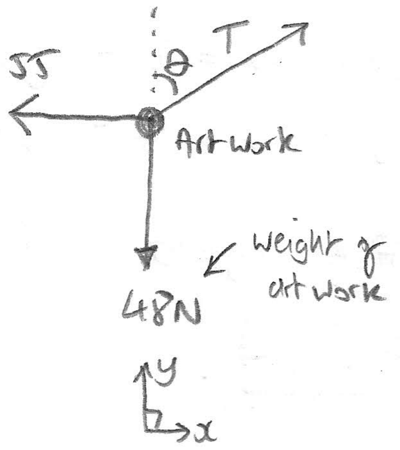
FORCES & MOMENTS

1/ (i)



System in equilibrium.

Newton II:

$/\!/x: \quad 0 = T\sin\theta - 55$

$/\!/y: \quad 0 = T\cos\theta - 48$

$\therefore \quad T\sin\theta = 55$

$T\cos\theta = 48$

$\therefore \quad \tan\theta = \frac{55}{48}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{55}{48}\right)$

$= \boxed{48.9°}$

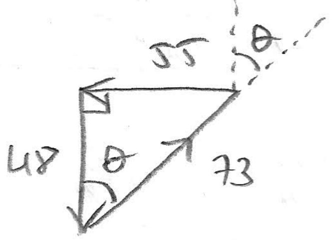
$T^2\sin^2\theta + T^2\cos^2\theta = 55^2 + 48^2$

$\therefore \quad T^2(\sin^2\theta + \cos^2\theta) = 5329$ ← (the bracket) $= 1$

$\therefore \quad T = \sqrt{5329}$

$\therefore \quad \boxed{T = 73\ N}$

Note:

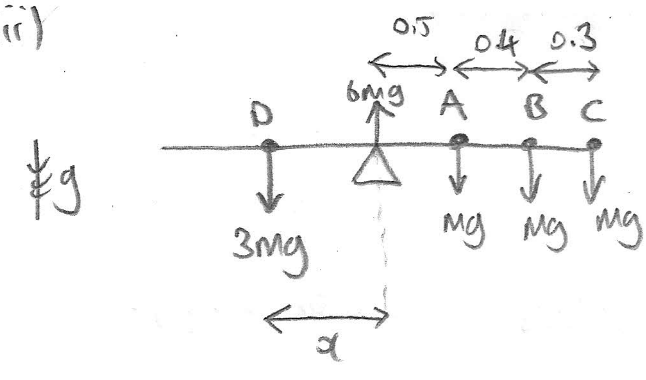


ie vector sum of forces is a closed triangle since system is in equilibrium

Also $73^2 = 55^2 + 48^2 \Rightarrow 73, 55, 48$ is a Pythagorean Triple.

And since the artwork is in Samos, so is this question!

(ii)



Net turning moment about pivot = 0

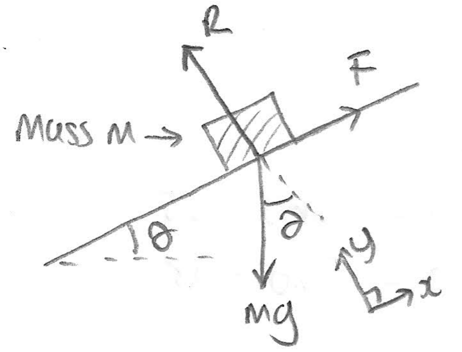
$\therefore \quad 3mgx = 0.5Mg + 0.9Mg + 1.2Mg$

$3x = 2.6$

Hence:

$\boxed{x = 0.87m}$

(iii)



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Newton II, assuming equilibrium

$/\!/x: \quad 0 = F - Mg\sin\theta$

$/\!/y: \quad 0 = R - Mg\cos\theta$

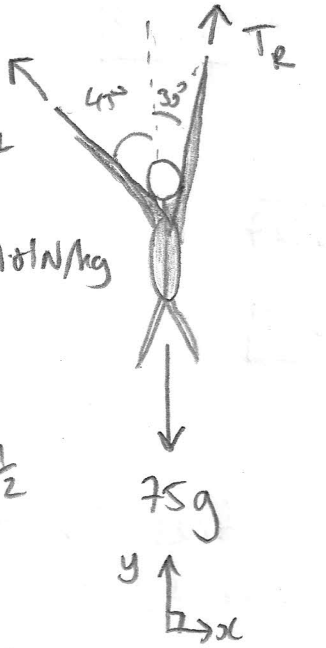
So $F = Mg\sin\theta \qquad R = Mg\cos\theta$

Now mass won't slide if $\boxed{F < \mu R}$.

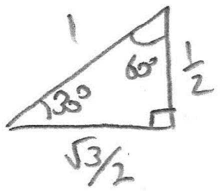
$\therefore \quad Mg\sin\theta < \mu Mg\cos\theta$

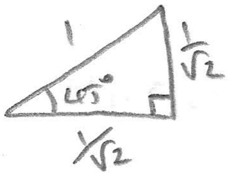
$\boxed{\tan\theta < \mu}$

(iv)



¥g = 9.81N/kg





Climber is in equilibrium

$\therefore$ Newton II

$/\!/x: \quad 0 = T_R\sin 30^\circ - T_L\sin 45^\circ$

$/\!/y: \quad 0 = T_R\cos 30^\circ + T_L\cos 45^\circ - 75g$

$\therefore \quad T_R/2 = T_L/\sqrt{2} \Rightarrow \boxed{T_R = T_L\sqrt{2}}$

$\therefore \quad 75g = T_L\sqrt{2}\frac{\sqrt{3}}{2} + T_L/\sqrt{2}$

$75g = \frac{T_L\sqrt{3}}{\sqrt{2}} + \frac{T_L}{\sqrt{2}}$

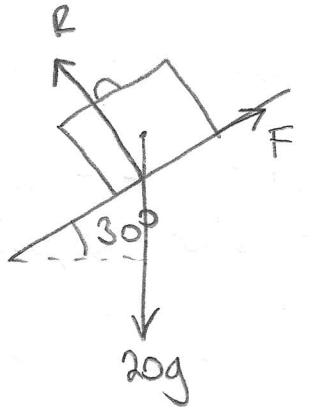
$75\sqrt{2}g = T_L(\sqrt{3}+1)$

$$\boxed{\frac{75\sqrt{2}g}{\sqrt{3}+1} = T_L}$$

$\therefore \quad T_R = \sqrt{2}T_R = \boxed{\frac{150g}{\sqrt{3}+1}}$

$\therefore \quad T_R = \boxed{381N} \qquad T_L = \boxed{537N}$ (to 3s.f).

(v)



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on the point of sliding. So $F = \mu R$

and $\mu = \tan 30^\circ = \frac{1}{\sqrt{3}} \approx$ **0.577**

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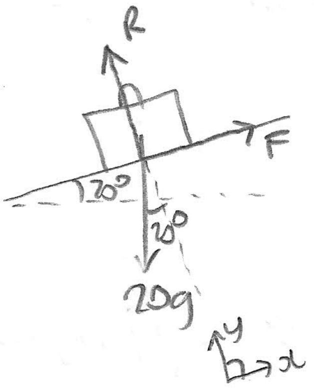


Since elevation of ramp is < 30°, assume static equilibrium.

∴ by Newton II

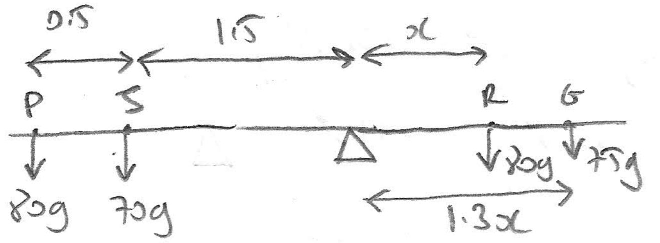
$// x: \quad 0 = F - 20g\sin 20^\circ$

$// y: \quad 0 = R - 20g\cos 20^\circ$

$\Rightarrow \quad F = 20g\sin 20^\circ = $ **67.1 N**

$R = 20g\cos 20^\circ = $ **184.4 N**

(vi)



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Balancing turning moments about the see-saw pivot

$$2 \times 80 + 1.5 \times 70 = 80x + 75 \times 1.3x$$

$$265 = 177.5x$$

$$\boxed{1.49 = x}$$

∴ separation between Paul and George is:

$0.5 + 1.5 + 1.3 \times x = $ **3.94 m**