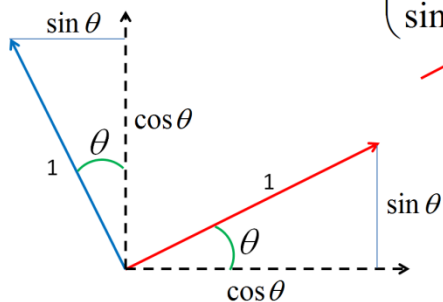


Rotation anticlockwise about (0,0) by angle  $\theta$

$$\mathbf{R} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$



Special cases of rotation anticlockwise about (0,0) by angle  $\theta$

$$\mathbf{R}_{30^\circ} = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$\mathbf{R}_{45^\circ} = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{R}_{60^\circ} = \begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

Special cases of rotation anticlockwise about (0,0) by angle  $\theta$

$$\mathbf{R}^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix} \quad \text{i.e. } \mathbf{R}_\theta^n = \mathbf{R}_{n\theta}$$

$$\mathbf{R}^{-n} = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}$$

i.e. the *inverse* of a clockwise rotation by  $\theta$  is an anticlockwise rotation by  $\theta$ .

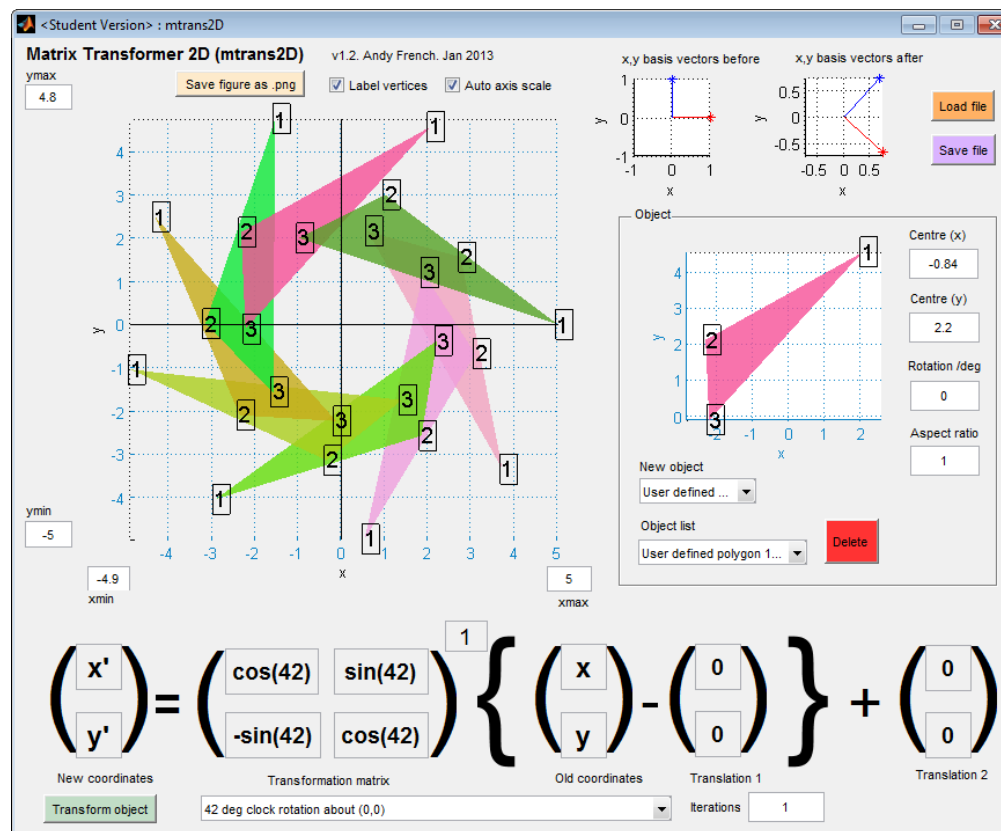
Use of rotation matrices to derive sine and cosine addition formulae

$$\mathbf{R}_{A \pm B} = \mathbf{R}_{\pm B} \mathbf{R}_A$$

$$\begin{pmatrix} \cos(A \pm B) & -\sin(A \pm B) \\ \sin(A \pm B) & \cos(A \pm B) \end{pmatrix} = \begin{pmatrix} \cos B & \mp \sin B \\ \pm \sin B & \cos B \end{pmatrix} \begin{pmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{pmatrix}$$

$\therefore$

$$\begin{aligned} \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \end{aligned}$$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(42) & \sin(42) \\ -\sin(42) & \cos(42) \end{pmatrix} \left\{ \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

New coordinates Transformation matrix Old coordinates Translation 1 Translation 2  
Transform object 42 deg clock rotation about (0,0) Iterations 1