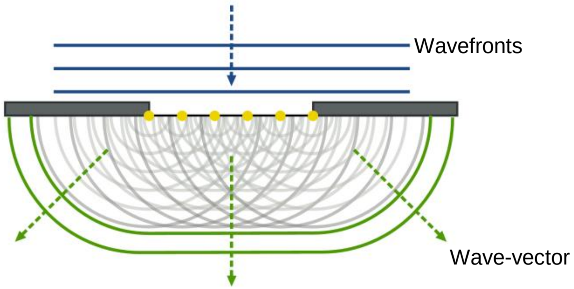
Geometric optics

Light is an *electromagnetic wave,* and an extended source of light can be thought of as consisting of a set of *point sources*, which radiate in a spherical manner until an obstruction is reached. This is, in essence, *Huygen's Principle*.

Christiaan Huygens
1629-1695

This idea can be used to describe wave effects such as *diffraction*

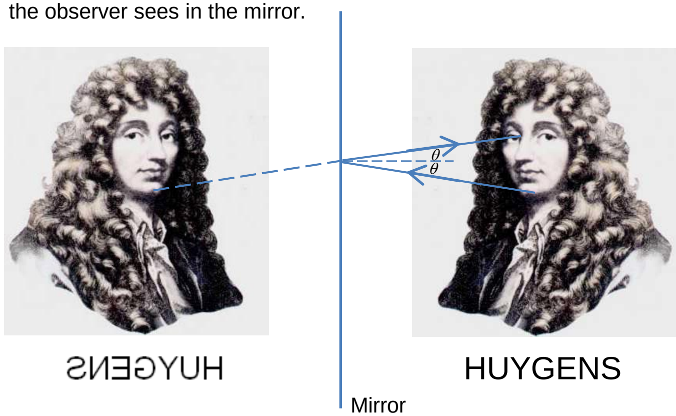


Sufficiently far away from a wave source, the *wavefronts* will tend to be straight. The direction of propagation of the wave can therefore be reduced to a single *wave-vector.* In *Geometric Optics*, we model the propagation of light using straight lines (or *rays*) from the source along these wave-vectors.
i.e. we assume we are in the 'far-field' of the source. In practice, this means a distance of much more than one wavelength away. For visible light of wavelength 500nm, this is not a problem for human-sized scenarios!

Reflection and mirrors

The *Law of Reflection* states the angle of incidence of a ray from the normal to a reflective surface equals the angle of the reflected ray, measured from the normal to the surface.

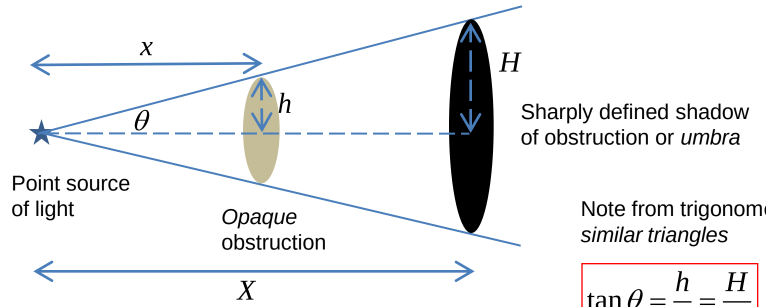
Virtual image i.e. what the the observer sees in the mirror.



If we follow the straight line path of light from Huygen's chin through the mirror, we will arrive at its position within the virtual image. In reality, the light arrives at Huygen's eyes via the reflected ray path shown.

Shadows and eclipses

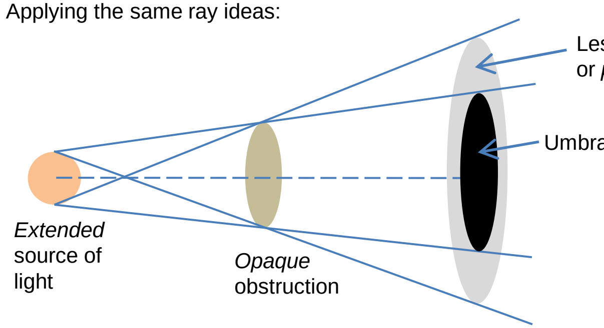
The ray model of light propagation explains the nature of shadows. A point source of light should produce a sharply defined shadow or *umbra*.



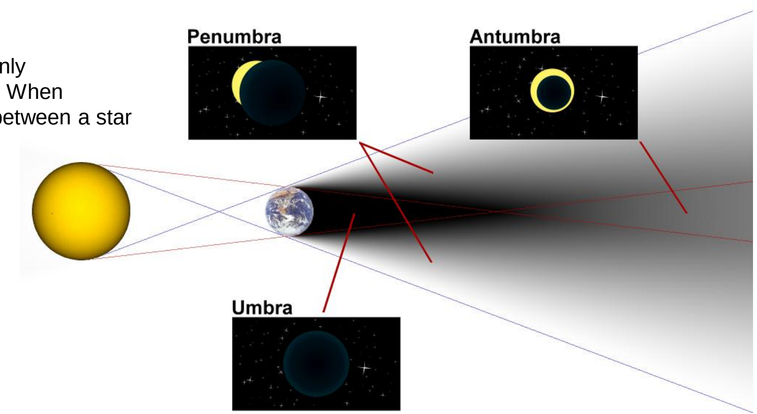
Note from trigonometry or the idea of *similar triangles*

$$\boxed{\tan\theta = \frac{h}{x} = \frac{H}{X}}$$

The situation is, literally, less clearly defined (!) for an *extended object.* Applying the same ray ideas:



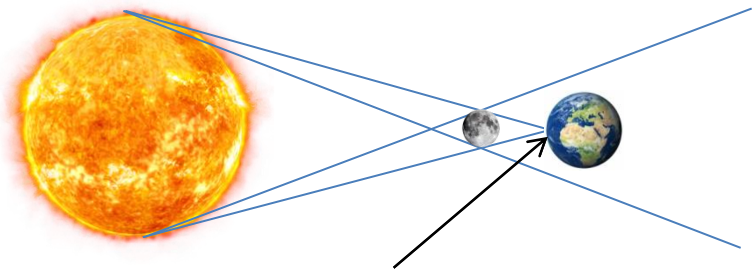
The terms umbra and penumbra are most commonly associated with *eclipses* i.e. When a moon (or planet) passes between a star and an observer.



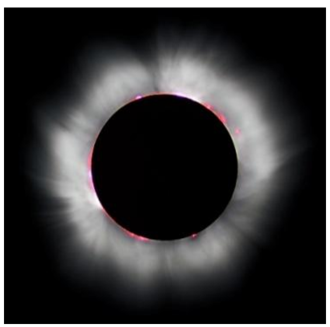
Solar and lunar eclipses

A total lunar eclipse is when the moon is positioned in the umbra of the Sun's light, obscured by the Earth. i.e. Light cannot reach the surface of the moon from the Sun as the Earth is in the way. This means no light is reflected back to Earth from the Moon's surface, which is why it appears black during a lunar eclipse.

Solar Eclipse

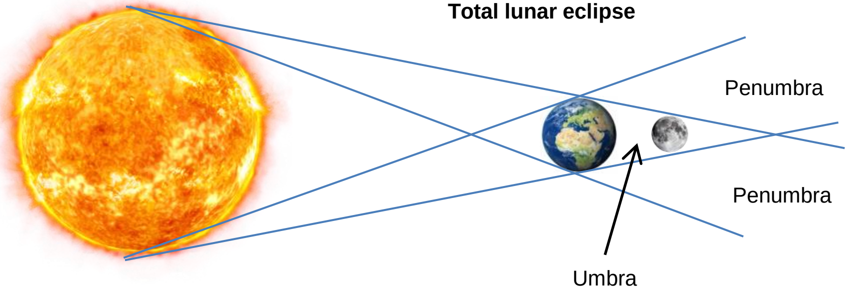


'*Totality*' i.e. at this point on Earth the Moon blocks the sun. Only the *solar corona* will be seen.



Solar flares and corona observed from Earth during a total solar eclipse

Total lunar eclipse

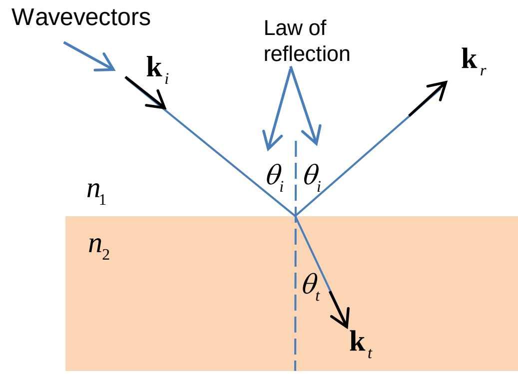


Refraction and critical angles

Refraction is the bending of a ray when it passes through a change in refractive index n. The latter is defined to be the speed of light in a vacuum divided by the speed of light in a medium. For air, $n=1$ since the relatively low density of air molecules means the propagation of light is not impeded significantly. Other materials such as glass contain much greater densities of charge carrying (and hence light perturbing) molecules, forcing light to take a more torturous path and consequently 'slowing it down' in a straight line sense.

Refractive indices

vacuum 1	air 1.00	ice 1.31	water 1.33
human cornea 1.37-1.40	human lens 1.39-1.41	plexiglas 1.49	
crown glass 1.52	sapphire 1.76-1.78	diamond 2.42	



Snell's Law of refraction

$$\boxed{\begin{aligned} n_1 \sin\theta_i &= n_2 \sin\theta_t \\ \therefore \theta_t &= \sin^{-1}\left(\frac{n_1 \sin\theta_i}{n_2}\right) \end{aligned}}$$

Real solutions for the transmitted ray angle when

$$0 \le \theta_t \le 90^\circ$$

$$\therefore 0 \le \sin\theta_t \le 1 \quad \therefore 0 \le \frac{n_1 \sin\theta_i}{n_2} \le 1$$

$\theta_i = \theta_t$ If $n_2 = n_1$

i.e. no refraction since the medium hasn't changed in an optical sense!

$0 < \frac{n_1 \sin\theta_i}{n_2} < 1$ *Always true* if $n_2 > n_1$ so full range of refracted angles are possible

$$n_1 > n_2, \quad 0 < \frac{n_1 \sin\theta_i}{n_2} < 1$$

$$\therefore \sin\theta_i < \frac{n_2}{n_1} \Rightarrow \theta_i < \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Critical angle

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

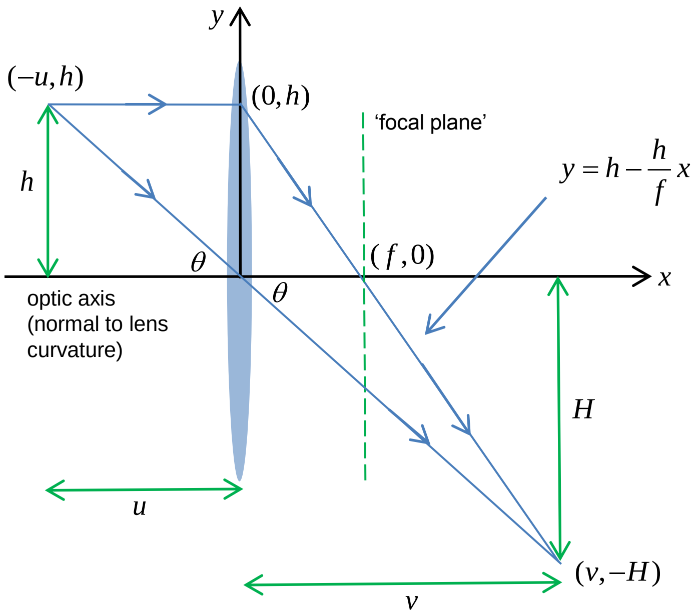
So EM waves propagating from a high refractive index medium to a lower refractive index medium will be **internally reflected** (i.e. no transmission) when the angle of incidence exceeds a ***critical angle*** θ_c

This effect explains why **optical fibres** can be so useful for communications. Light is orientated such that it is internally reflected within the fibre until it emerges. The lack of transmission at each reflection significantly reduces the overall propagation loss. For a glass-air interface with a refractive index ratio of 1/1.5, the critical angle is 41.8°

Lenses*

A lens is an optical device which focuses light via *refraction*. An idealized *thin lens* (i.e. where the physical width has a negligible effect upon the ray paths) will converge rays parallel to the lens normal (the 'optic axis') to a single point. The distance of this point from the lens is called the *focus*.

Lenses are typically ground into spherical shapes, so a ray which intersects the thin lens on its optical axis will not be refracted – this is because the angle of incidence from the normal to the lens surface will be zero, since all radials of a sphere are also normals.



Consider light rays sourced at *(-u,h)* from the centre point of the thin lens. Rays from this point converge at *(v,-H)*. The diagram above might represent a **projector** or a **macro camera lens** – i.e. a small object (such as the detail on the eye of an insect) becomes magnified and therefore able to be resolved using the CCD of a digital camera.

$$\tan\theta = \frac{h}{u} = \frac{H}{v} \quad \therefore H = \frac{v}{u}h$$

$$y = h - \frac{h}{f}x \text{ at } (v,-H)$$

$$\therefore -H = h - \frac{h}{f}v$$

$$\therefore -\frac{v}{u}h = h - \frac{h}{f}v \Rightarrow -\frac{v}{u} = 1 - \frac{v}{f}$$

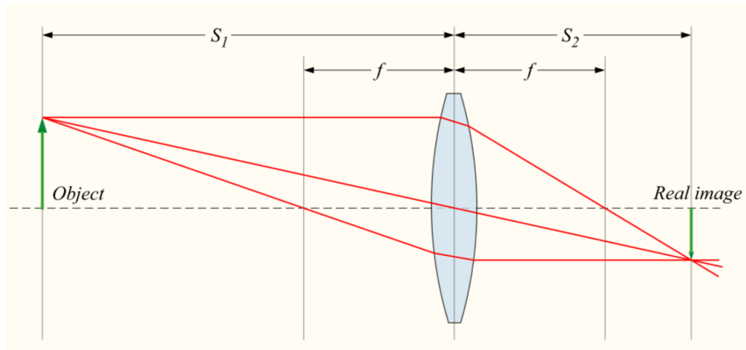
$$-\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

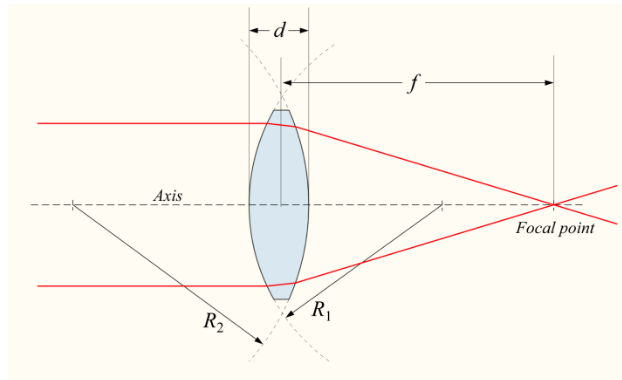
Thin Lens Formula

The reciprocal of focal length *f* of a lens is the *lens power,* measured in *dioptres*

Note the symbols *u* and *v* for the object and image distances from the lens are also typically written as S_1 and S_2.



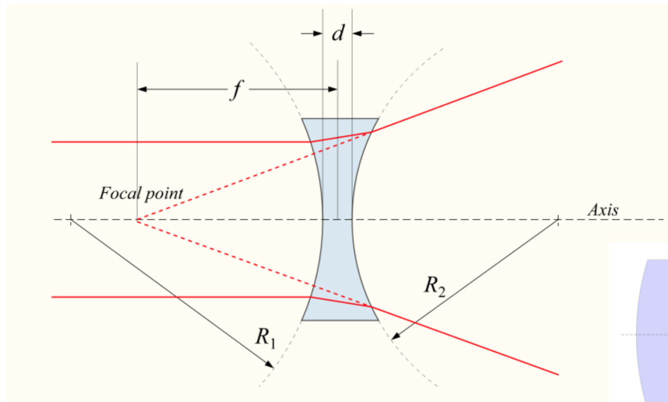
Note the image is *'real and inverted'*. This is an example of a **camera** setup, where the image is typically smaller than the object. The converse situation would be a **projector** or **macro lens**, as described earlier in the derivation of the thin lens formula.



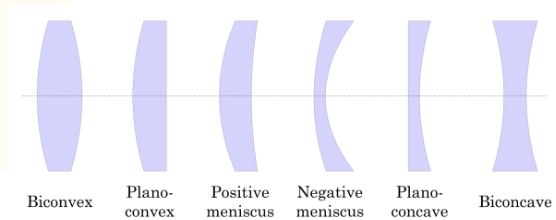
A more precise model of a lens takes into account the radii of curvature of the (spherical) surfaces, the physical width *d* and the refractive index *n* of the lens.

If the lens is used in air (i.e. with a refractive index of unity), the **Lensmaker's formula** determines the focal length in terms of these geometric and physical parameters.

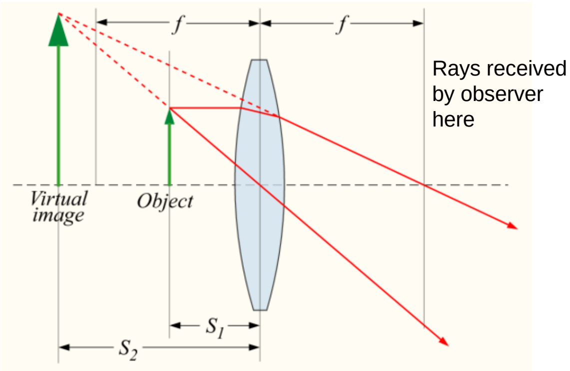
$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right)}$$



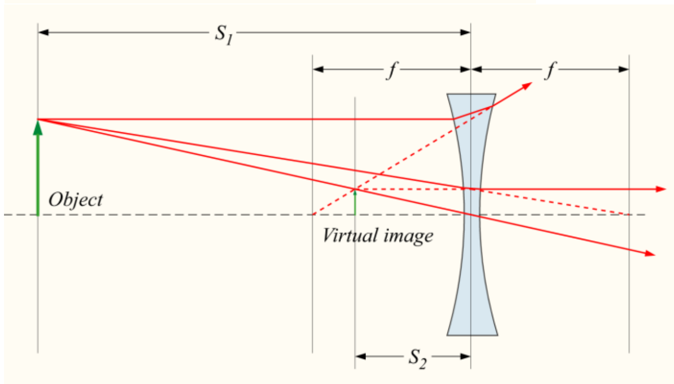
A lens need not be converging. A *concave* curvature will result in *diverging* rays, i.e. the focal point is on the opposite side of the optic axis.



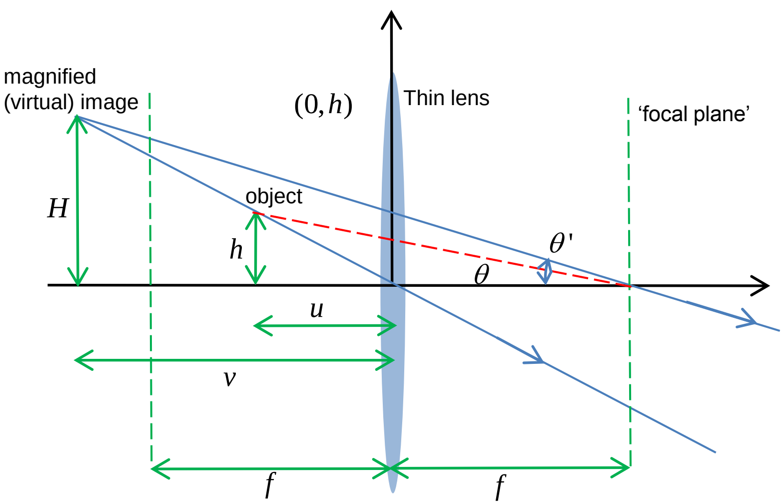
* Several illustrations from https://en.wikipedia.org/wiki/Lens_(optics)



Biconvex lenses can also be used as a magnifying glass. In this case the object is magnified, from the perspective of the observer, when viewed through the lens. This can be explained by following the ray paths to their convergence point. From the observer's perspective, this is equivalent to a magnified object at this location. It is a 'virtual image' since the image is not actually projected anywhere, the observer (in the *right focal plane*) instead interprets the source of the diverging rays it receives.



A similar virtual image effect can be seen for biconcave lenses. In this case the lens is 'negatively magnified' i.e. the virtual image is smaller than the object.



For a magnifying optical device the **linear magnification factor** is: $M=\frac{H}{h}$

From geometry: $H=\frac{v}{u}h \quad \therefore M=\frac{v}{u}$

Hence for a thin lens:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \therefore 1+\frac{u}{v}=\frac{u}{f}$$

$$M=\frac{v}{u}=\frac{1}{1-\frac{u}{f}}$$

$$\therefore \boxed{M=\frac{f}{f-u}}$$

An alternative (and perhaps more practically useful) measure is the **angular magnification factor.**

$$M_\theta=\frac{\theta'}{\theta}=\frac{\tan^{-1}(u+f)}{\tan^{-1}(v+f)}$$

e.g. a 10x lens will make the angular width of the Moon (0.5º to the naked eye) appear to be 5º.

If the angles are small (and measured in radians)

$$M_\theta=\frac{\theta'}{\theta}=\frac{\tan^{-1}(u+f)}{\tan^{-1}(v+f)}\approx\frac{u+f}{v+f}$$

$$M_\theta\approx\frac{1+f/u}{v/u+f/u}$$

For a thin lens $\frac{v}{u}=\frac{f}{f-u}$

$$M_\theta\approx\frac{1+f/u}{\frac{f}{f-u}+f/u}=\frac{u/f+1}{\frac{u}{f-u}+1}$$

The small angle approximation is valid if :

$u \ll f$ Magnification scenario

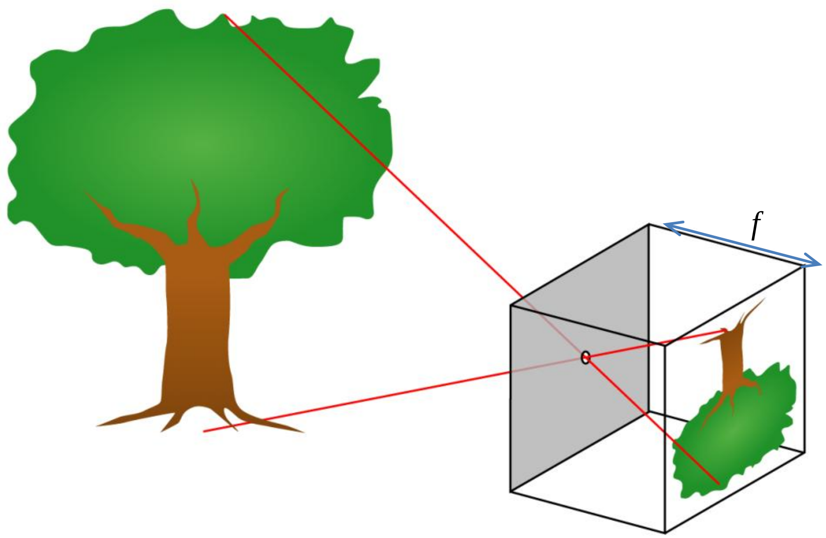
$u \gg f$ Camera or refracting telescope scenario

Hence $\boxed{M_\theta\approx\frac{u}{f}+1}$

This is typically how zoom magnification is specified in photography. i.e. angular magnification factor is "object distance x lens power (in dioptres) + 1"

* Several illustrations from https://en.wikipedia.org/wiki/Lens_(optics)

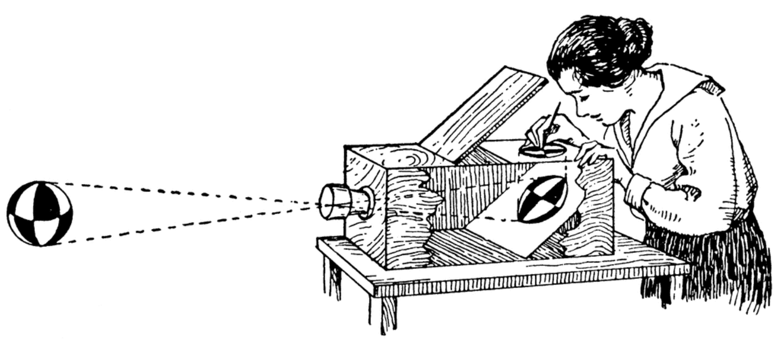
Pinhole camera



Joseph Petzval (1807-1891) showed that the pinhole radius which delivers the optimum image resolution is:

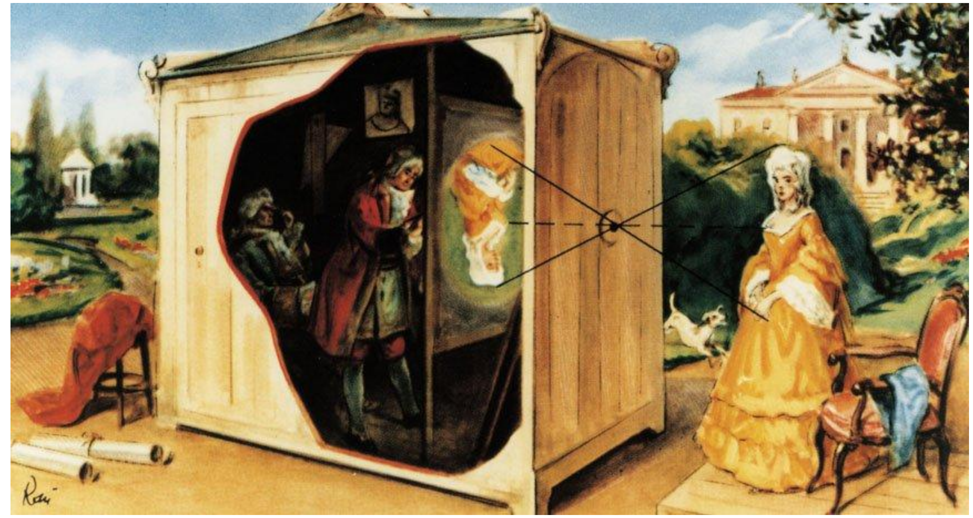
$$\boxed{r = \sqrt{f\lambda}}$$

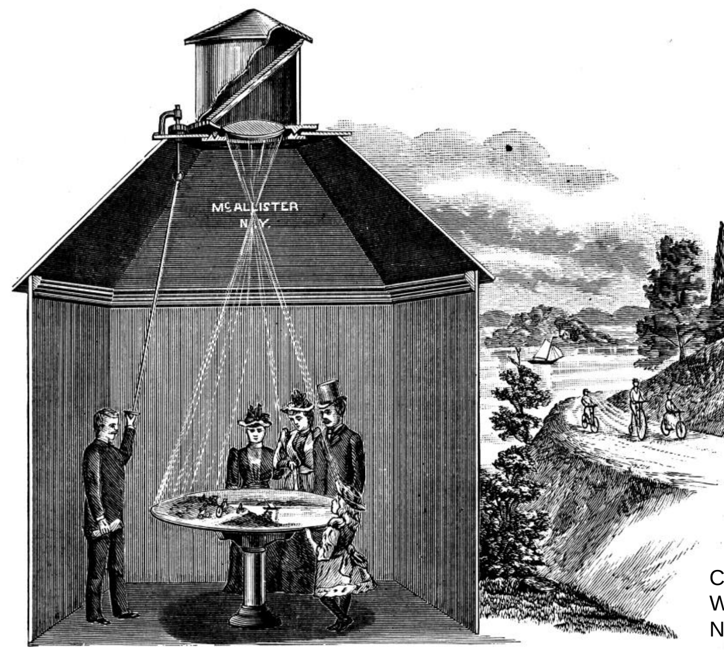
Note f is the *focal length*, and not the frequency of the light! For the pinhole camera the focal length is the distance from the pinhole to the image plane.



A portable pinhole camera or *camera obscura*

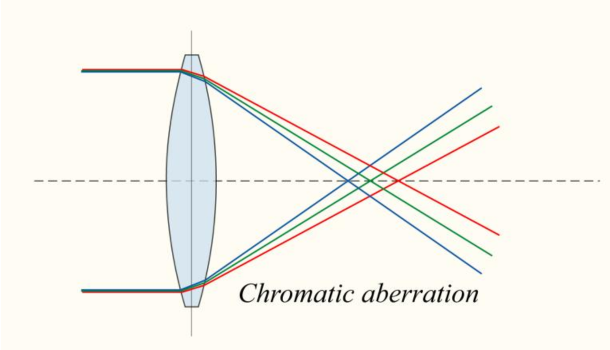
A *Camera obscura* is a name for a device (or building!) which utilizes the pinhole camera idea. In pre-photography days, it could provide a mechanism for an artist to make an accurate tracing of a scene.

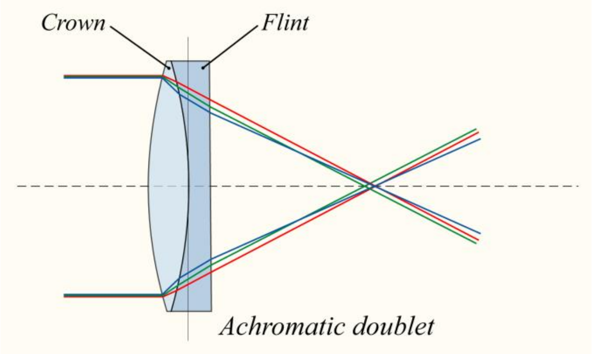




Camera Obscura
William Y. McAllister
New York, c1890

Optical aberrations



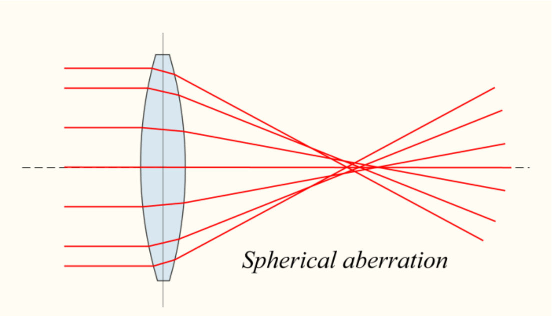


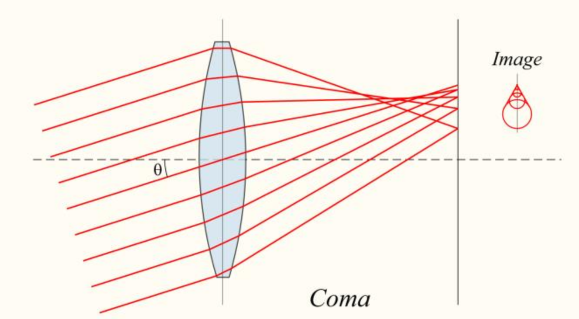
For two thin lenses separated by a short air-gap d, the combined focal length is given by

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

The refractive index of a lens material such as glass will typically vary with the wavelength of light. This means different colours will have different focal lengths given the same lens. The resulting image distortion (i.e. blurring of certain hues) is called a **chromatic aberration.**

Chromatic aberration can be minimized by adding a second lens - an ‘achromatic doublet’





Unlike a parabolic reflector, a spherical lens will not perfectly converge parallel rays to a single focus. This results in image blurring called **spherical aberration.**

Parallel rays at an angle to the optic axis will also not be focused to a single point in the focal plane, This results in a comet-like effect called **comatic aberration**.

For simulated viewfinder examples of the aberrations above:
http://www.nikon.com/products/sportoptics/how_to/guide/binoculars/technologies/technologies_08.htm