Newton's laws of motion:

I Unless acted upon by an external **force**, the center of mass of an object will continue to move at constant velocity. This is a special case of Newton II where there is *zero vector sum of force.*

II mass × acceleration = vector sum of force. $m\mathbf{a} = \sum_i \mathbf{f}_i$. Assuming mass m is constant, his means the ***rate of change of momentum*** $\frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} =$ **the vector sum of force** $\sum_i \mathbf{f}_i$.

Forces are measured in *Newtons* (N), which in SI units are: $1\text{N} = 1\text{kgms}^{-2}$.

III Force all contact forces, "for every action there is an equal and opposite reaction." i.e. if body A is in contact with body B and exerts a force upon it, the force upon A due to B is equal in magnitude and opposite in direction.

Systems involving **objects in contact** can be treated as (i) a whole system from a Newton II sense, and (ii) look at the component parts and use Newton III to relate the contact forces. e.g. trains and carriages, cars and trailers with tow bars…

Systems of **pulleys** can often be modelled as having *light, frictionless pulleys, connected by light inextensible string.* To solve these systems, relate the accelerations of any masses, assuming the string doesn't stretch.

The force of gravity is **weight**. For an object of mass m the weight is: $\mathbf{W} = m\mathbf{g}$. On the Earth's surface, $\mathbf{g}$ acts towards the center of the Earth with strength $g = 9.81\text{Nkg}^{-1}$. Outside a spherical body of mass M (e.g. planet, star etc) $g = \frac{GM}{r^2}$ where r is the distance to the center of mass of the body, and the *Universal Gravitational Constant* is: $G = 6.67\times10^{-11}\text{m}^3\text{s}^{-2}\text{kg}^{-1}$. **Electrical forces** between (static) charges work in a similar *inverse-square* way. However, unlike gravity (where mass is always positive, and all masses attract each other), charges can have *both signs,* and *opposite charges attract, alike charges repel*. e.g. electrons are negatively charged, and will be attracted to positively charged protons, but protons will repel protons, and electrons will repel electrons.

Frictional forces: An object won't slide if the friction force $F \le \mu R$ where R is the normal contact force and μ is the coefficient of friction between the object and the surface that it is in contact with. When it does slide, the friction force is $F = \mu_D R$. The 'dynamic' coefficient of friction should be similar to μ, but may change in certain scenarios (e.g. ice skating, when a surface heats up during sliding).

Drag forces are typically of the form $F = \frac{1}{2}c_D \rho A v^2$ where c_D is the *drag coefficient*, ρ is the density of fluid (e.g. air, water), A is the cross sectional area of the moving body, and v is its speed. For low velocity drag, especially involving viscous forces, $F \propto v$. Viscous drag ('Stokes' drag') on a sphere of radius r is given by $F = 6\pi\eta r v$ where η is the dynamic viscosity.

If an object is immersed in a fluid, the **upthrust** on it (Archimedes' principle) is equal to the weight of fluid displaced.

Question 1 (In all questions, take $g = 9.81\text{Nkg}^{-1}$ unless otherwise stated)

(i) A car of mass 2000kg accelerates at 1.2m/s^2. Calculate the driving force on the car, if the total resistive forces sum to 600N.

(ii) The four winds blow upon the ancient Macedonean meteorologist Andronicus of Cyrrhus (mass 90kg). *Boreas* blows with 30N *from* the North, *Zephryos* blows with 60N from the West, *Notus* blows with 50N from the South and *Eurus* blows with 42N from the East. Calculate the magnitude and direction (in terms of an angle clockwise from North) of the acceleration of Andronicus.

(iii) Jennifer Bond escapes from a burning airplane and deploys a parachute. Her total mass is 65kg and she falls through air of density $\rho = 1.23\text{kgm}^{-3}$. If the drag coefficient is $c_D = 0.8$ and her desired terminal speed is 4.00m/s, calculate the radius of her (circular cross section) parachute (in m).

(iv) A large suitcase (mass 30kg) slides down a metal ramp in Heathrow airport. The coefficient of friction is μ and the dynamic version is the same. The suitcase slides with acceleration 6.0m/s^2 when the slope angle is 45°. Calculate μ. Ignore air resistance.

(v) A tractor of mass 11,000kg pulls a trailer of mass 21,000kg. They are connected by a rigid metal rod. Both the tractor and the trailer experience a resistive force equal to 10% of their weight. Both the tractor and the trailer accelerate at 3.14m/s^2. (a) Calculate the driving force of the tractor. (b) Calculate the tension in the metal rod.

(vi) A 1.23kg mass is connected to mass m via a single frictionless, light pulley via a light inextensible string. Calculate mass m if it accelerates downwards at $\frac{1}{10}g$, and the tension (in N) in the string.

(vii) A locomotive of mass M provides a net force to itself and the N carriages (each of mass m). The acceleration of the whole train (which moves as a single rigid object) is a. Show that the force of the carriages on the locomotive is: $f_1 = Nma$. Hence show that the tension in the cable pulling the n^{th} carriage is $f_n = (N - n + 1)ma$. If the locomotive has a mass of $M = 110,000\text{kg}$ and each of the thirty carriages has a mass of 131,000kg, what is the required driving force if the whole train accelerates at 0.1m/s^2?

(viii) A uniform solid plastic sphere of density 920kg/m^3 and radius 5cm is immersed in glycerol of density 1260 kg/m^3. Calculate the acceleration of the sphere if it rises at speed 0.5m/s. The dynamic viscosity of glycerol is $\eta = 1.07\text{Nsm}^{-2}$. Also calculate the magnitudes of the forces acting on the sphere: (i) weight; (ii) upthrust; (iii) viscous drag. Ignore any 'kinematic drag' proportional to the square of speed.[1]

(ix) Calculate the mass of the Earth (in kg), given it is approximately spherical with a radius of 6371km.

(x) Rather than enter orbit, a spacecraft of mass 100,000kg wishes to maintain its position and watch the Earth rotate, i.e. the spacecraft is stationary relative to the center of mass of the Earth. Calculate the distance from the *surface* of the Earth, *in Earth radii,* if the spacecraft engines provide a thrust of 4.0kN.

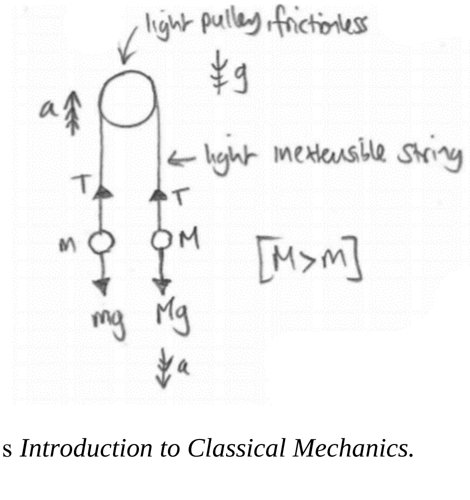
Question 2 A mass of 123kg is to be dragged up a slope of 30° via a pulley system. If the coefficient of friction is $\mu = 0.2$, the mass accelerates uniformly from rest, and the mass travels 22m up the slope in 6.0s, calculate the mass m (in kg) on the end of the pulley, which drops vertically without resistance.

Question 3 Consider the idealized pulley system on the right:

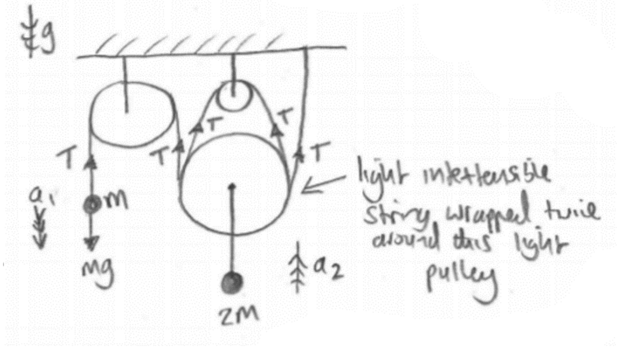
(a) Show that:

$$a = \frac{1 - m/M}{1 + m/M} g \quad \text{and} \quad T = \frac{2mg}{1 + m/M}$$

(b) Sketch graphs of a and T vs m/M.



Question 4 The following '*Atwood machine'* is problem 3.27 in Morin's *Introduction to Classical Mechanics.*

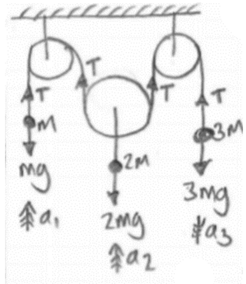


Show that:

$$a_1 = \tfrac{4}{9}g,\ a_2 = \tfrac{1}{9}g,\ T = \tfrac{5}{9}mg.$$

[1] This would probably be more important than the viscous drag as the sphere rises faster.

Question 5



The following ‘Atwood machine’ is problem 3.29 in Morin’s *Introduction to Classical Mechanics.*

Show that: $a_1 =$ $a_1 = \frac{1}{5}g,\ a_2 = \frac{1}{5}g,\ a_3 = \frac{3}{5}g$ and that the string tension is $T = \frac{6}{5}mg$.

Question 6 A skier of mass 100kg is sliding downhill on a slope of elevation 30°. The coefficient of friction with the slope is $\mu = 0.05.$

(a) Calculate the acceleration (in m/s^2) down the slope, if air resistance can be ignored.

The skier then deploys a parachute at an angle of 42° to the slope.

(b) Calculate the parachute force on the skier resulting from the parachute which results in dynamic equilibrium, with *friction acting uphill.* i.e. the skier is sliding downhill at constant velocity.

(c) Calculate the parachute force on the skier such that the skier is in dynamic equilibrium, but the *friction force acts downhill.* Note this means the skier *slides uphill*, and an uphill gust of wind provides the force!

(d) What will be the instantaneous acceleration uphill, if the parachute force is 900N, but the skier is sliding downhill?

Question 7 During a mountain rescue operation, a sled is dragged up a 40° slope with coefficient of friction $\mu = 0.5$, *at constant speed*, using a rope, inclined at 30° to the slope, providing a constant tension of 700N. Calculate the mass of the sled (in kg). What is the minimum mass such that the sled starts to accelerate *down* the slope?

Question 8 A projectile is launched radially at speed u from the surface of an atmosphere-free spherical planet of radius R and uniform density ρ. Using the calculus trick $dv/dt = v dv/dx$ where x is displacement and velocity $v = dx/dt$, show that the velocity v at distance x from the surface of the planet is: $v = \sqrt{u^2 - \frac{8}{3}G\pi\rho R^2\left(1 - \frac{R}{x+R}\right)}$.

Hence show that $u_E = \sqrt{\frac{8}{3}G\pi\rho}R$ represents the *escape velocity*, i.e. when $x \to \infty$, the projectile is still moving away from the planet if $u > u_E$. Assume the only force acting on the projectile is the gravitational attraction of the planet.

If the escape velocity is the speed of light $c = 2.998\times10^8\,\text{ms}^{-1}$, show that $R = 2GM/c^2 = \dfrac{c}{\sqrt{\frac{8}{3}G\pi\rho}}$, where M is the mass of the planet. (This is called the *Schwarzschild radius,* and has the same mathematical expression as the event horizon of a *Black Hole,* obtained using General Relativity!). Calculate the radius of a Black Hole with $\rho = 1000\text{kgm}^{-3}$.

Question 9 ** This question requires some computer programming, or spreadsheet construction. **

If the atmosphere is of constant temperature[2], the density ρ of air varies (approximately) with altitude h as $\rho \approx \rho_0 e^{-h/H}$ where $\rho_0 = 1.225\text{kgm}^{-3}$ and $H = 10{,}400\text{m}$. A skydiver of mass $m =$ 100kg falls from rest from $h_0 =$ 10km. The cross-sectional area of the skydiver is $A =$ 2.0m^2 and the drag coefficient is $c_D = 0.4$. Use Newton II: $ma = mg - \frac{1}{2}c_D\rho Av^2$ and the following *Verlet* method with a fixed time step of $\Delta t = 0.1\text{s}$ to compute (and plot) displacement, velocity and acceleration until the skydiver is 2.0km from the ground. (At this point a parachute is deployed). The Verlet iterative method (which uses the idea of constant acceleration motion between small, fixed, time steps) is:

$$t_{n+1} = t_n + \Delta t, \quad h_n = h_0 - x_n\ , \quad \alpha = a\left(x_n, v_n\right),\ x_{n+1} = x_n + v_n\Delta t + \tfrac{1}{2}\Delta t^2\ ,$$
$$V = v_n + \alpha\Delta t,\ \ \beta = a\left(x_{n+1}, V\right),\ \ v_{n+1} = v_n + \tfrac{1}{2}\left(\alpha + \beta\right)\Delta t$$

For a more realistic model, increase A in a linear fashion over a set deployment time such as 10s.

Investigate how large A must be when the parachute is deployed, in order to achieve a landing speed of about 5.0m/s. This means continuing the simulation until $h_n = 0$, but with a larger A when $h < 2.0\text{km}$.

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[2] Depending which layer you are referring to, there is a lapse rate in the atmosphere. In the *troposphere*, temperature reduces between 5 and 10 degrees every km in altitude. The lower limit is for more humid air, the upper limit for dry air.