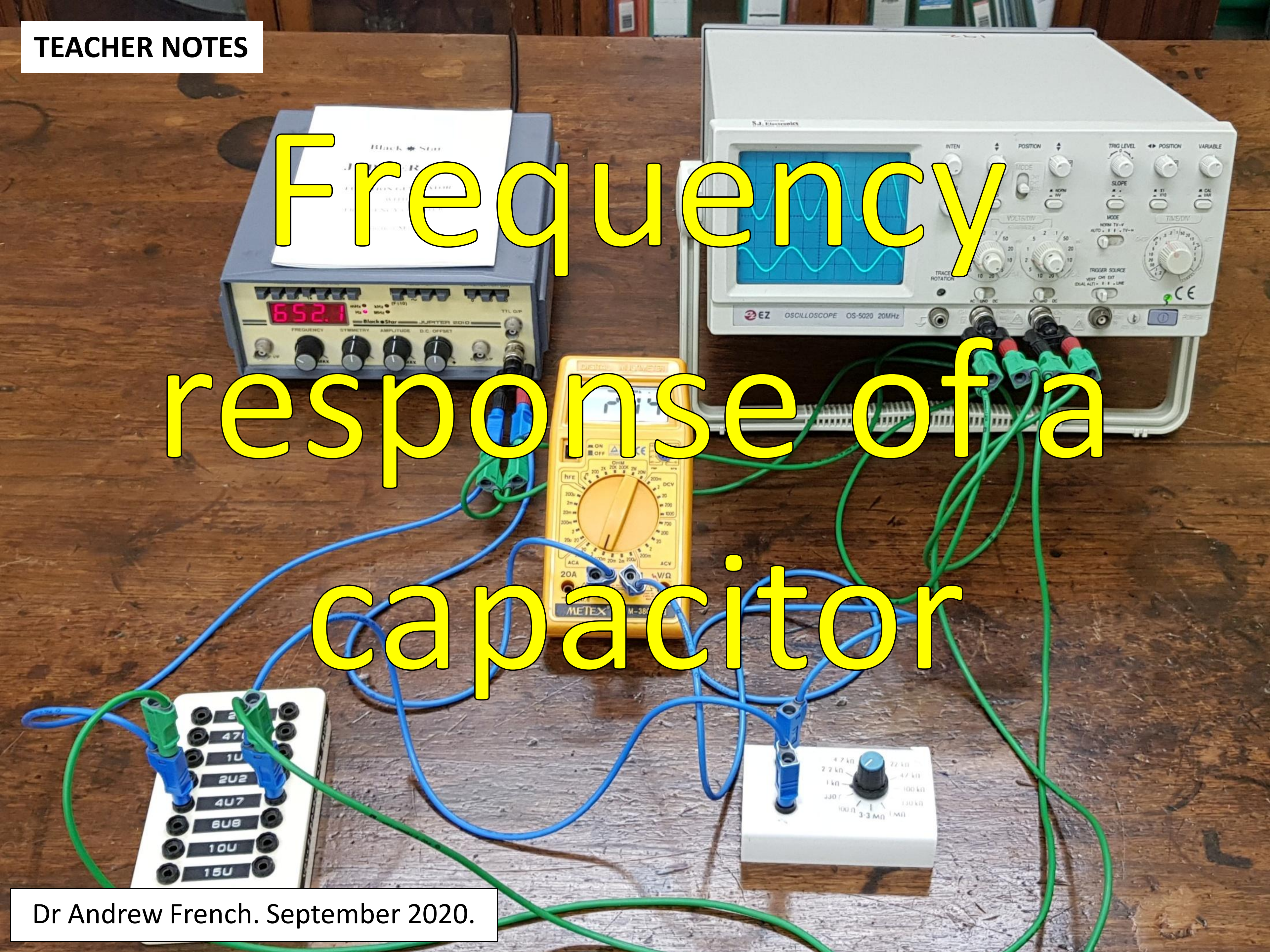
TEACHER NOTES

Frequency response of a capacitor



Dr Andrew French. September 2020.

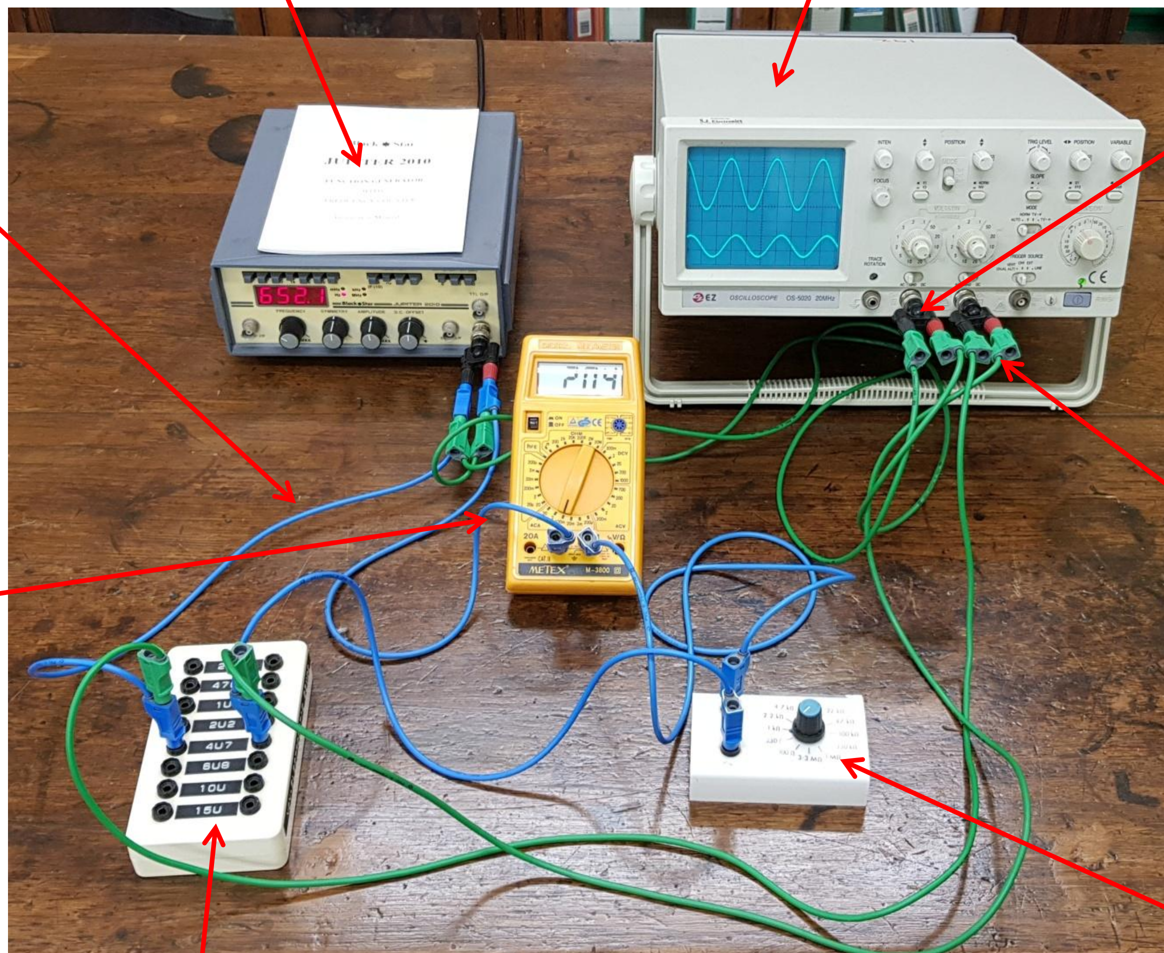
Equipment

Signal generator + mains cable kettle lead + BNC to 4mm converter

Dual input oscilloscope + mains cable kettle lead + 2 x BNC to 4mm converter

4 x blue wires (main RC circuit loop)

2x green wires (PD across AC input, connected to oscilloscope CH1)



Multimeter (set in ACA mode i.e. gives RMS amplitude of (AC) current)

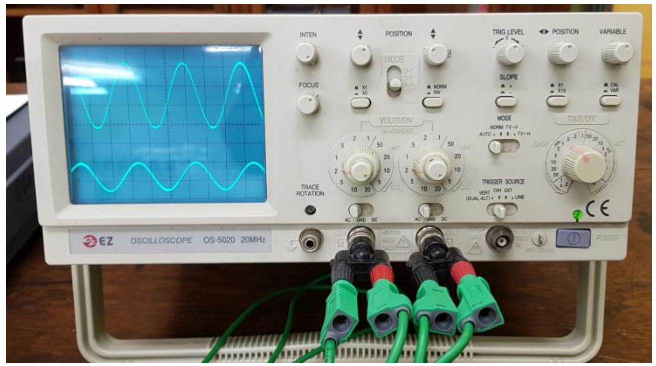
2x green wires (PD across capacitor, connected to oscilloscope CH2)

Fixed resistors (only 100ohm will be used)

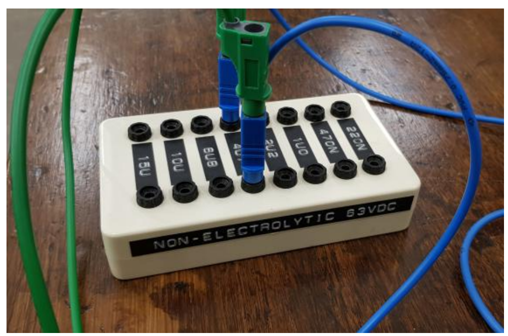
Capacitor block (non-electrolytic) range 0.220μF to 15μF.

Signal generator + mains cable kettle lead + BNC to 4mm converter

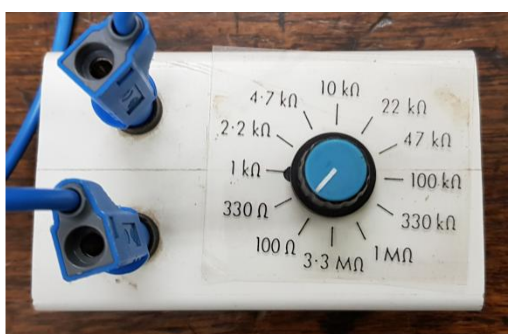
Dual input oscilloscope + mains cable kettle lead + 2 x BNC to 4mm converter



Capacitor block (non-electrolytic) range 0.220µF to 15µF.

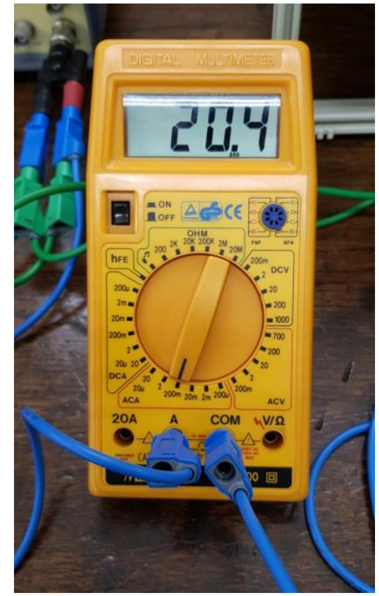


Fixed resistors (only 100ohm will be used)



Kit list

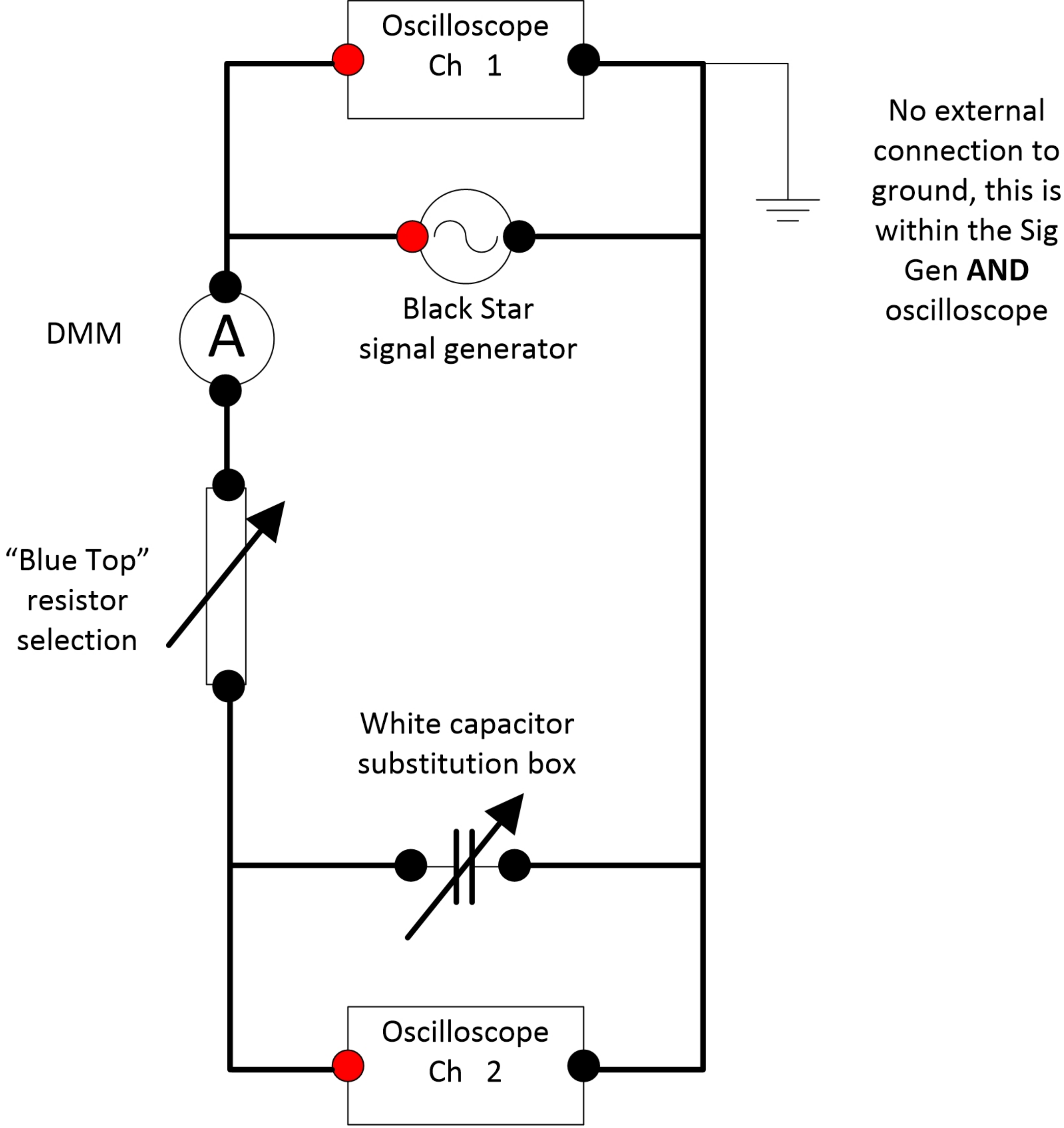
Multimeter
(set in ACA mode i.e. gives RMS amplitude of (AC) current)



4 x blue wires (main RC circuit loop)

2x green wires
(PD across AC input, connected to oscilloscope CH1)

2x green wires
(PD across capacitor, connected to oscilloscope CH2)


Oscilloscope
Ch 1
No external connection to ground, this is within the Sig Gen **AND** oscilloscope
Black Star signal generator
DMM
A
“Blue Top” resistor selection
White capacitor substitution box
Oscilloscope
Ch 2

Suggested experiments

1. Fix V_{in} at about **3.0V** (the middle value of the signal generator amplitude).

2. Fix R at about **100Ω** (measure it accurately with a multi-meter).

3. Fix capacitances at **1.0μF, 2.2μF and 4.7μF**. Do the middle value only if time permits.

4. Record and plot gain $|V_c/V_{\text{in}}|$ vs frequency f for ten to twenty measurements over the frequency range **0** to **3,000Hz.**

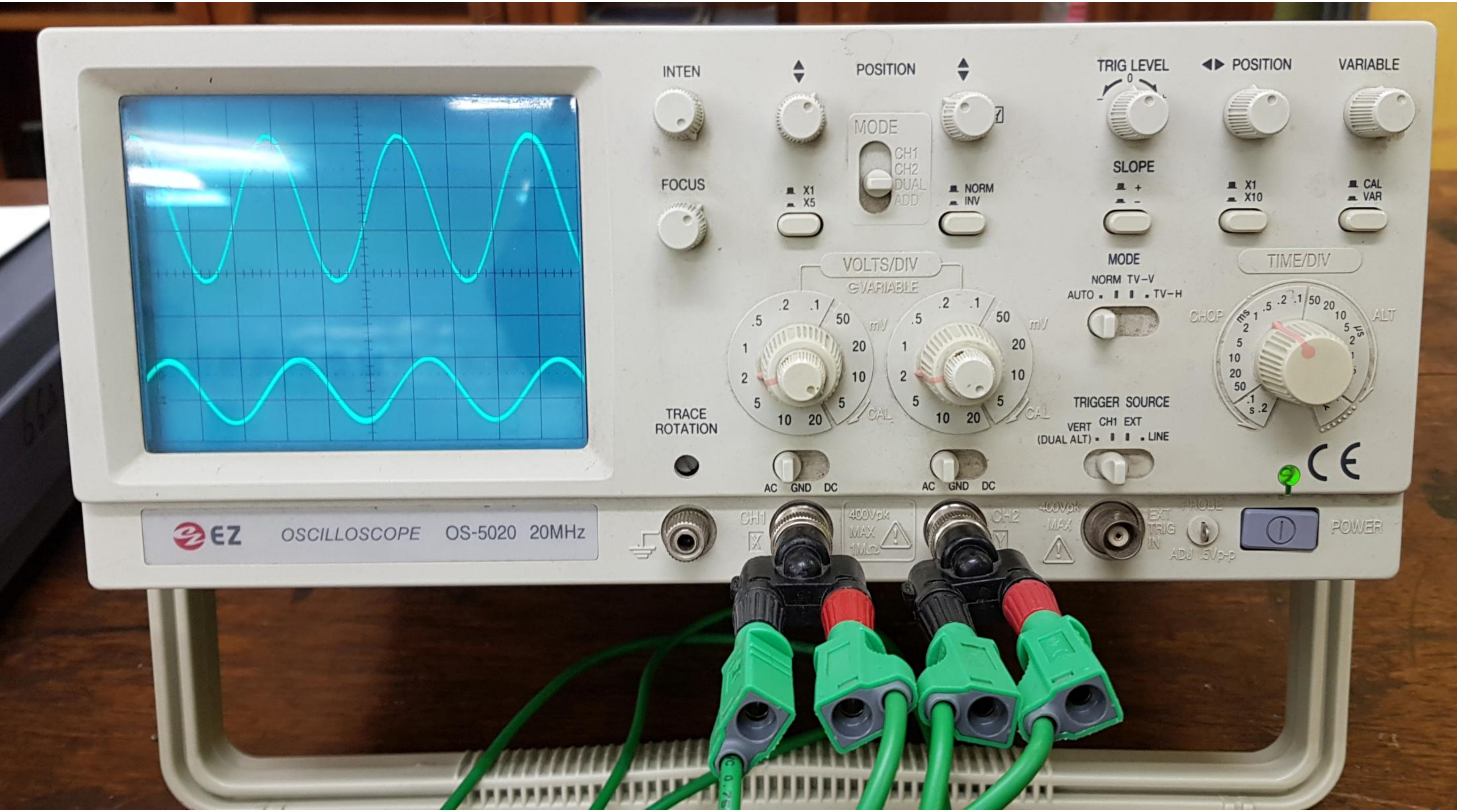
5. Also record and plot the RMS current vs frequency.

6. Fixing the frequency at **200Hz**, find the RMS current for the *full range of capacitances.* **You should find that current is proportional to capacitance.**

7. For a 4.7μF capacitor, keep the frequency at 3,000Hz and switch to a square wave, and then a triangle wave output from the signal generator. Observe that the RC circuit *integrates* the input, if the output is taken across the capacitor. i.e. an output of triangle waves or parabolae, respectively.

8. See if you can observe a *differentiation* effect for low frequencies (i.e. about 100Hz) when the output is taken across the resistor instead of the capacitor.

Dual channel **oscilloscope**

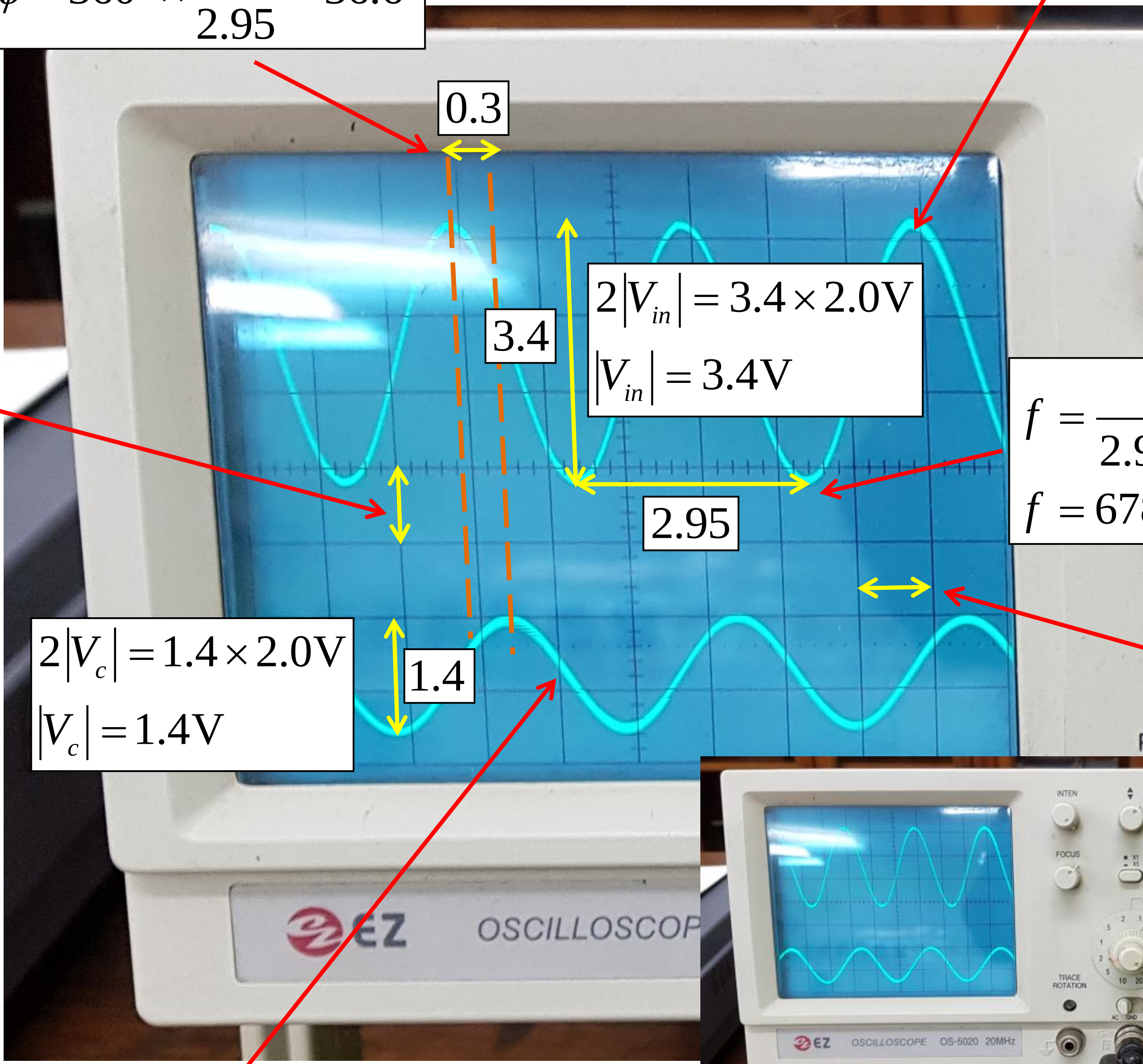


Channel 1 (CH1): Potential difference across output of **signal generator** (upper trace).
Channel 2 (CH2): Potential difference across **capacitor** (lower trace).

Phase difference

$$\phi = 360^{o} \times \frac{0.30}{2.95} = 36.6^{o}$$

Input voltage from signal generator (CH1)



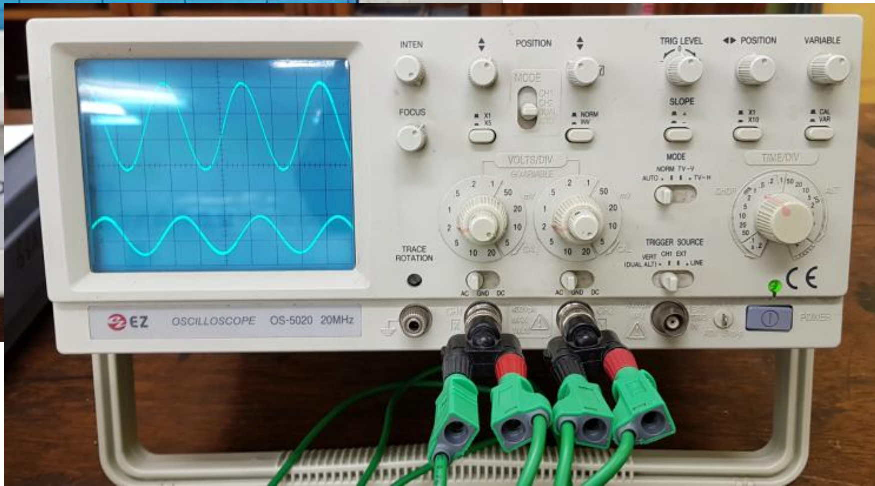
Voltage divisions = 2.0V

$$2\left|V_{in}\right| = 3.4 \times 2.0\text{V}$$
$$\left|V_{in}\right| = 3.4\text{V}$$

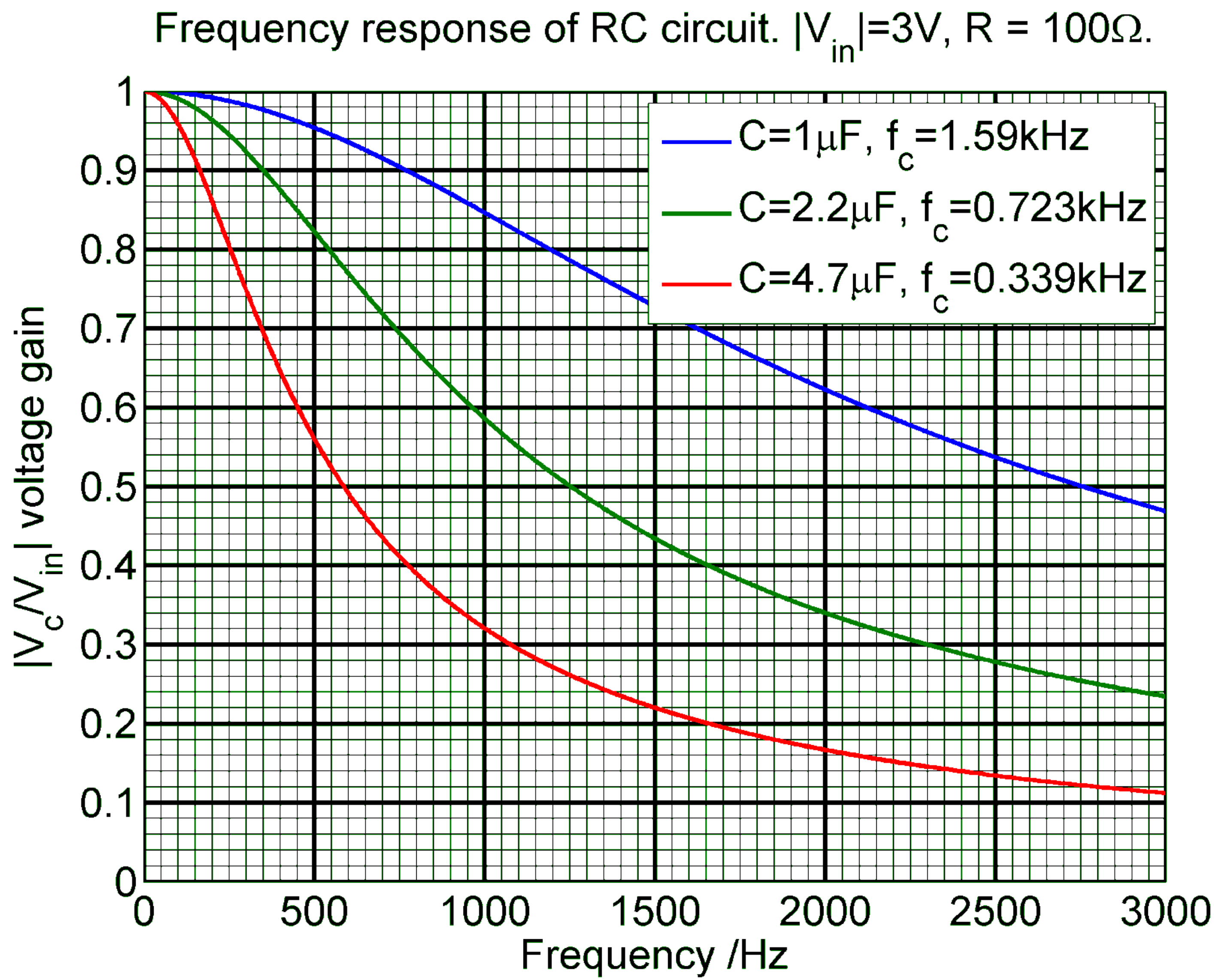
$$f = \frac{1}{2.95 \times 0.5 \times 10^{-3}\text{s}}$$
$$f = 678\text{Hz}$$

$$2\left|V_{c}\right| = 1.4 \times 2.0\text{V}$$
$$\left|V_{c}\right| = 1.4\text{V}$$

Timebase = 0.5ms



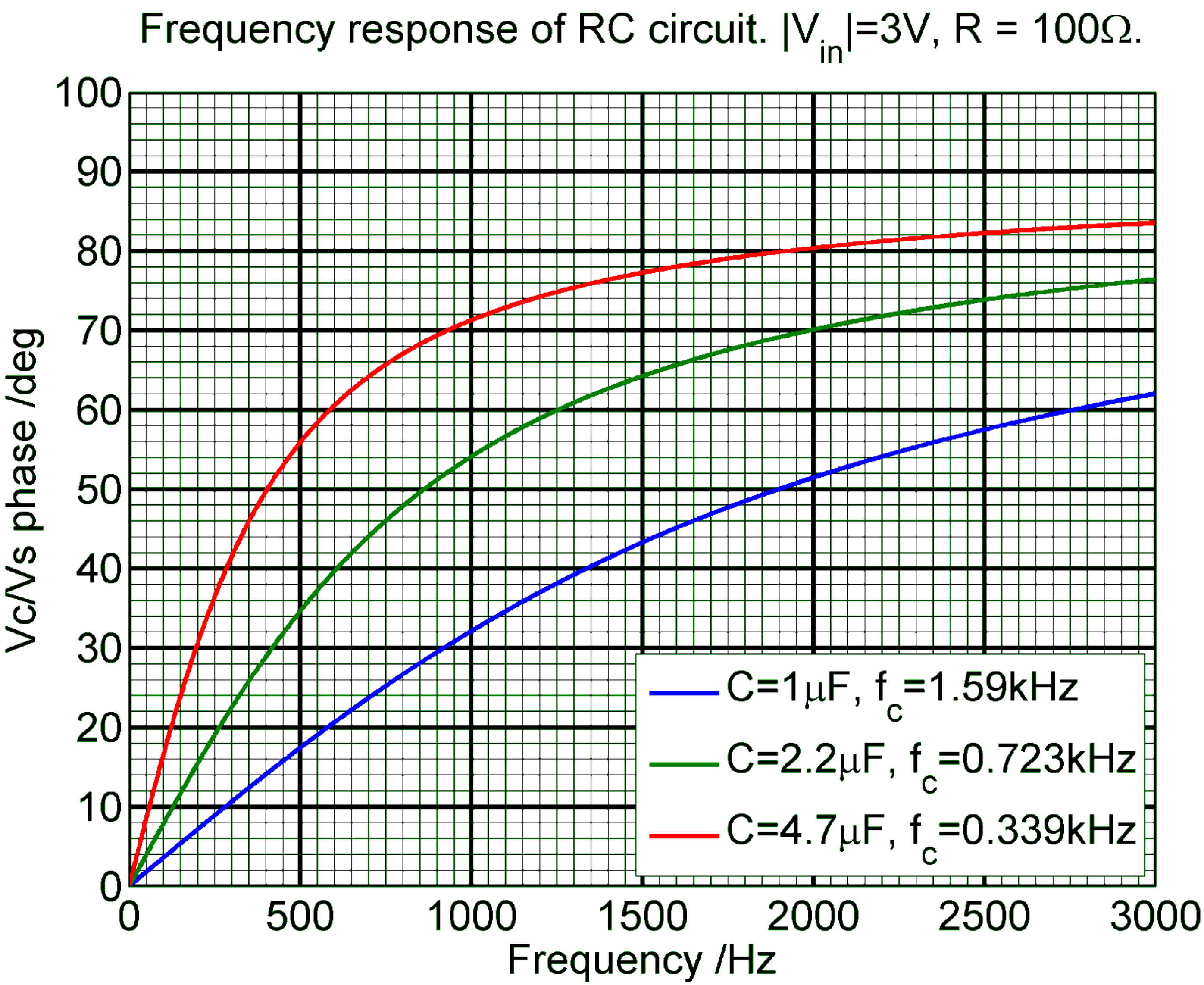
Potential difference across the capacitor (CH2)



$$f_c = \frac{1}{2\pi RC}$$ Characteristic frequency

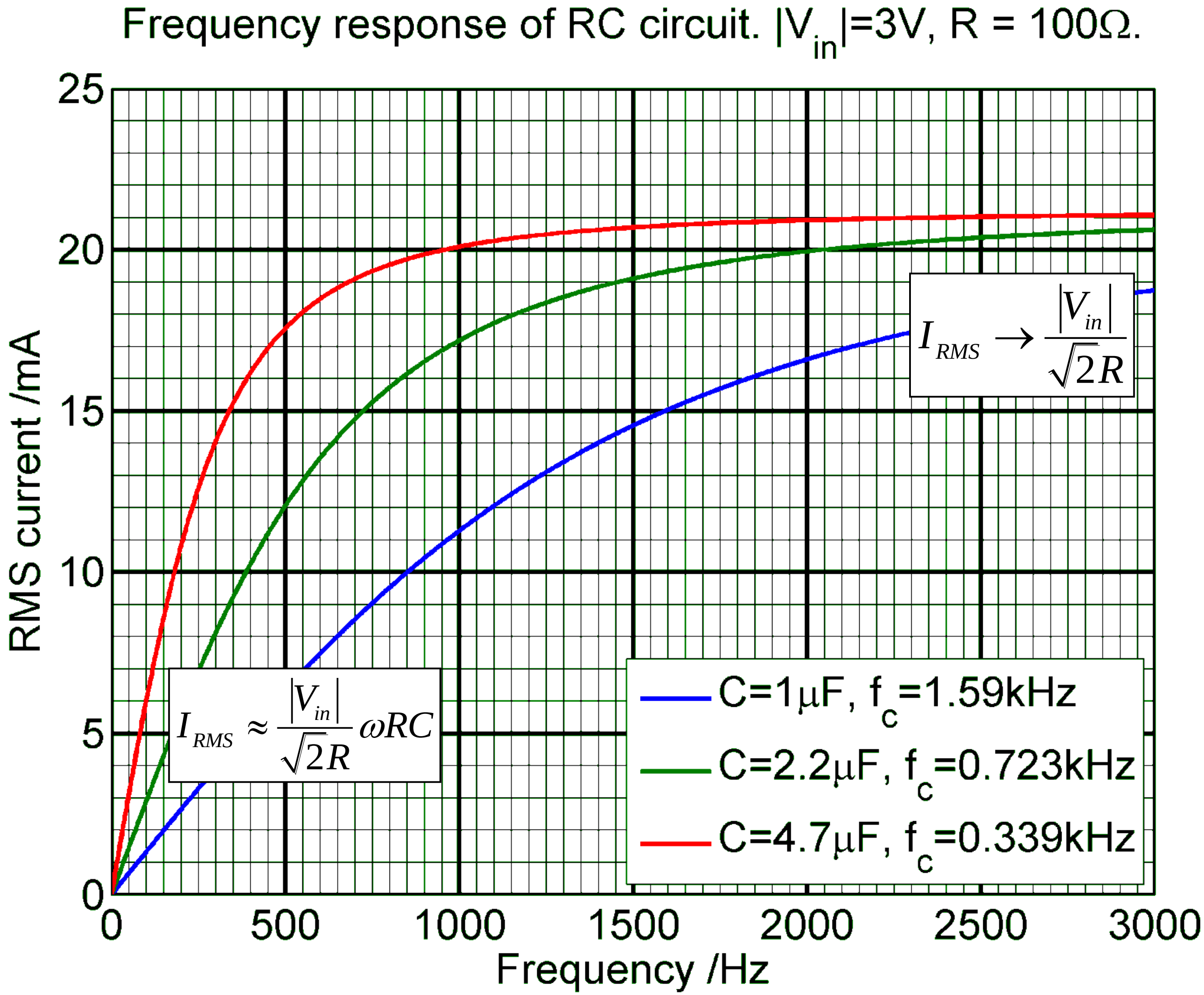
Gain reduces as frequency increases.

Phase shift tends to 90^o as frequency increases



$$f_c = \frac{1}{2\pi RC}$$ Characteristic frequency

When frequencies are much greater than the characteristic frequency, current tends to a *constant*:

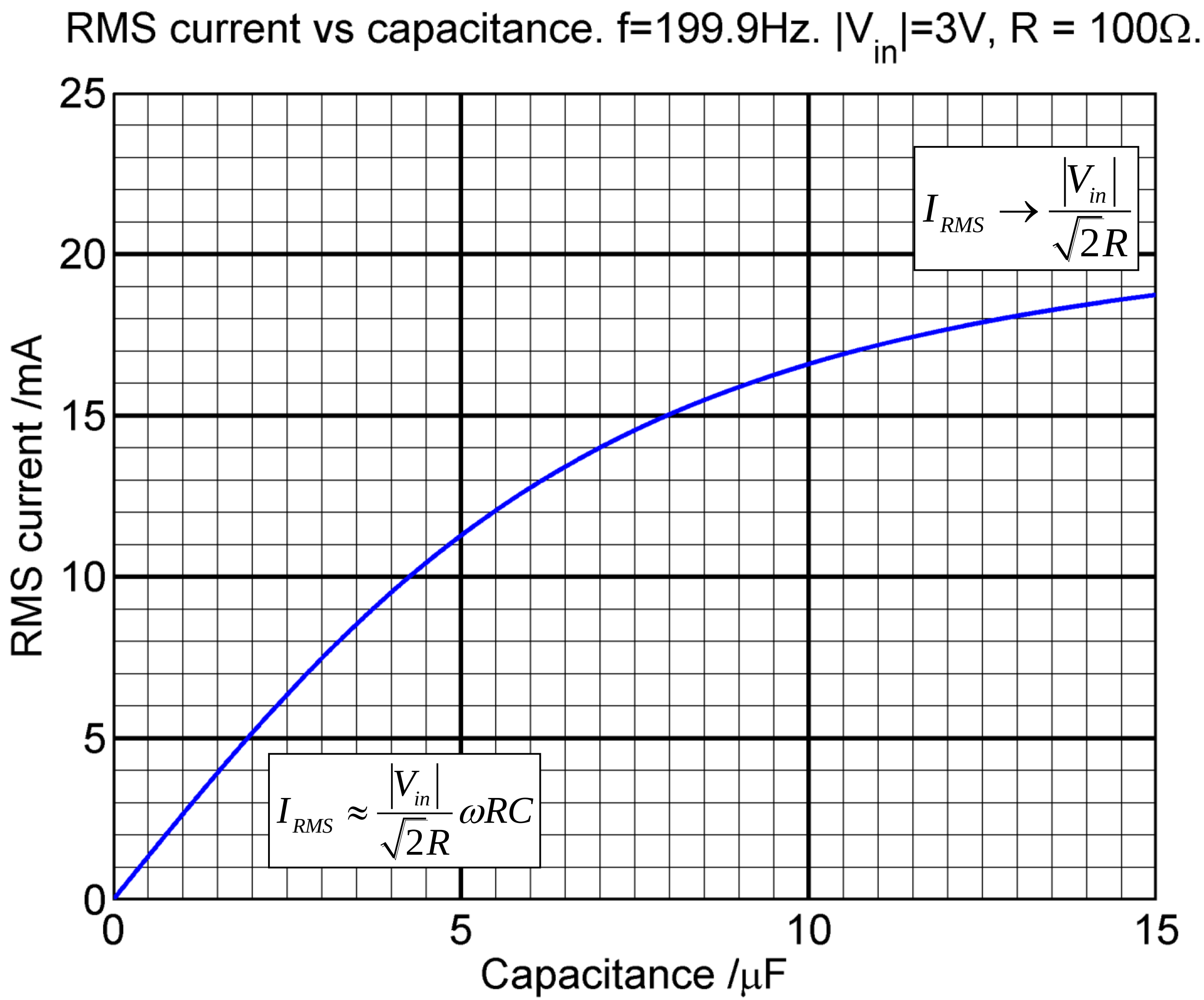


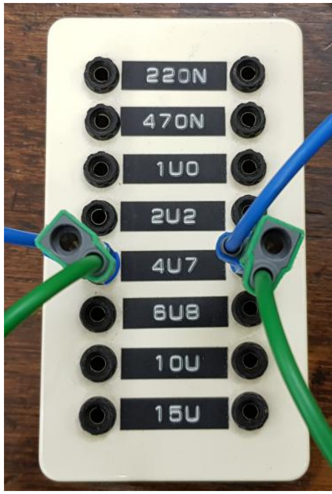
Characteristic frequency:

$$f_c = \frac{1}{2\pi RC}$$

Current is *proportional* to frequency (and capacitance) when frequencies are much less than the characteristic frequency.

When frequencies are much greater than the characteristic frequency, current tends to a *constant*:





Current is *proportional* to capacitance when frequencies are much less than the characteristic frequency.

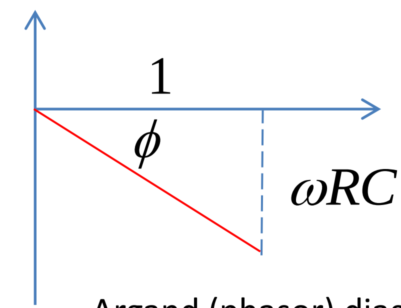
Mathematical model of voltage gain vs frequency

$$\omega = 2\pi f \qquad Z_R = R$$

$$\frac{V_C}{V_{in}} = \frac{Z_C}{Z_C + Z_R}$$ Potential divider

Complex impedence $$Z_C = \frac{1}{i\omega C}$$

$$\frac{V_C}{V_{in}} = \frac{1/i\omega C}{1/i\omega C + R} = \frac{1}{1 + i\omega RC} = \frac{1 - i\omega RC}{1 + \omega^2 R^2 C^2}$$

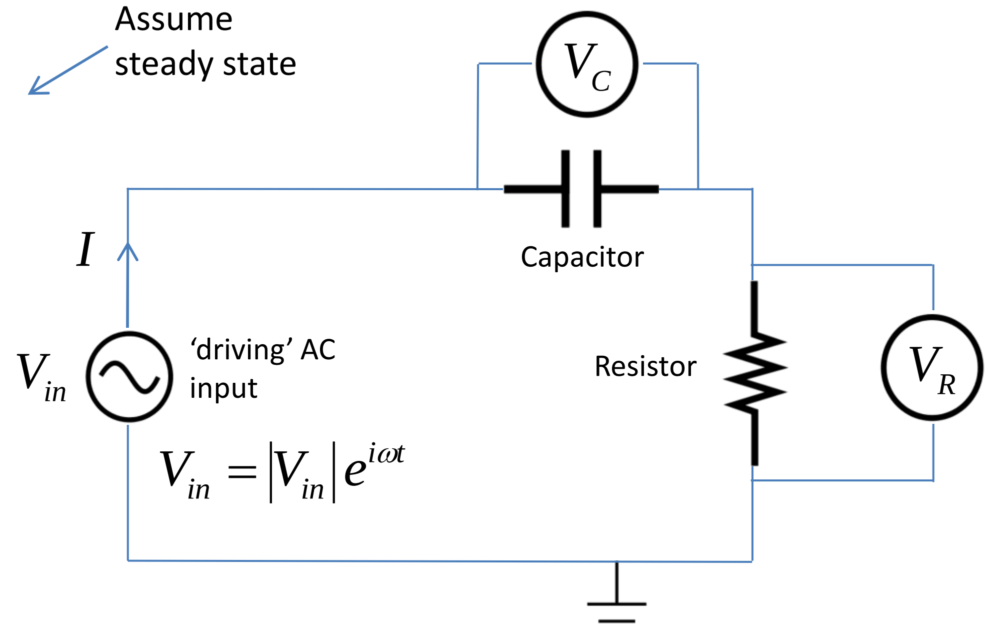


Argand (phasor) diagram

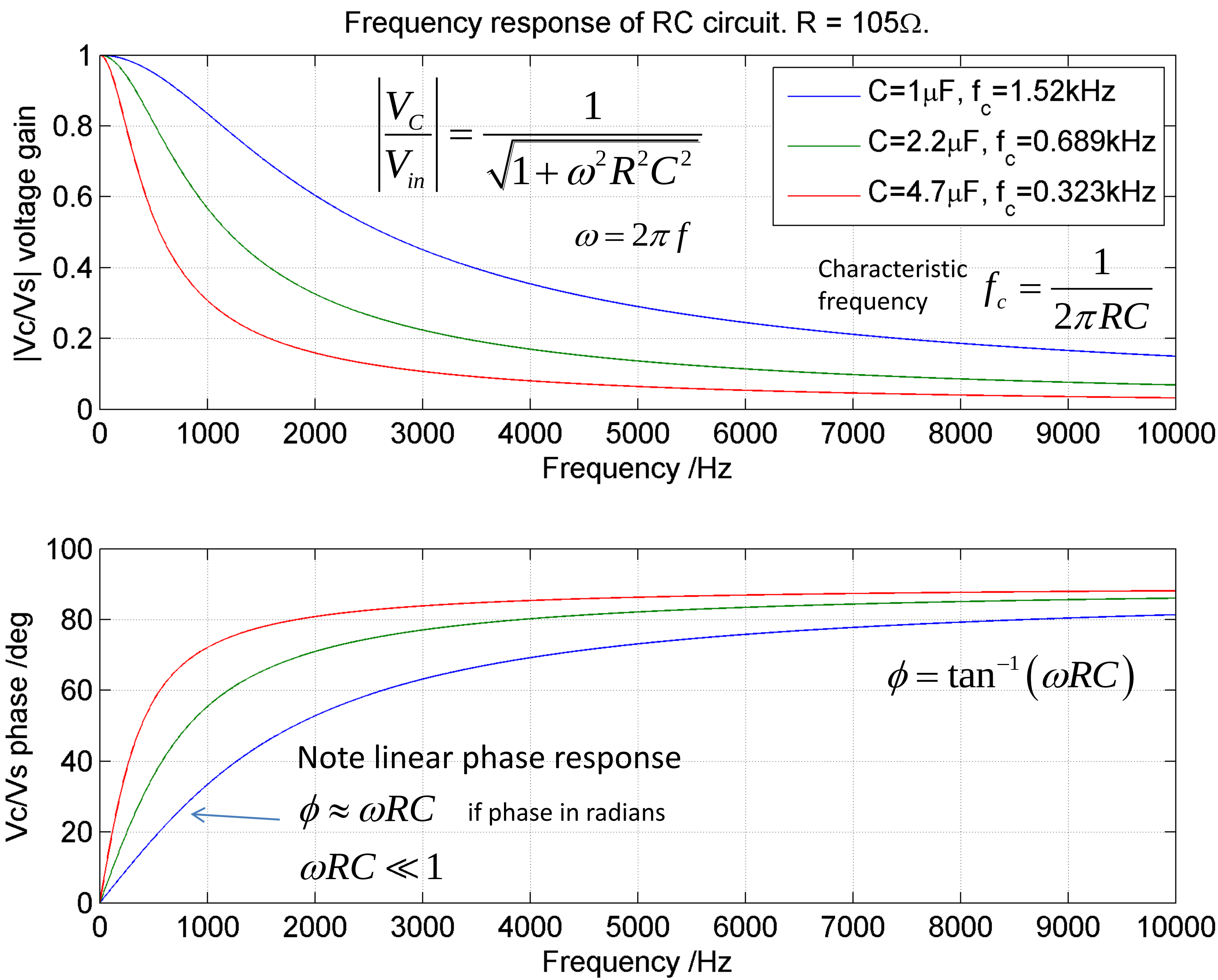
$$\therefore \frac{V_C}{V_{in}} = \frac{1}{1 + \omega^2 R^2 C^2} \sqrt{1 + \omega^2 R^2 C^2}\, e^{-i \tan^{-1}(\omega RC)}$$

$$V_C = |V_C| e^{i(\omega t - \phi)} \qquad V_{in} = |V_{in}| e^{i\omega t}$$

Assume steady state



$$\boxed{\left|\frac{V_C}{V_{in}}\right| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \qquad \phi = \tan^{-1}(\omega RC)}$$



Frequency response of RC circuit. R = 105Ω.
|Vc/Vs| voltage gain
$\left|\frac{V_C}{V_{in}}\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$
$\omega = 2\pi f$
C=1μF, f_c=1.52kHz
C=2.2μF, f_c=0.689kHz
C=4.7μF, f_c=0.323kHz
Characteristic frequency
$f_c = \frac{1}{2\pi RC}$
Frequency /Hz
Vc/Vs phase /deg
$\phi = \tan^{-1}(\omega RC)$
Note linear phase response
$\phi \approx \omega RC$
if phase in radians
$\omega RC \ll 1$
Frequency /Hz

Finding the RMS current

$$\omega = 2\pi f$$

$$V_C = \frac{|V_{in}|}{\sqrt{1+\omega^2 R^2 C^2}} e^{i\{\omega t - \tan^{-1}(\omega RC)\}}$$

$$V_C = I Z_C$$

$$V_C = I / i\omega C \quad \therefore I = i\omega C V_C$$

$$\therefore I = \frac{i\omega C |V_{in}|}{\sqrt{1+\omega^2 R^2 C^2}} e^{i\{\omega t - \tan^{-1}(\omega RC)\}}$$

$$\therefore |I| = \frac{|V_{in}|}{R} \times \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$$

$$\therefore \boxed{I_{RMS} = \frac{|V_{in}|}{\sqrt{2}R} \times \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}}$$

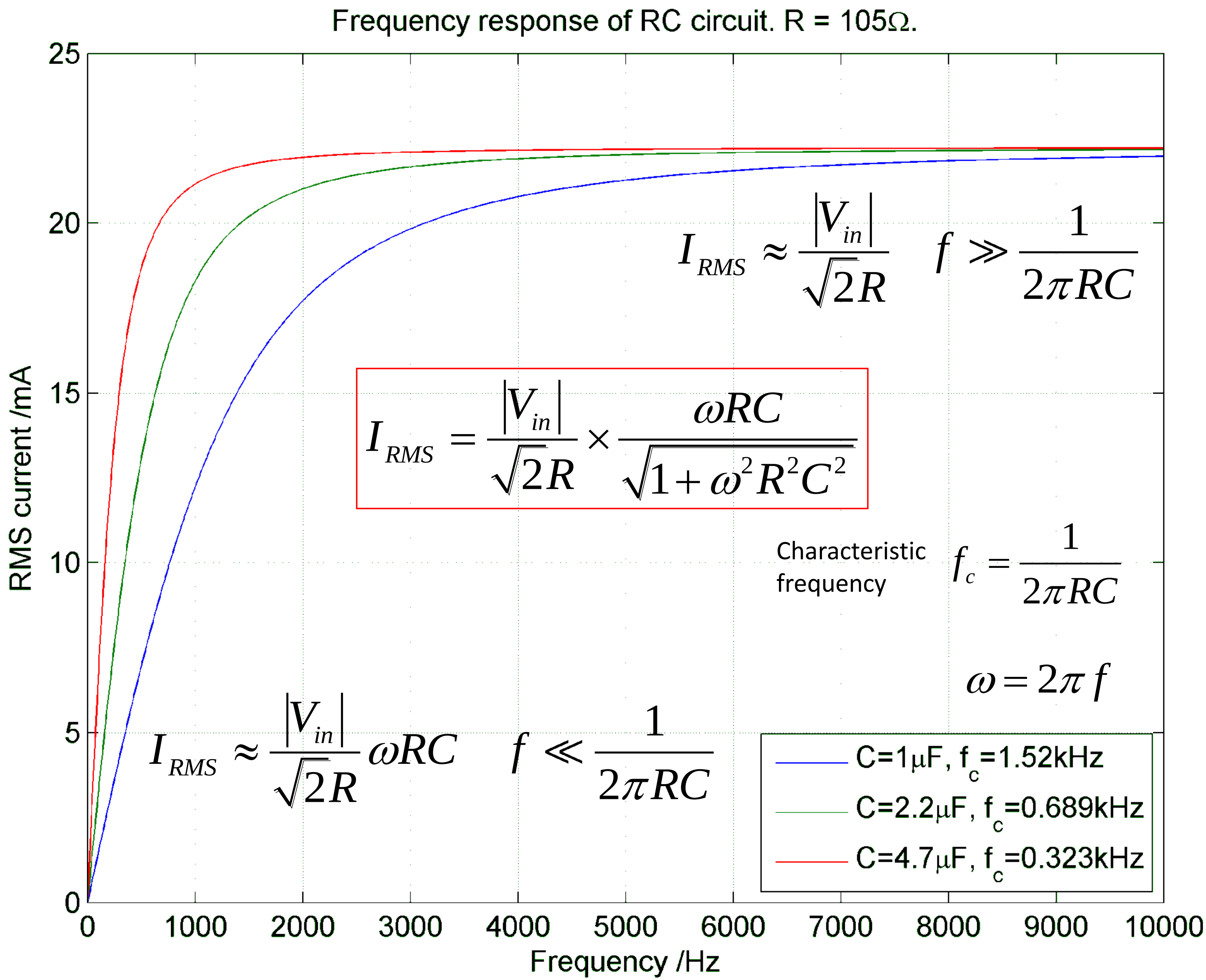
Characteristic frequency $$f_c = \frac{1}{2\pi RC}$$

Low frequency limit:
Current is proportional to frequency and capacitance.

$$I_{RMS} \approx \frac{|V_{in}|}{\sqrt{2}R} \omega RC \qquad f \ll \frac{1}{2\pi RC}$$

High frequency limit:
Current tends to a constant, as one would expect for a DC circuit.

$$I_{RMS} \approx \frac{|V_{in}|}{\sqrt{2}R} \qquad f \gg \frac{1}{2\pi RC}$$



Frequency response of RC circuit. R = 105Ω.
RMS current /mA
Frequency /Hz
0
5
10
15
20
25
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
$I_{RMS} \approx \frac{|V_{in}|}{\sqrt{2}R} \quad f \gg \frac{1}{2\pi RC}$
$I_{RMS} = \frac{|V_{in}|}{\sqrt{2}R} \times \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$
Characteristic frequency $f_c = \frac{1}{2\pi RC}$
$\omega = 2\pi f$
$I_{RMS} \approx \frac{|V_{in}|}{\sqrt{2}R}\omega RC \quad f \ll \frac{1}{2\pi RC}$
C=1μF, f_c=1.52kHz
C=2.2μF, f_c=0.689kHz
C=4.7μF, f_c=0.323kHz

RC circuit as an *integrator*

High frequency limit

$$f \gg \frac{1}{2\pi RC}$$

$$V_C = \frac{|V_{in}|}{\sqrt{1+\omega^2 R^2 C^2}} e^{i\left\{\omega t - \tan^{-1}(\omega RC)\right\}}$$

$$\omega RC \gg 1$$

$$\Rightarrow V_C \approx \frac{|V_{in}|}{\omega RC} e^{i\left\{\omega t - \frac{\pi}{2}\right\}} = \frac{|V_{in}|}{i\omega RC} e^{i\omega t} = \frac{V_{in}}{i\omega RC}$$

$$e^{i\frac{\pi}{2}} = i$$

$$e^{-i\frac{\pi}{2}} = 1/i$$

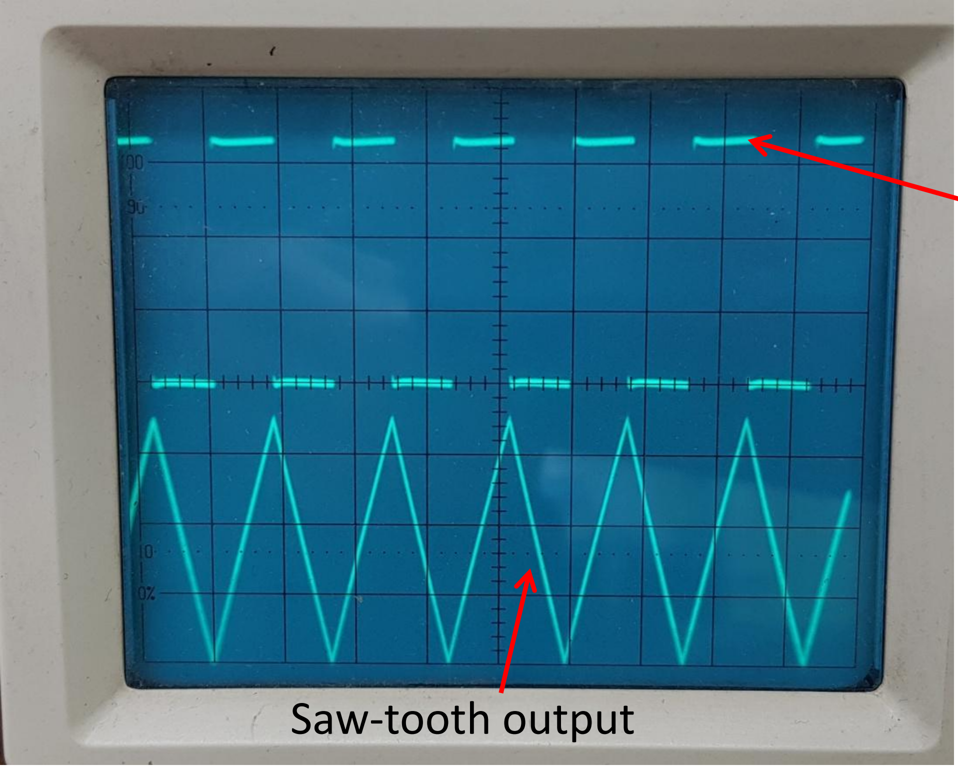
$$V_{in} = |V_{in}| e^{i\omega t}$$

$$\therefore \int V_{in} dt = |V_{in}| \int e^{i\omega t} dt = \frac{|V_{in}|}{i\omega} e^{i\omega t} + c$$

$$\therefore \boxed{V_C \approx \tfrac{1}{RC} \int V_{in} dt}$$

So a square wave input will produce a 'sawtooth' (triangular wave) capacitor voltage.

RC circuit as an *integrator*



Square (ish!)-wave input

Saw-tooth output

$$C = 15\mu F$$

$$R = 107\Omega$$

$$|V_{in}| = 3.4\text{V}$$

$$f = 3{,}125\text{Hz}$$

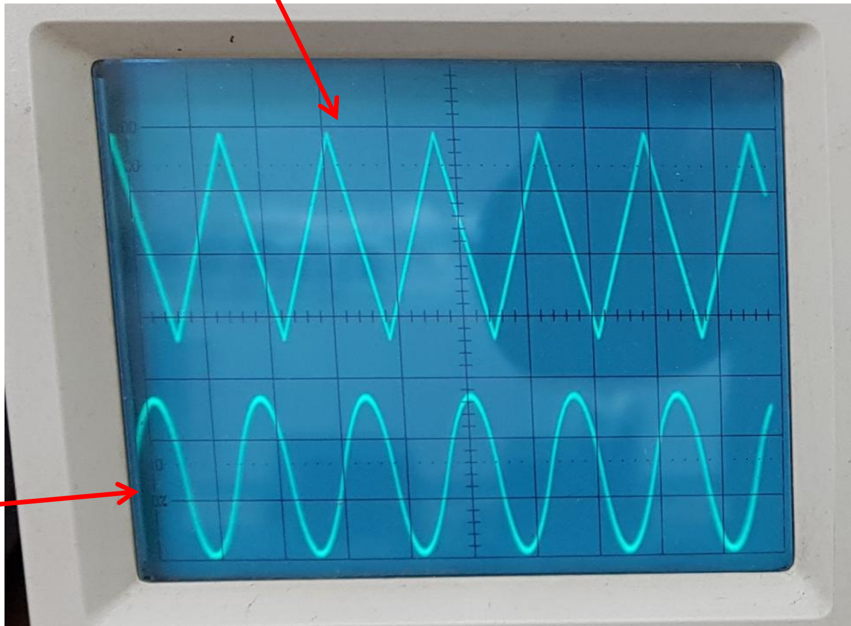
$$\frac{1}{2\pi RC} = 99\text{Hz}$$

Saw-tooth input

$$\boxed{V_C \approx \frac{1}{RC}\int V_{in}\,dt}$$

High frequency limit

$$f \gg \frac{1}{2\pi RC}$$



Parabola output

RC circuit as an *differentiator*

Low frequency limit

$$f \ll \frac{1}{2\pi RC}$$

$$\frac{V_R}{V_{in}} = \frac{R}{1/i\omega C + R} = \frac{i\omega RC}{1 + i\omega RC}$$

$$\omega RC \ll 1$$

$$\Rightarrow V_R \approx i\omega RCV_{in}$$

$$V_{in} = |V_{in}|e^{i\omega t}$$

$$\therefore \frac{dV_{in}}{dt} = i\omega V_{in}$$

$$\therefore \boxed{V_R = RC\frac{dV_{in}}{dt}}$$

Note: For an unknown reason I couldn't get this to work, even when using the 0.22μF capacitor and frequency turned down to 100Hz. In this case f_c = 6,786Hz.

AF 26/09/2020

So a 'sawtooth' (triangular wave) input will result in a square wave resistor potential difference.

Excel sheet of measurements taken using RC circuit setup

Vin = 3.4V R = 106.6 ohms

RC circuit experiment. Dr A French. Winchester College laboratory P5. 26/09/2020.

For all data points below: Vin = 3.4V. R = 106.7 ohms

C = 4.7muF

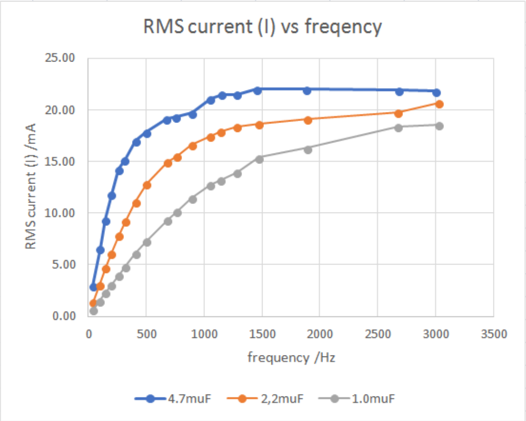
f /hz	I RMS /mA	Vc /volts
43	3.03	3.30
102.7	6.60	3.25
150	9.38	3.00
200.2	11.84	2.80
264.2	14.29	2.60
316.4	15.20	2.30
412.9	17.00	2.00
503.1	17.88	1.73
678.6	19.15	1.40
761.5	19.30	1.23
894.1	19.72	1.10
1058	21.10	1.00
1155	21.50	0.93
1285	21.50	0.85
1457	22.00	0.75
1884	22.00	0.60
2684	21.90	0.45
3006	21.80	0.40

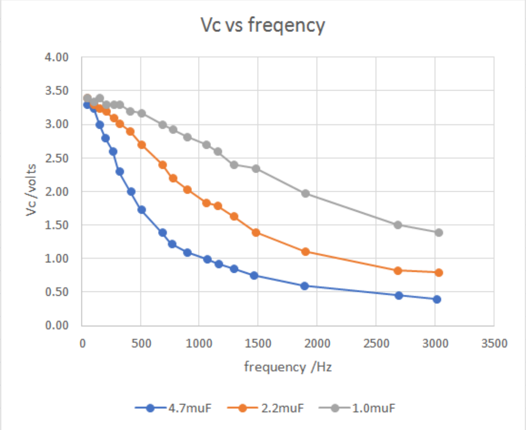
C = 2.2muF

f /hz	I RMS /mA	Vc /volts
42.44	1.39	3.40
99.1	3.12	3.30
151.3	4.76	3.25
202.3	6.13	3.20
265.9	7.85	3.10
321	9.22	3.02
408.5	11.10	2.90
504.7	12.86	2.70
681.8	14.99	2.40
768.1	15.49	2.20
893.9	16.63	2.03
1056	17.47	1.84
1146	17.93	1.80
1284	18.39	1.63
1474	18.63	1.40
1890	19.09	1.12
2674	19.79	0.83
3025	20.70	0.80

C = 1.0muF

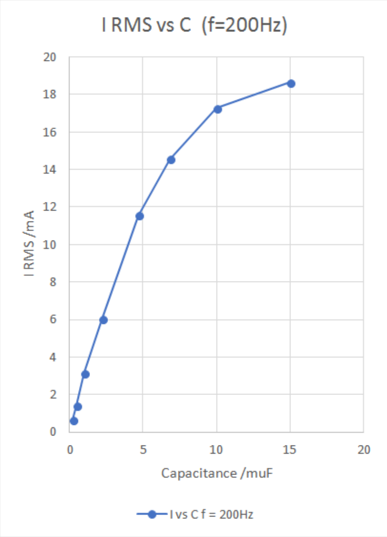
f /hz	I RMS /mA	Vc /volts
42.44	0.68	3.40
99.1	1.54	3.35
151.3	2.34	3.40
202.3	3.10	3.30
265.8	4.01	3.30
321	4.87	3.30
408.5	6.13	3.20
504.7	7.30	3.17
681.8	9.36	3.00
768.1	10.17	2.93
893.9	11.43	2.82
1056	12.74	2.70
1146	13.24	2.60
1284	13.95	2.40
1474	15.38	2.35
1890	16.30	1.98
2674	18.37	1.51
3025	18.58	1.40



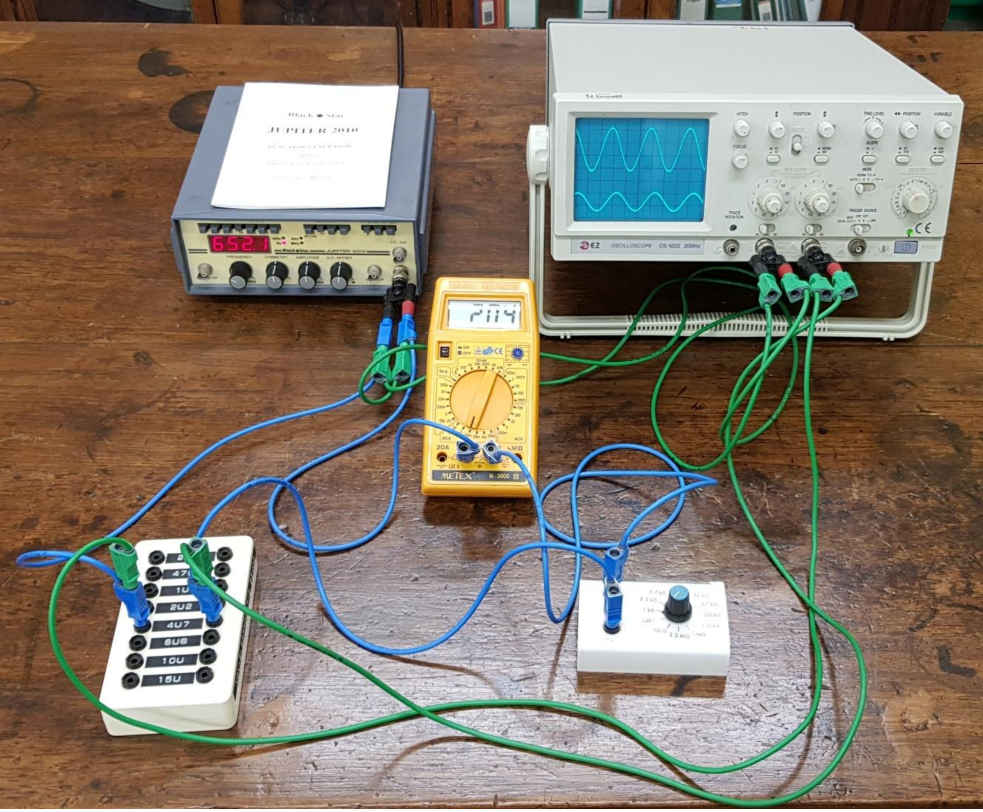


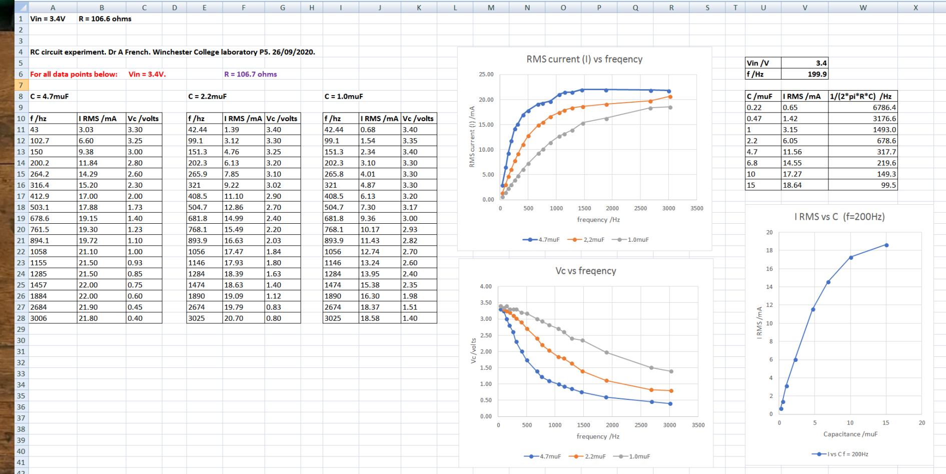
Vin /V	3.4
f /Hz	199.9

C /muF	I RMS /mA	1/(2*pi*R*C) /Hz
0.22	0.65	6786.4
0.47	1.42	3176.6
1	3.15	1493.0
2.2	6.05	678.6
4.7	11.56	317.7
6.8	14.55	219.6
10	17.27	149.3
15	18.64	99.5



As frequency and capacitance changes, *the amplitude of the signal generator also changes slightly.* In this example, the signal generator amplitude has to be changed for every measurement to ensure a consistent value of 3.4V. The latter was measured using the oscilloscope.





Vin = 3.4V	R = 106.6 ohms

RC circuit experiment. Dr A French. Winchester College laboratory P5. 26/09/2020.

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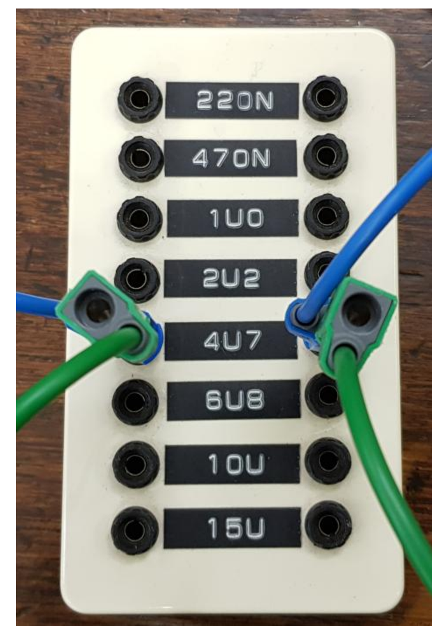
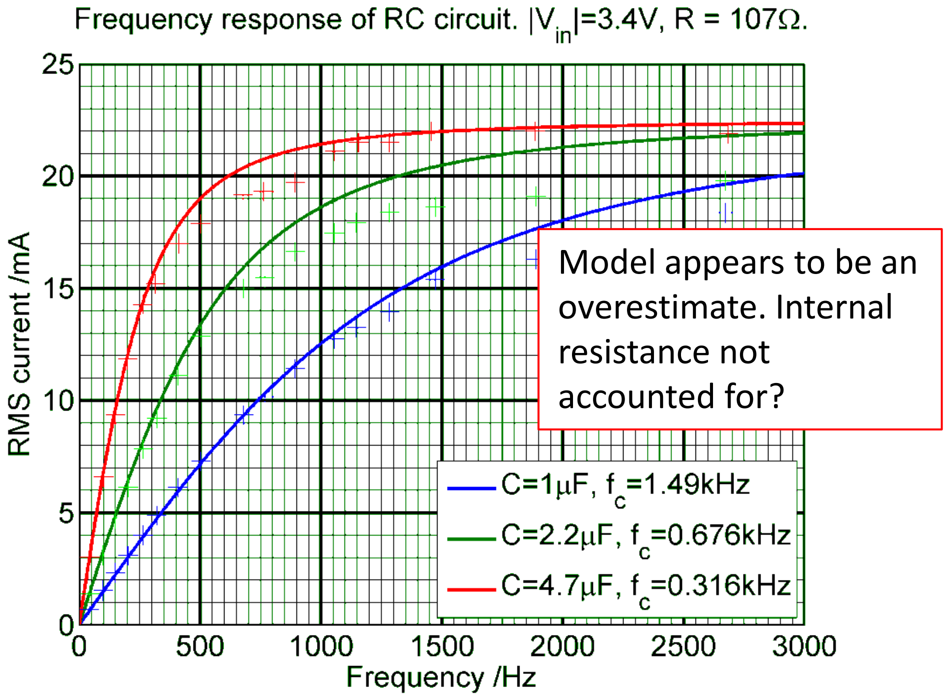
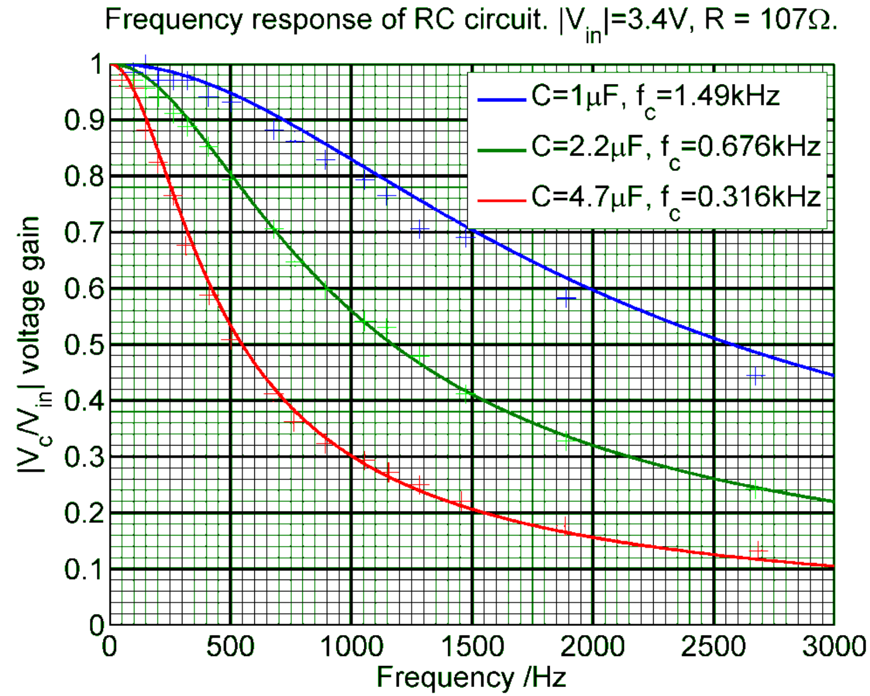
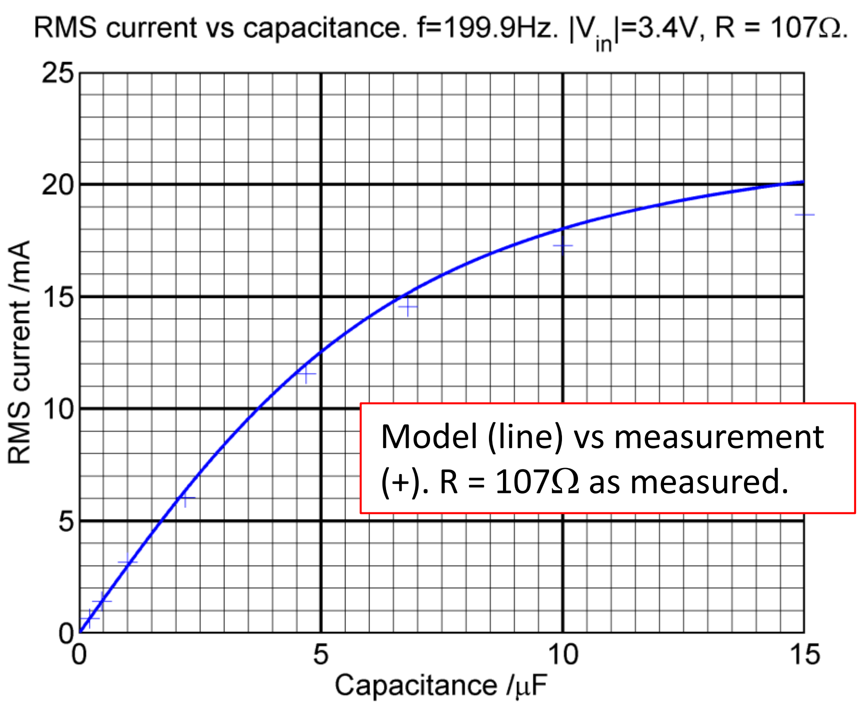
2.2	6.05	678.6
4.7	11.56	317.7
6.8	14.55	219.6
10	17.27	149.3
15	18.64	99.5

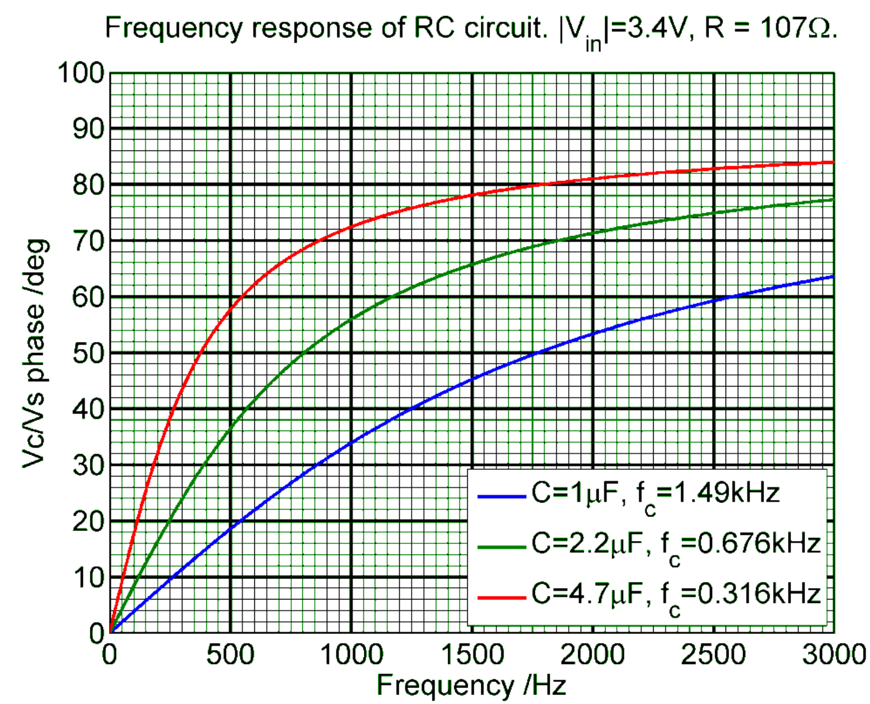
Experimental data *underlaid* with model curves



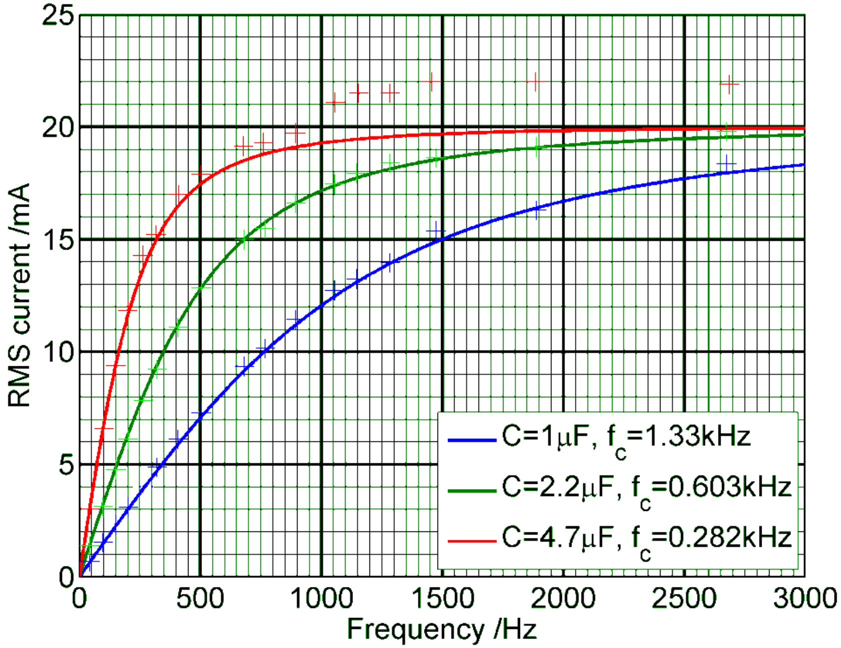

Frequency response of RC circuit. |V_in|=3.4V, R = 107Ω.
RMS current /mA
Frequency /Hz
C=1μF, f_c=1.49kHz
C=2.2μF, f_c=0.676kHz
C=4.7μF, f_c=0.316kHz
Model appears to be an overestimate. Internal resistance not accounted for?


Frequency response of RC circuit. |V_in|=3.4V, R = 107Ω.
|V_c/V_in| voltage gain
Frequency /Hz
C=1μF, f_c=1.49kHz
C=2.2μF, f_c=0.676kHz
C=4.7μF, f_c=0.316kHz

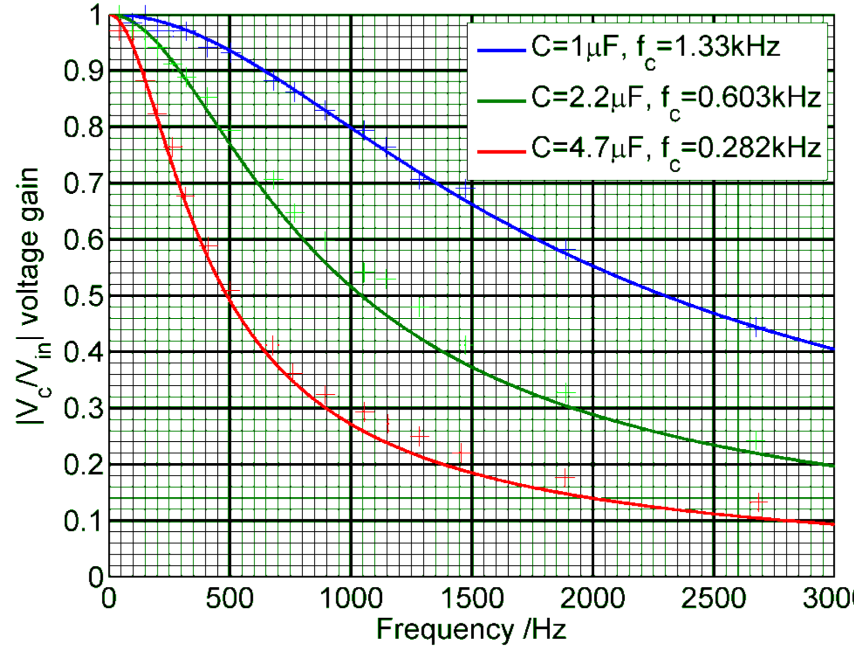

RMS current vs capacitance. f=199.9Hz. |V_in|=3.4V, R = 107Ω.
RMS current /mA
Capacitance /μF
Model (line) vs measurement (+). R = 107Ω as measured.


Frequency response of RC circuit. |V_in|=3.4V, R = 107Ω.
Vc/Vs phase /deg
Frequency /Hz
C=1μF, f_c=1.49kHz
C=2.2μF, f_c=0.676kHz
C=4.7μF, f_c=0.316kHz

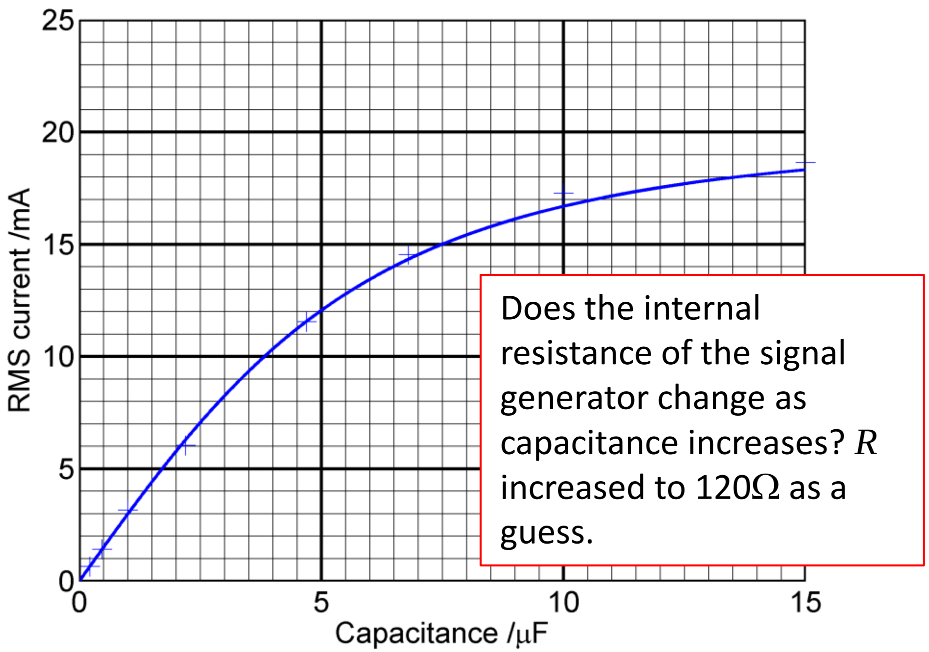
Frequency response of RC circuit. $|V_{in}|$=3.4V, R = 120Ω.



Frequency response of RC circuit. $|V_{in}|$=3.4V, R = 120Ω.



RMS current vs capacitance. f=199.9Hz. $|V_{in}|$=3.4V, R = 120Ω.



Frequency response of RC circuit. $|V_{in}|$=3.4V, R = 120Ω.