Polygons

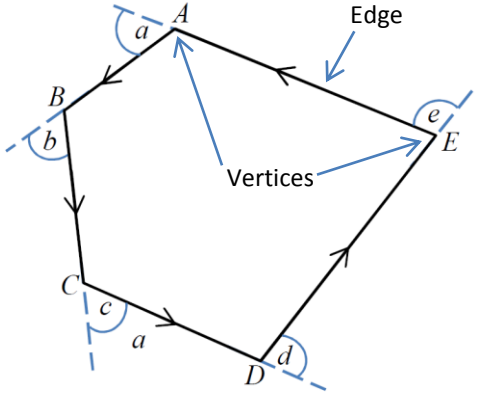
An *N*-gon is a *closed* shape (without any internal holes) comprising of *N* straight line *edges* which join at *N vertices*.

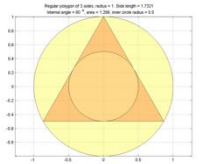
A *regular* N-gon is a polygon with sides of the *same length*. This also means *every* internal angle is the same.

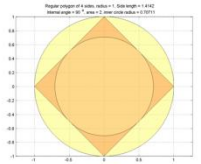
By considering a polygon as a cycle of vectors (i.e. arrows), **the total angle of turn must sum to 360º.**

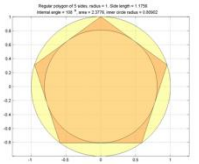
Hence for the irregular pentagon below

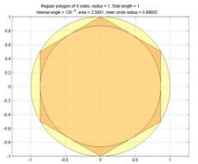
$$a+b+c+d+e=360^o$$



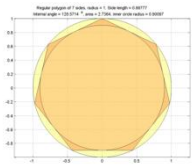

Triangle N=3

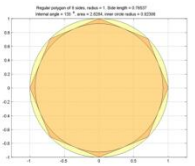

Square N=4

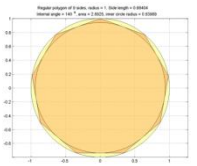

Pentagon N=5

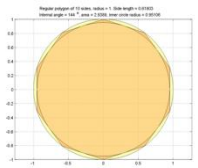

Hexagon N=6

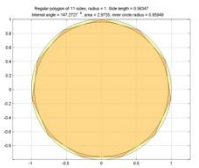
Regular polygons *inscribed* and *circumscribed* by circles. As *N* becomes large, the polygon will tend to a *circle* and hence all three shapes will coalesce.

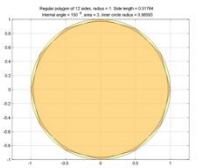

Heptagon N=7

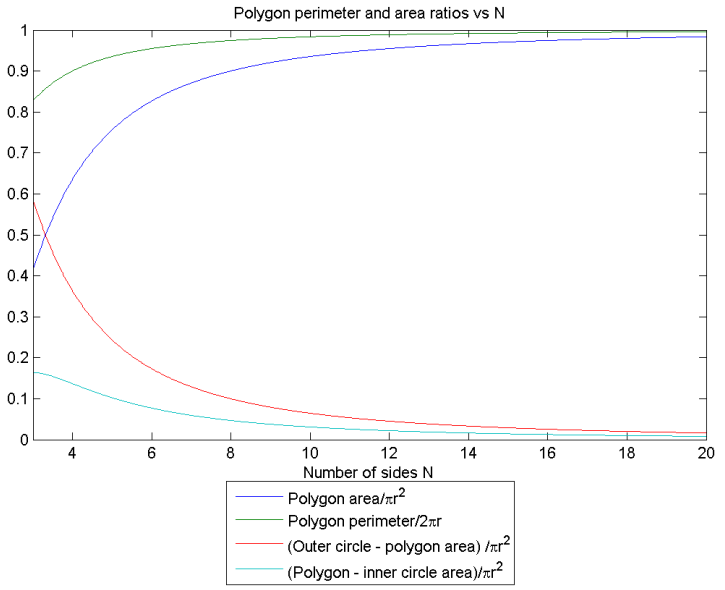

Octagon N=8


Nonagon N=9

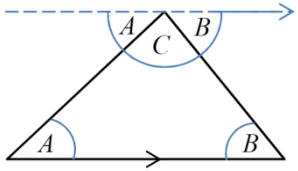

Decagon N=10


Hendecagon N=11


Dodecagon N=12



The internal angles of a triangle sum to 180º

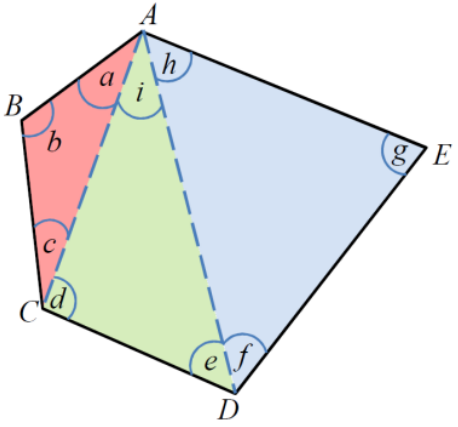


$$A+B+C=180^o$$

We can divide an *N*-gon into *N* – 2 triangles, which meet at a single vertex.

Hence the sum of the internal angles of an *N*-gon is:

$$\Theta=(N-2)\times 180^o$$



e.g. $a+b+c+d+e+f+g+h+i=(5-2)\times 180^o=540^o$

The angle of turn for a regular N-gon is $360^o/N$

The sum of internal angles is

$$\Theta=N\left(180^o-\frac{360^o}{N}\right)=(N-2)\times 180^o$$

which agrees with the general result for any N-gon