.7135\mathrm{MeV} - 7.9707\mathrm{MeV} $$

$$ \therefore \boxed{\Delta E = 17.5893\mathrm{MeV}} $$

The idea of the **difference in total binding energy being the energy released** can make nuclear energy calculations a little easier than having to deal with masses, and their conversion to energies.

But note the discrepancies between NUBASE and SEMF ("Liquid Drop") calculations. SEMF is particularly bad for the fusion calculation.