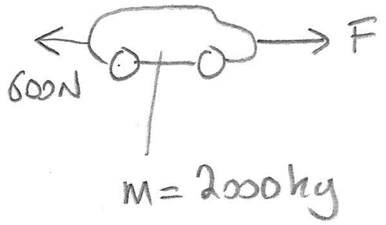
FORCES & ACCELERATION

1/ (i)



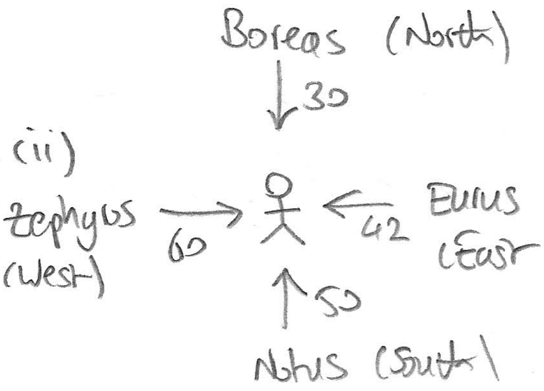
$a = 1.2\,m/s^2$

NII: $ma = F - 600$

$\therefore \quad F = 2000 \times 1.2 + 600 \quad (N)$

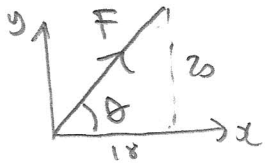
$\boxed{F = 3000N}$

(ii)



Vector sum of force is: (in N)

$$\begin{pmatrix}60\\0\end{pmatrix} + \begin{pmatrix}0\\-30\end{pmatrix} + \begin{pmatrix}-42\\0\end{pmatrix} + \begin{pmatrix}0\\50\end{pmatrix} = \begin{pmatrix}18\\20\end{pmatrix}$$



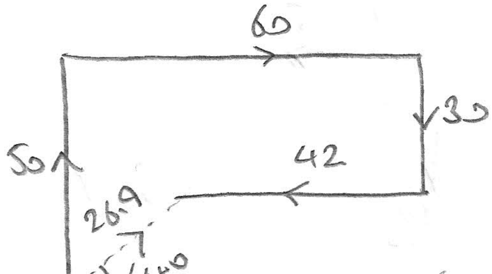
Net force ("resultant force") is of magnitude:

$F = \sqrt{18^2 + 20^2} = \boxed{26.9N}$

at angle $\theta = \tan^{-1}\left(\frac{20}{18}\right) = 48°$

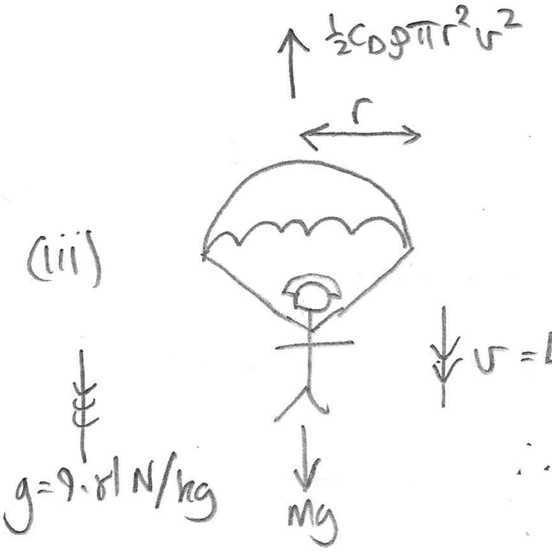
So bearing (ie clockwise from N) is $\boxed{042°}$

Note vector addition is:



⇢ is the resultant force.

(iii)



$\downarrow v = 4.00\,m/s$ in eq drag = weight

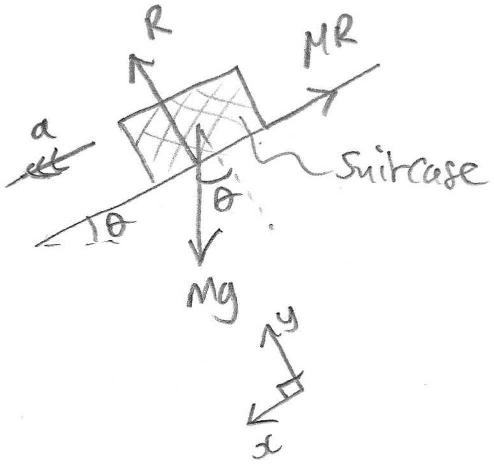
$\therefore \quad Mg = \frac{1}{2} C_D \rho \pi r^2 v^2$

$\therefore \quad \sqrt{\frac{2Mg}{C_D \rho \pi v^2}} = r$

$\therefore\ r = \sqrt{\dfrac{2\times 65\times 9.81}{0.8\times 1.23\times \pi\times 4.00^2}} = \boxed{5.1\text{ m}}$

(ie the parachute radius)

(iv)



$g = 9.81$ N/kg

NII $/\!/x$: $Ma = Mg\sin\theta - \mu R$

$/\!/y$: $0 = R - Mg\cos\theta$

$\therefore\ R = Mg\cos\theta$

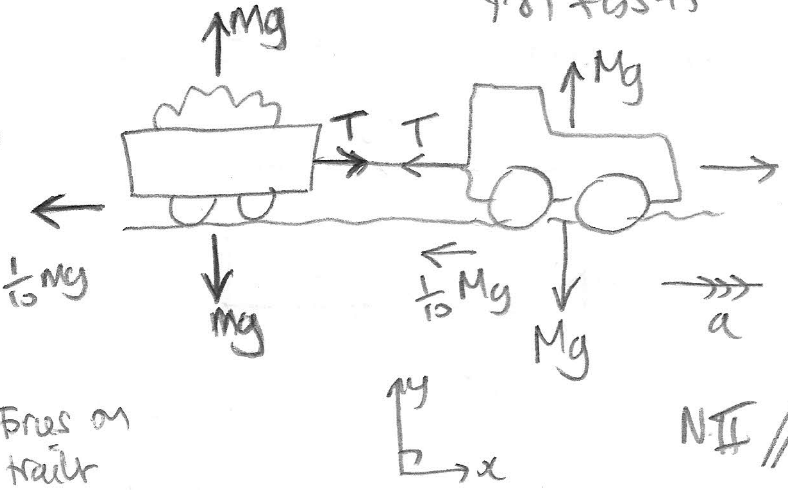
$\therefore\ Ma = Mg\sin\theta - \mu Mg\cos\theta$

$\therefore\ \mu Mg\cos\theta = Mg\sin\theta - Ma$

$\therefore\ \boxed{\mu = \tan\theta - \dfrac{a}{g\cos\theta}}$

$\therefore\ \mu = \tan 45^\circ - \dfrac{6.0}{9.81\times\cos 45^\circ} = \boxed{0.14}$

(v)



a) whole system of tractor + trailer

↗ Forces on trailer

↗ Forces on tractor

NII $/\!/x$: $(M+m)a = F - \frac{1}{10}(m+M)g$

$\therefore\ F = (M+m)\left(a + \frac{g}{10}\right)$

$F = (11{,}000 + 21{,}000)\left(3.14 + \frac{9.81}{10}\right)$

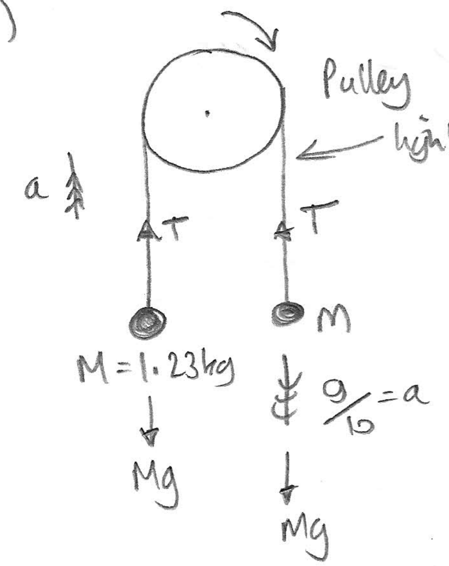
$= \boxed{131{,}900\text{ N}}$

b) NII for trailer:

$ma = T - \frac{1}{10}mg \quad \therefore\ T = m\left(a + \frac{1}{10}g\right) = 21000\left(3.14 + \frac{9.81}{10}\right)$

$= \boxed{86{,}500\text{ N}}$

(vi)



NII for mass m: $ma = mg - T$ ①

" " " M: $Ma = T - Mg$ ②

$\therefore T = M(a+g)$

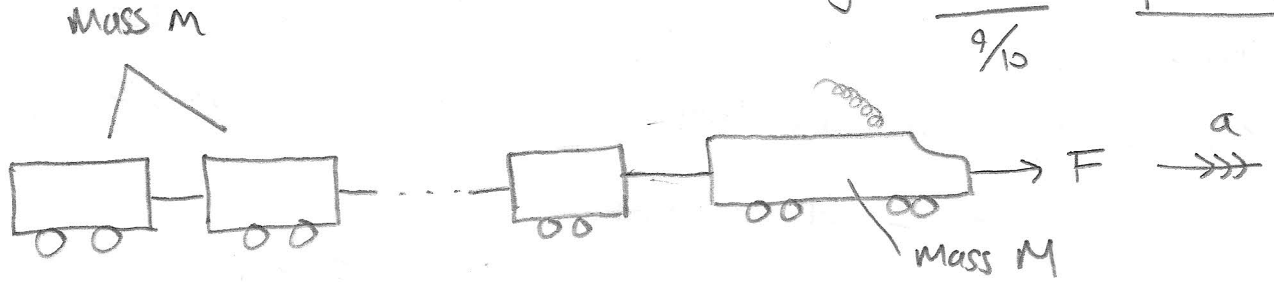
$\therefore m(g-a) = T = M(a+g)$

$$\boxed{m = \frac{M(a+g)}{g-a}}$$

So $m = 1.23\text{kg} \times \dfrac{1+\frac{1}{10}}{1-\frac{1}{10}}$

$m = 1.23\text{ kg} \times \dfrac{11/10}{9/10} = \boxed{1.51\text{ kg}}$

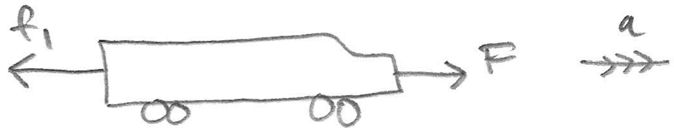
(vii)



Whole system of locomotive + N carriages

NII: $\boxed{F = (Nm + M)a}$

Forces on locomotive:

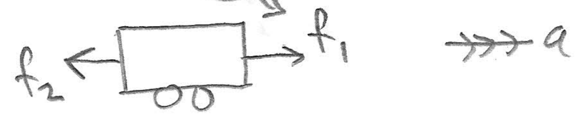


NII: $F - f_1 = Ma$ $\therefore f_1 = F - Ma$

$\therefore \boxed{f_1 = Nma}$

(Note NIII explains why this force is f_1)

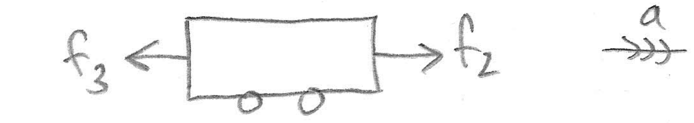
First carriage:



NII: $f_1 - f_2 = ma$ $\therefore f_2 = f_1 - ma$

$\boxed{f_2 = (N-1)ma}$

Second Carriage:



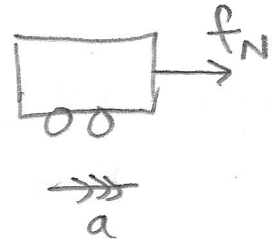
NII: $ma = f_2 - f_3$ $\therefore f_3 = f_2 - ma$

$\therefore \boxed{f_3 = (N-2)ma}$

So to generalize:

$$\boxed{f_n = (N-n+1)ma}$$

This makes good sense as the final carriage pulls no other



NII: $f_N = ma$

is $f_N = (N-N+1)ma = ma$ ✓

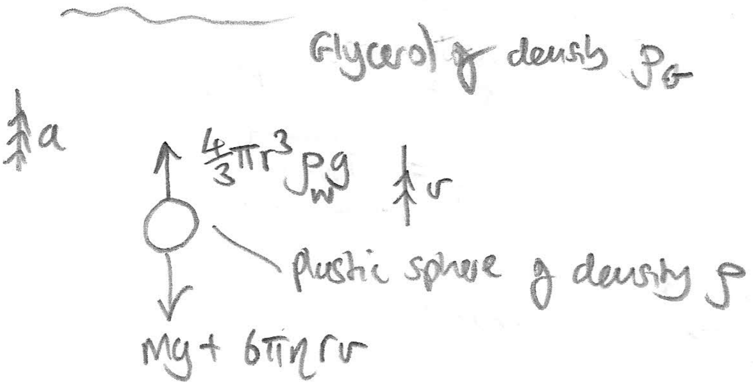
Now let $M = 110{,}000$ kg, $m = 131{,}000$ kg, $a = 0.1\ m/s^2$,
{think an Australian Iron ore train!} $N = 30$

$$\therefore F = (Nm + M)a$$

$$F = (30 \times 131{,}000 + 110000) \times 0.1$$

$$= \boxed{404{,}000\ N}$$

(viii)



NII upwards:

$$\underbrace{\tfrac{4}{3}\pi r^3 \rho a}_{\text{mass} \times \text{acc.}} = \underbrace{\tfrac{4}{3}\pi r^3 \rho_G g}_{U} - \underbrace{\tfrac{4}{3}\pi r^3 \rho g}_{W} - \underbrace{6\pi\eta r v}_{D}$$

$M = \frac{4}{3}\pi r^3 \rho$

U pthrust Weight Drag (viscous)

So $\boxed{a = \left(\frac{\rho_G}{\rho} - 1\right)g - \frac{6\pi\eta r v}{\frac{4}{3}\pi r^3 \rho}}$

$v = 0.5\,m/s$
$r = 5\times10^{-2}\,m$
$\eta = 1.07\,Nsm^{-2}$

$$a = \left(\frac{1260}{920} - 1\right)\times 9.81 - \frac{6\pi \times 1.07 \times 5\times10^{-2} \times 0.5}{4/3\,\pi \times (5\times10^{-2})^3 \times 920}$$

$$= 3.625 - 1.047$$

$$= \boxed{2.58\,m/s^2}$$

Forces:

Upthrust $U = \frac{4}{3}\pi r^3 \rho_G g$
$= \boxed{6.47N}$

Weight $W = \frac{4}{3}\pi r^3 \rho g$
$= \boxed{4.73N}$

Drag $D = 6\pi\eta r v$
$= \boxed{0.50\,N}$

so net force upwards is $\boxed{1.24N}$

$\therefore$ if sphere mass is $M = \frac{4}{3}\pi r^3 \rho = 0.48\,kg$

$\Rightarrow$ upward acceleration is $\frac{1.24\,N}{0.48\,kg}$

$= \boxed{2.58\,m/s^2}$ ✓

Note terminal velocity v_T is when $a=0$

$\therefore$ $6\pi\eta r v_T = \left(\frac{\rho_G}{\rho} - 1\right)g \times \frac{4}{3}\pi r^3 \rho$

$\Rightarrow$ $v_T = \left(\frac{\rho_G}{\rho} - 1\right)\frac{g\rho r^2}{\eta} \times \frac{4}{3\times6}$

$$\therefore \boxed{v_T = \left(\frac{\rho_G}{\rho} - 1\right)\frac{g\rho r^2}{\eta} + \frac{2}{9}}$$

which in this case is:

$$v_T = \left(\frac{1260}{920} - 1\right)\frac{\times 9.81 \times 920 \times (5\times10^{-2})^2}{1.07} + \frac{2}{9}$$

$$= \boxed{1.173 \text{ m/s}}$$

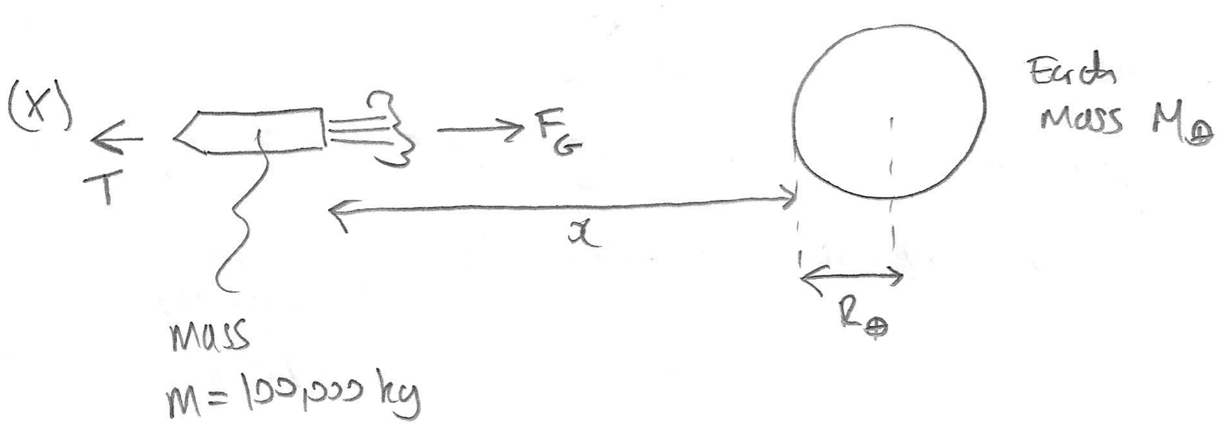
(This is quite fast, so kinematic drag effects will be significant ie "kv^2" force terms).

(ix) $$g = \frac{GM_\oplus}{R_\oplus^2} \quad \therefore \quad M_\oplus = \frac{gR_\oplus^2}{G}$$

$$M_\oplus = \frac{9.81 \times (6371\times10^3)^2}{6.67\times10^{-11}} \quad \text{(kg)}$$

$$= \boxed{5.97\times10^{24} \text{ kg}}$$

(x)



In eq $T = F_G = \dfrac{GmM_\oplus}{(x+R_\oplus)^2} \quad \therefore \quad x + R_\oplus = \sqrt{\dfrac{GmM_\oplus}{T}}$

rocket thrust

$$\therefore \quad x = \sqrt{\frac{GmM_\oplus}{T}} - R_\oplus$$

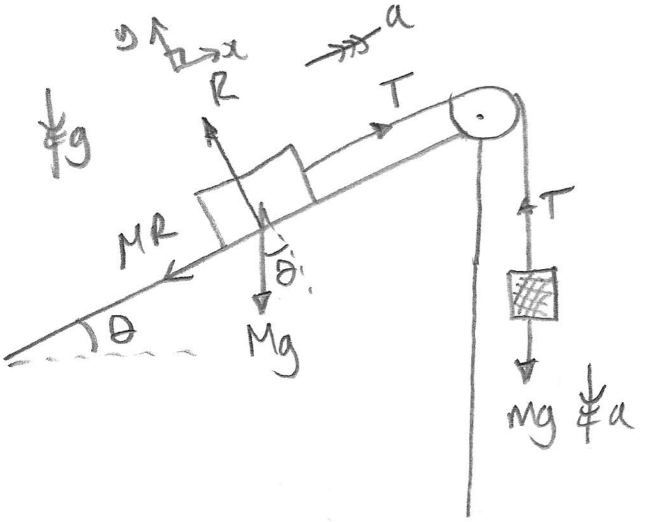
$$x = \sqrt{\frac{6.67\times10^{-11}\times 100{,}000\times 5.97\times10^{24}}{4000}} - 6371\times10^{3} \quad (m)$$

4000 ↑ Thrust is 4kN

$= 9.34\times10^{7}\,m$

$\simeq \boxed{14.7\ R_{\oplus}}$

2/



NII for mass on slope

$\parallel x: \quad Ma = T - Mg\sin\theta - \mu R \quad (1)$

$\parallel y \quad 0 = R - Mg\cos\theta \quad (2)$

NII for mass m on end of pulley

$ma = mg - T \quad (3)$

So combining (1) and (2)

$$Ma = T - Mg\sin\theta - \mu Mg\cos\theta \quad (4)$$

$$ma = -T + mg \quad (3)$$

$$(M+m)a = mg - Mg\sin\theta - \mu Mg\cos\theta \quad (4)+(3)$$

$$M(a + g\sin\theta + \mu g\cos\theta) = m(g-a)$$

$$\therefore \quad m = \frac{M(a + g\sin\theta + \mu g\cos\theta)}{g-a}$$

$$\boxed{m = M\frac{\left(a/g + \sin\theta + \mu\cos\theta\right)}{1 - a/g}}$$

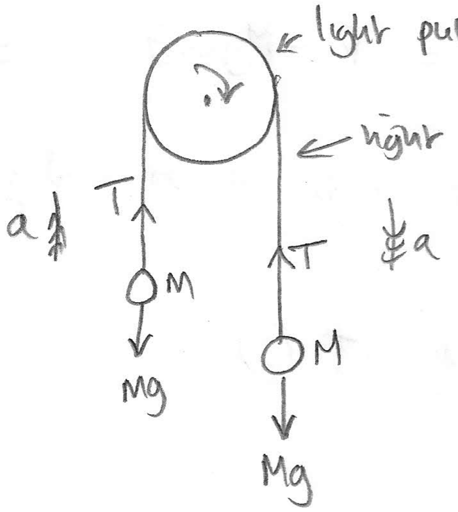
kinematics : $\boxed{x = \frac{1}{2}at^2}$ so $a = \frac{2x}{t^2}$

$a = \frac{2 \times 22}{6.0^2} = \boxed{1.22\ m/s^2}$

$\therefore \quad m = 123kg \times \frac{\left(\frac{1.22}{9.81} + \sin 30^\circ + 0.2 \times \cos 30^\circ \right)}{1 - 1.22/9.81}$

$\boxed{m = 112kg}$

3/



$(M > m)$

NII: $Ma = Mg - T$ ①

$ma = T - mg$ ②

① + ② : $(M+m)a = (M-m)g$

$\therefore a = \frac{M-m}{M+m} g$

$$\boxed{a = \frac{1 - m/M}{1 + m/M} g}$$

[and $T = Mg - Ma$ from ①

$\Rightarrow T = Mg - \frac{M-m}{1+m/M} g$

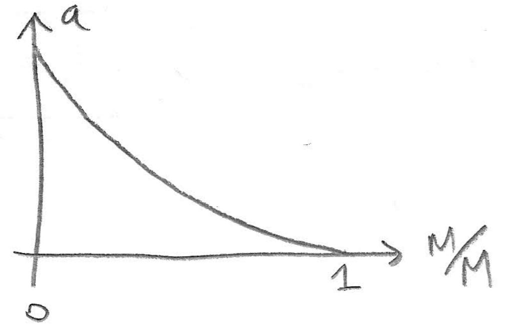
$T = \frac{(M+m)g - (M-m)g}{1 + m/M} = \boxed{\frac{2mg}{1 + m/M}}$

{ This makes sense, since if $M \gg m$ M is in free fall at $a = g$. $\therefore$ m must accelerate upwards at g. $\therefore T = 2 \times mg$ }

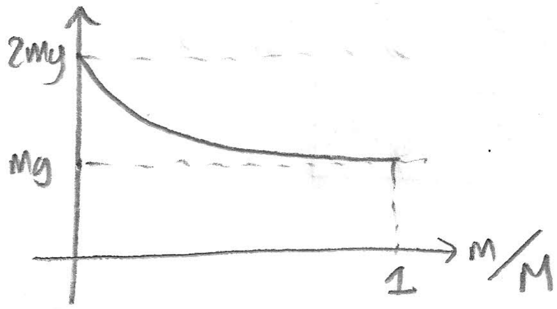
b) $\boxed{a = \dfrac{1 - m/M}{1 + m/M} g}$

when $m/M \to 0$ $\quad a \to g$

when $m/M \to 1$ $\quad a \to 0$ $\qquad$ (Note $M > m$)



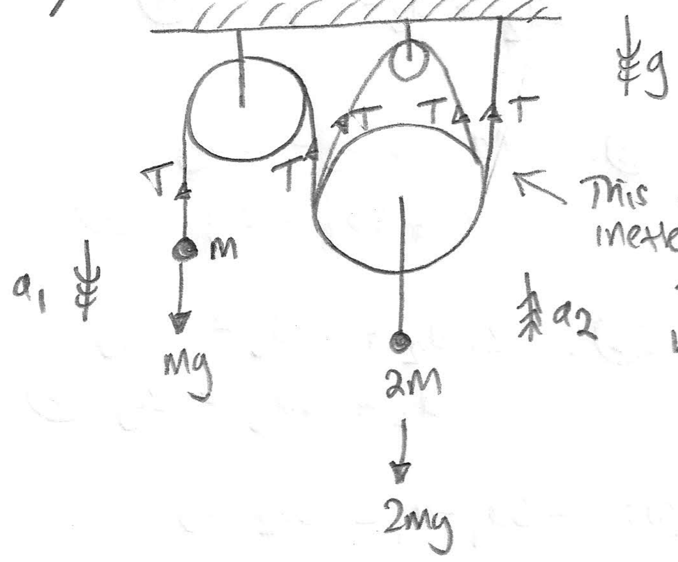
$$T = \frac{2mg}{1 + m/M}$$



when $m/M \to 0$, $T \to 2mg$

when $m/M \to 1$, $T \to mg$

4/



This string (light & inextensible) is wrapped twice round the lower pulley.

NII: $m a_1 = mg - T$ ①

$2M a_2 = 4T - 2Mg$ ②

"Conservation of String"

$a_1 = 4a_2$ ③

(i.e. If lower pulley moves up by x, since there are four strings connected to it, all four will be shortened by x, $\therefore$ left mass will move down by $4x$ $\therefore$ $a_1 = 4a_2$)

Since string is inextensible $\rightarrow$

So: $4(1) + (2)$: $4ma_1 + 2ma_2 = 4mg - 2mg$

Using (3): $4m(4a_2) + 2ma_2 = 2mg$

$$18a_2 = 2g$$

$$\boxed{a_2 = \tfrac{1}{9}g}$$

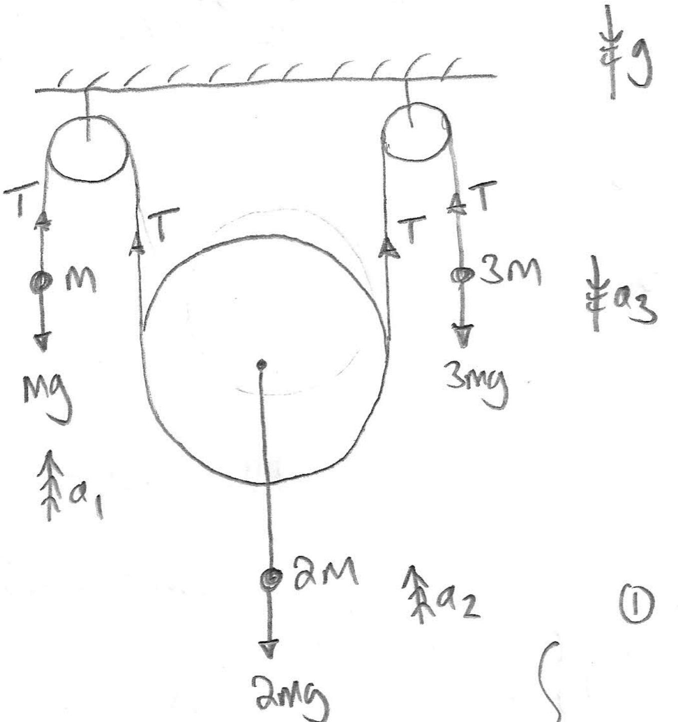
∴ Since $a_1 = 4a_2 \Rightarrow \boxed{a_1 = \tfrac{4}{9}g}$

$T = mg - ma_1$ from (1)

∴ $T = mg - \frac{4}{9}mg$

$\boxed{T = 5/9\, mg}$

5/



NII: $ma_1 = T - mg$ (1)

$2ma_2 = 2T - 2mg$ (2)

$3ma_3 = 3mg - T$ (3)

"conservation of string"

$2a_2 = a_3 - a_1$ (4)

(1) + (3): $ma_1 + 3ma_3 = 2mg$

∴ $a_1 + 3a_3 = 2g$ (5)

(2) + 2(3): $2ma_2 + 6ma_3 = 4mg$

$a_2 + 3a_3 = 2g$ (6)

Ellimnate T first to get 3 eqs in a_1, a_2, a_3

3(4) − (5): $-3a_1 - a_1 = 6a_2 - 2g$

$2g = 6a_2 + 4a_1$

$g = 3a_2 + 2a_1$ (7)

(5) − (6): $a_1 - a_2 = 0 \Rightarrow \boxed{a_1 = a_2}$ (8)

$\therefore$ in (7): $g = 3a_1 + 2a_1 \Rightarrow \boxed{g/5 = a_1}$

$\therefore \boxed{a_2 = g/5}$ since $a_1 = a_2$

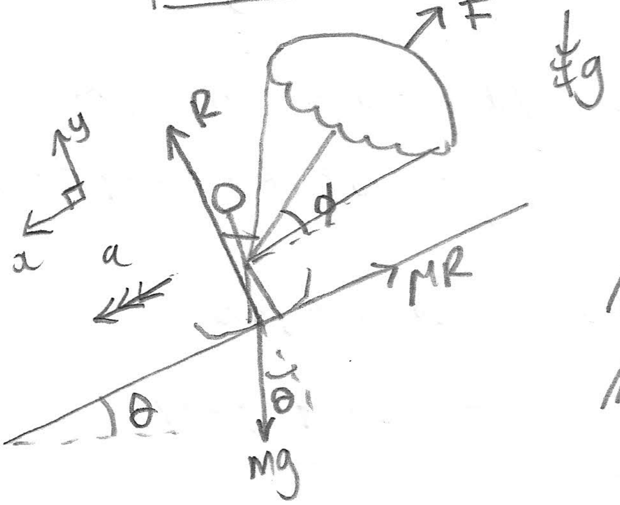
and in (4): $a_3 = 2a_2 + a_1 = 3a_1 = \boxed{3g/5}$

In (1): $T = ma_1 + mg$

$\therefore T = \frac{mg}{5} + mg$

$\boxed{T = \frac{6}{5} mg}$

6/



$\mu = 0.05$
$\theta = 30°$
$\phi = 42°$
$m = 100kg$

{ignore air resistance, apart from parachute drag, when deployed}

a) Initially no parachute deployed. ($F = 0$).

So NII:

$//x: \quad ma = mg\sin\theta - \mu R$

$//y: \quad 0 = R - mg\cos\theta$

$\therefore \boxed{a = g\sin\theta - \mu g\cos\theta}$

$\therefore a = 9.81(\sin 30° - 0.05\cos 30°)$
$= \boxed{4.48 \text{ m/s}^2}$

b) If friction acts uphill, and $a = 0$ after parachute deployed (ie, skier is sliding downhill at constant velocity)

NII $//x: \quad 0 = mg\sin\theta - \mu R - F\cos\phi$ (1) {since sliding $F = \mu R$}

$//y: \quad 0 = -mg\cos\theta + R + F\sin\phi$ (2)

So $R = mg\cos\theta - F\sin\phi$ from (2)

$\therefore \mu(mg\cos\theta - F\sin\phi) = mg\sin\theta - F\cos\phi$ (1)

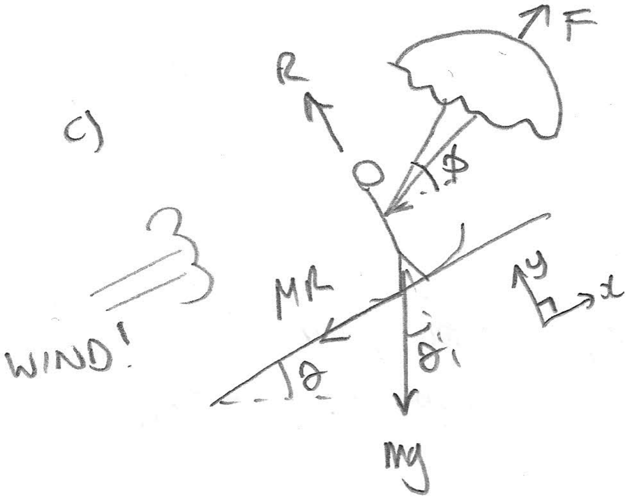
$\Rightarrow \mu mg\cos\theta - mg\sin\theta = F(\mu\sin\phi - \cos\phi)$

$$\therefore \quad \boxed{F = \frac{mg(\mu\cos\theta - \sin\theta)}{\mu\sin\phi - \cos\phi}}$$

So $$F = \frac{100 \times 9.81 \times (0.05\cos 30^\circ - \sin 30^\circ)}{0.05\sin 42^\circ - \cos 42^\circ}$$

$$= \boxed{631\ \text{N}}$$

c)



Now assume a wind gust pulls the skier up the slope at constant velocity.

NII: $/\!/x: \quad 0 = F\cos\phi - \mu R - mg\sin\theta$

$/\!/y: \quad 0 = R + F\sin\phi - mg\cos\theta$

$$\therefore \quad \mu(mg\cos\theta - F\sin\phi) = F\cos\phi - mg\sin\theta$$

$$\mu mg\cos\theta + mg\sin\theta = F(\cos\phi + \mu\sin\phi)$$

$$\therefore \quad \boxed{F = \frac{mg(\sin\theta + \mu\cos\theta)}{\mu\sin\phi + \cos\phi}}$$

$$\therefore \quad F = \frac{100 \times 9.81(\sin 30^\circ + 0.05\cos 30^\circ)}{0.05\sin 42^\circ + \cos 42^\circ}$$

$$= \boxed{686\ \text{N}}$$

{ So between F = 631N and 686N, Skier would be in static equilibrium, and during this range the friction force would → 0 and then change direction }

d) Returning to the downhill sliding situation, but let a act uphill

NII: $/\!/x$: $-ma = mg\sin\theta - \mu R - F\cos\phi$ ①

Acts downhill

$/\!/y$: $0 = R + F\sin\phi - mg\cos\theta$ ②

$\therefore$ $-ma = mg\sin\theta - \mu\left(mg\cos\theta - F\sin\phi\right) - F\cos\phi$

$a = -g\sin\theta + \mu\left(g\cos\theta - \frac{F}{m}\sin\phi\right) + \frac{F}{m}\cos\phi$

$$\boxed{a = g\left(-\sin\theta + \mu\cos\theta + \frac{F}{mg}\left(\cos\phi - \mu\sin\phi\right)\right)}$$

$\therefore$ if $F = 100N$

$a = 9.81\left(-\sin 30^\circ + 0.55\cos 30^\circ + \frac{900}{100 \times 9.81}\left(\cos 42^\circ - 0.55\sin 42^\circ\right)\right)$

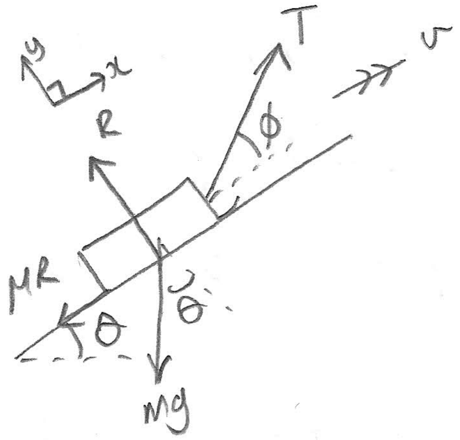
$= \boxed{1.91\ m/s^2}$

[Note slope.m gives the wrong answer, since it assumes sliding uphill if F acts downhill. In our case, skier slides downhill, pops the parachute and experiences a deceleration, but still slides downhill!]

* This question illustrates the care you must take for inclined plane problems!

You can't just 'plug numbers into formulae'! You have to think about the scenario first *

7/



Constant speed up the slope.

$\phi = 30^\circ \quad \theta = 40^\circ \quad \mu = 0.5$

$T = 700$ N.

NII $/\!/x$: $0 = T\cos\phi - mg\sin\theta - \mu R$

$/\!/y$: $0 = R - mg\cos\theta + T\sin\phi$

$\therefore \quad R = mg\cos\theta - T\sin\phi$

$\therefore \quad mg\sin\theta = T\cos\phi - \mu(mg\cos\theta - T\sin\phi)$

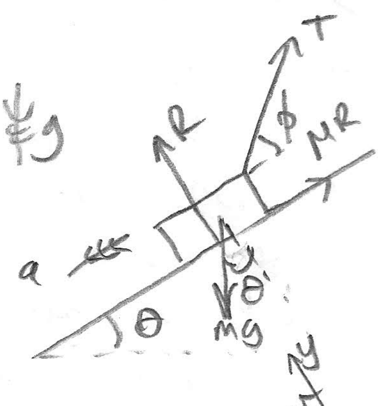
$\therefore \quad mg(\sin\theta + \mu\cos\theta) = T(\cos\phi + \mu\sin\phi)$

$$\therefore \quad \boxed{m = \frac{T(\cos\phi + \mu\sin\phi)}{g(\sin\theta + \mu\cos\theta)}}$$

$$\therefore \quad m = \frac{700 \times (\cos 30^\circ + 0.5\sin 30^\circ)}{9.81 \times (\sin 40^\circ + 0.5\cos 40^\circ)}$$

$$= \boxed{77.6 \text{ kg}}$$

Now let mass accelerate down the slope.



NII $/\!/x$: $ma = mg\sin\theta - \mu R - T\cos\phi$

$/\!/y$: $0 = R + T\sin\phi - mg\cos\theta$

$\therefore \quad ma = mg\sin\theta - \mu(mg\cos\theta - T\sin\phi) - T\cos\phi$

Let $a > 0$ (and obviously $m > 0$)

$a > 0$

$\Rightarrow \quad mg(\sin\theta - \mu\cos\theta) + T(\mu\sin\phi - \cos\phi) > 0$

$$\Rightarrow \quad \boxed{m > \frac{T}{g}\frac{(\cos\phi - \mu\sin\phi)}{\sin\theta - \mu\cos\theta}}$$

if $\sin\theta - \mu\cos\theta > 0$

$\therefore$ $\boxed{\tan\theta > \mu}$ (Note this is the sliding criteria if $T=0$).

In our case $\tan 40^\circ = 0.839$
$\mu = 0.5$ so ok.

$$\therefore \quad M > \frac{700}{9.81} \frac{(\cos 30^\circ - 0.5\sin 30^\circ)}{\sin 40^\circ - 0.5\cos 40^\circ}$$

$\boxed{M > 169.2 \text{ kg}}$

So if $M > 169.2$kg, it will accelerate down the slope. If $77.6 < M < 169.2$ (kg) it will be in static equilibrium. If $M < 77.6$ kg it will slide up the slope. [Assuming T, θ, ϕ, μ remain the same].

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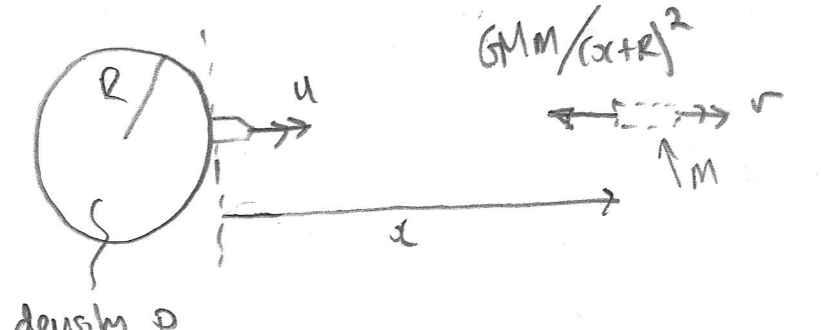


$M = \frac{4}{3}\pi R^3 \rho$

(ie assume uniform planet)

NII $// x$: $\quad m\frac{dv}{dt} = -\frac{GMm}{(x+R)^2}$

$\frac{dv}{dt} = v\frac{dv}{dx}$

$\left[\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v\right]$

$$\therefore \quad v\frac{dv}{dx} = -\frac{G\frac{4}{3}\pi R^3\rho}{(x+R)^2}$$

$$\therefore \int_u^v v\,dv = -\frac{4}{3}G\pi R^3\rho \int_0^x \frac{dx}{(x+R)^2}$$

Now $\frac{d}{dx}\left(\frac{1}{x+R}\right) = \frac{-1}{(x+R)^2}$ so $\int_0^x \frac{dx}{(x+R)^2} = -\int_0^x d\left(\frac{1}{x+R}\right)$

$$\therefore \quad \tfrac{1}{2}v^2 - \tfrac{1}{2}u^2 = \tfrac{4}{3} G\pi R^3 \rho \left[\frac{1}{x+R} \right]_0^x$$

$$v^2 - u^2 = \tfrac{8}{3} G\pi \rho R^3 \underbrace{\left(\frac{1}{x+R} - \frac{1}{R} \right)}_{}$$

$-ve$ since $x \geqslant 0$

$$v^2 = u^2 - \tfrac{8}{3} G\pi \rho R^3 \left(\frac{1}{R} - \frac{1}{x+R} \right)$$

$$\therefore \quad \boxed{v = \sqrt{u^2 - \tfrac{8}{3} G\pi \rho R^3 \left(\frac{1}{R} - \frac{1}{x+R} \right)}}$$

So as $x \uparrow$, $v \downarrow$

$v=0$ represents the furthest x the projectile attains from the planet before gravity attracts it back.

ie $$u^2 = \tfrac{8}{3} G\pi \rho R^3 \left(\frac{1}{R} - \frac{1}{x+R} \right)$$

ie $$\frac{3u^2}{8G\pi \rho R^3} = \frac{1}{R} - \frac{1}{x+R}$$

$$\frac{1}{x+R} = \frac{1}{R} - \frac{3u^2}{8G\pi \rho R^3}$$

$$\therefore \quad \boxed{x_{max} = \left(\frac{1}{R} - \frac{3u^2}{8G\pi \rho R^3} \right)^{-1} - R}$$

Now when $\frac{1}{R} = \frac{3u^2}{8G\pi \rho R^3}$ $\Rightarrow x_{max} \to \infty$

ie the projectile escapes if $u^2 > \frac{8G\pi \rho R^2}{3}$

$\Rightarrow$ escape velocity $\boxed{u_E = \sqrt{\tfrac{8}{3} G\pi \rho R^2}}$

or better: $\boxed{u_E = \sqrt{\tfrac{8}{3}G\pi\rho}\ R}$

Now let $u_E = c$ (the speed of light)

$$M = \tfrac{4}{3}\pi R^3 \rho$$

so $\dfrac{2M}{R} = \tfrac{8}{3}\pi R^2 \rho$

$$u_E^2 = \tfrac{8}{3}G\pi\rho R^2$$

$\therefore$ $\boxed{u_E^2 = \dfrac{2GM}{R}}$

This is the more familiar expression for escape velocity, which you can obtain really easily if you know the GPE is $-\dfrac{GMm}{x+R}$

see fields course!

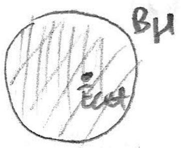
So if $u_E = c$

$\Rightarrow$ $\boxed{R = 2GM/c^2}$

"Schwarzschild radius" of a Black Hole.

Note this means $R \propto M$ so you don't need a really dense black hole for it to work as one!

$$R = \frac{u_E}{\sqrt{\tfrac{8}{3}G\pi\rho}}$$



So let $u_E = c$

$\Rightarrow$ $\boxed{R = \dfrac{c}{\sqrt{\tfrac{8}{3}G\pi\rho}}}$

if $\rho = 1000\,\mathrm{kg/m^3}$ (ie water density)

$$R = \frac{2.998 \times 10^8}{\sqrt{\tfrac{8}{3} \times 6.67 \times 10^{-11} \times \pi \times 1000}}$$

$= \boxed{4.01 \times 10^{11}\,\mathrm{m}} \approx \boxed{2.68\,\mathrm{AU}}$

$[1\,\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{m}]$

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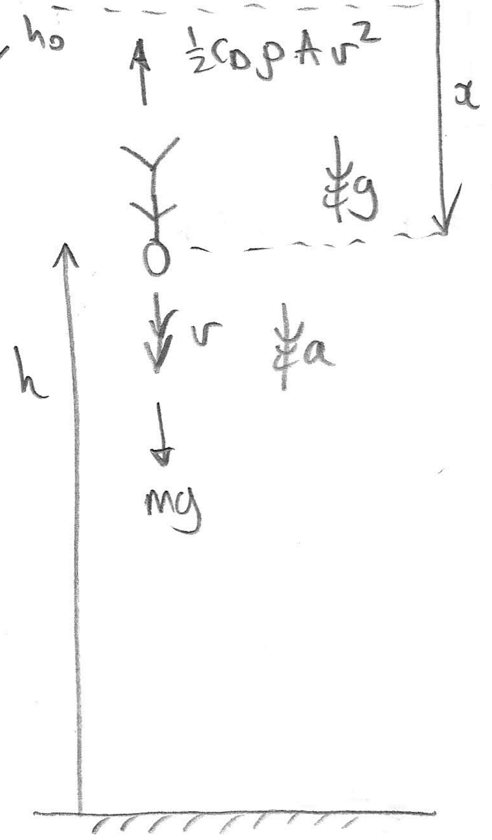

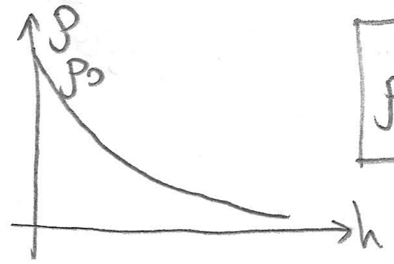

$$\boxed{\rho = \rho_0 e^{-h/H}}$$

$\rho_0 = 1.225$ kg/m^3

$H = 10{,}400$ m

air density and altitude (x)
(Assumes constant air temperature)

not true in the troposphere!

$A = 2.0\,\text{m}^2$
until height h_p
where parachute
is deployed.

Initial conditions:

$h_0 = 10{,}000$ m
$t_0 = 0$
$x_0 = 0$
$v_0 = 0$
$A = 2.0\,\text{m}^2$

let $g = 9.81$ N/kg
$C_D = 0.4$
be fixed constants
Also $M = 100$ kg

NII downwards:

$$ma = mg - \tfrac{1}{2}C_D\rho A v^2 \qquad \text{so} \qquad \boxed{a = g - \tfrac{1}{2}C_D\rho A v^2/m}$$

Verlet: ($\Delta t = 0.1$s)

$$x_{n+1} = x_n + v_n\Delta t + \tfrac{1}{2}a_n\Delta t^2$$

$$\rho = \rho_0 e^{-h_n/H}$$

$$t_{n+1} = t_n + \Delta t \qquad h_n = h_0 - x_n \qquad \alpha = a(x_n, v_n)$$

$$\beta = a(x_{n+1}, v_n) \qquad v_{n+1} = v_n + \tfrac{1}{2}(A_n + A_{n+1})\Delta t$$

$\hookrightarrow$ MATLAB code.

$\{A_p = \pi\times(7.14)^2 = 160\,\text{m}^2$
$t_{fall} = 506$s $\Rightarrow v_{land} = 5.0$ m/s$\}$

Aim is to pop parachute at $h \approx 2.0$ km and have $v < 5.0$ m/s at $h = 0$.