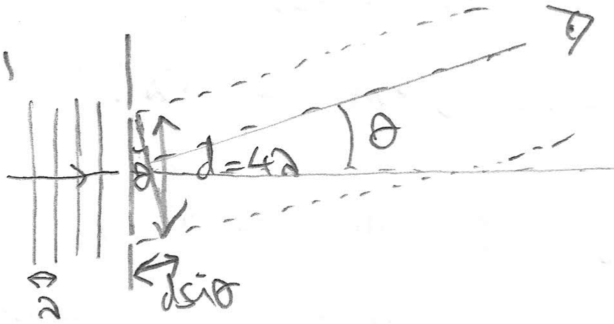
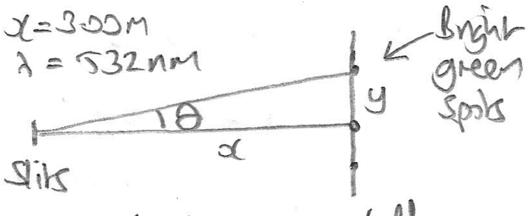
INTERFERENCE, DIFFRACTION, POLARIZATION, DOPPLER

1/ (i)





$y = x\tan\theta$

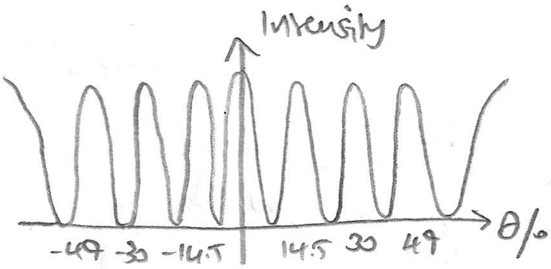
constructive interference : $d\sin\theta_n = n\lambda$ $\therefore$ $\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right)$

$\therefore$ $\boxed{y_n = x\tan\left(\sin^{-1}\left(\frac{n\lambda}{d}\right)\right)}$

(Should be close to $\frac{nx\lambda}{d}$ but only when $\theta \ll 1$ radian)

n	$n\lambda/d = n/4$	$\theta_n/°$	y_n/m
0	0	0	0
±1	±1/4	±14.5	0.77
±2	±1/2	±30°	1.73
±3	±3/4	±48.6°	3.40
±4	±1	±90°	∞

↑ you won't see this one!

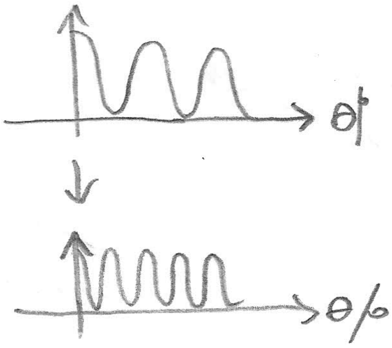


(ii) a) Before : $d = 4 \times 532$ nm. And this is still the same.

Now : $\frac{\lambda}{d} = \frac{445}{4 \times 532} = 0.209$ (i.e. with violet light)

n	$n\lambda/d$	$\theta_n/°$	y_n/m
0	0	0	0
±1	0.209	±12.1	0.64
±2	0.418	±24.7	1.38
±3	0.627	±38.9	2.42
±4	0.836	±56.8	4.58

So as $\lambda\downarrow$ and d is the same, the interference pattern bundles up.



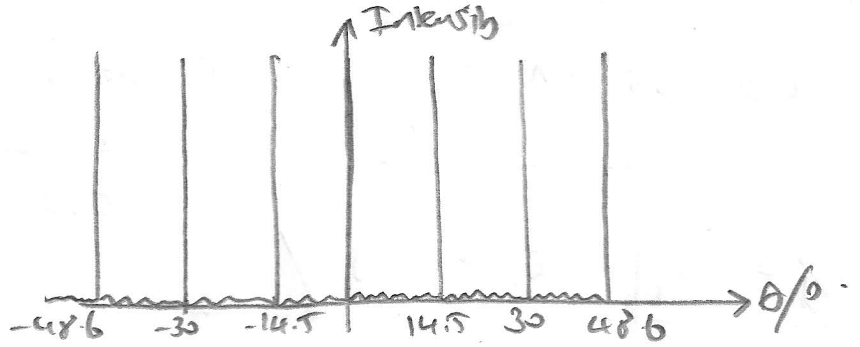
In this case you can now see the $n = \pm 4$ maxima

b) Slit widths $\ll \lambda$ and $\lambda = 532$ nm (green light) but # slits in 1 mm is now 470.

So $d = \dfrac{1 \times 10^{-3}\,m}{470} = 2.13 \times 10^{-6}\,m = 4\lambda$.

$[4 \times 532 \times 10^{-9} = 2.13 \times 10^{-6}\,m]$ ie same as in a).

Only difference is now many more slits in beam.



So width of peaks are now very thin ie spots are sharply defined. Very small subsidiary maxima

If illuminating beam is $\approx$ 1 mm wide, minima when $\theta_p = \sin^{-1}\left(\underbrace{\frac{p}{470} \times \frac{1}{4}}_{\lambda/d}\right)$, except when p is an integer multiple of 470.

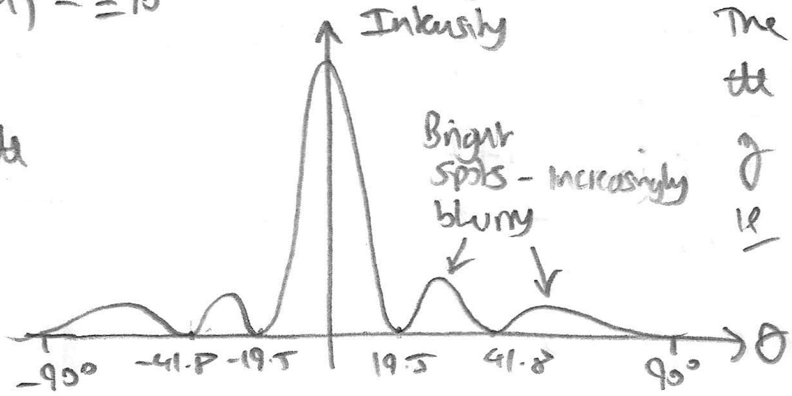
c) Single wide slit, $w = 3\lambda$.

zeros of "envelope" pattern when $\sin^{-1}(m/3) = \theta_m$

ie $\sin^{-1}(\pm\frac{1}{3}) = \pm 19.5°$, $\sin^{-1}(\pm\frac{2}{3}) = \pm 41.8°$

and $\sin^{-1}(\pm 1) = \pm 90°$

λ only sets the colour!

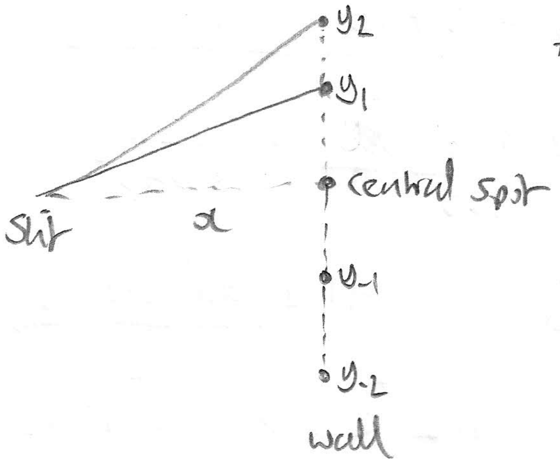


The pattern is the same regardless of λ, since $W = 3\lambda$ ie $\frac{\lambda}{d}$ is independent of λ.

(iii) $\lambda = 650\,nm$ (red)

$w = 100\,nm$ ie $\ll \lambda$

$$d = \frac{1\times10^{-3}\,m}{250} = 4.0\times10^{-6}\,m \Rightarrow \frac{\lambda}{d} = \frac{1}{6.154}$$



As in (i, ii) $y_n = x\tan\left(\sin^{-1}\left(\frac{n\lambda}{d}\right)\right)$

if $x = 5.0\,m$

$$y_n = 5.00\tan\left(\sin^{-1}\left(\frac{n}{6.154}\right)\right)$$

So $y_3 = 5.00\tan\left(\sin^{-1}\left(\frac{3}{6.154}\right)\right)$

$= \boxed{2.79\,m}$ (position of 3rd order maxima)

if $\frac{d}{\lambda} \rightarrow \frac{1\times10^{-3}}{500\times650\times10^{-3}} = 3.08$ (500 lines/mm)

$$\therefore\ y_3 = 5.00\tan\left(\sin^{-1}\left(\frac{3}{3.08}\right)\right) = \boxed{21.94\,m}$$

$$y_2 = 5.00\tan\left(\sin^{-1}\left(\frac{2}{3.08}\right)\right) = \boxed{4.28\,m}$$

$$y_1 = 5.00\tan\left(\sin^{-1}\left(\frac{1}{3.08}\right)\right) = \boxed{1.72\,m}$$

Too wide to see in most rooms!

(iv) Angular resolution $/^\circ$ is $\boxed{\delta\theta = \frac{180}{\pi}\frac{\lambda}{d}}$

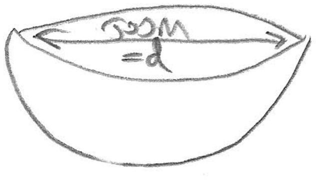
a) $\boxed{\text{Human eye}}$ $\delta\theta = \frac{180}{\pi}\times\frac{500\times10^{-9}}{7\times10^{-3}} = \boxed{4.1\times10^{-3}\,^\circ}$

(ie $\boxed{14.7}$ arc seconds, if 3600 arc seconds in 1°).

b) $\boxed{\text{HUBBLE SPACE TELESCOPE}}$ $\delta\theta = \frac{180}{\pi}\times\frac{500\times10^{-9}}{2.4} = \boxed{1.2\times10^{-5}\,^\circ}$

$= \boxed{0.043 \text{ arc seconds}}$

(v)



$$\delta\theta_1 = \frac{180}{\pi}\frac{c}{fd}$$

↑ in deg

FAST telescope:

$c = f\lambda$

$c/f = \lambda$

$$\delta\theta_1 = \frac{180}{\pi} \times \frac{2.998\times10^8}{500} \times \frac{1}{70\times10^6}$$

$$\boxed{\delta\theta_1 = 0.490} = \boxed{1770 \text{ arc-seconds}}$$

$$\delta\theta_2 = \frac{180}{\pi} \times \frac{2.998\times10^8}{500} \times \frac{1}{3.0\times10^9}$$

$$\boxed{= 0.011^\circ} \quad (41.2 \text{ arc seconds})$$

So resolution of FAST varies over $\boxed{41.2 < \delta\theta/\text{arc seconds} < 1770}$

As $f\uparrow$, resolution gets to be smaller angles. (i.e. finer details can be seen).

(vi) Bragg's law: $\boxed{2d\sin\theta = n\lambda}$

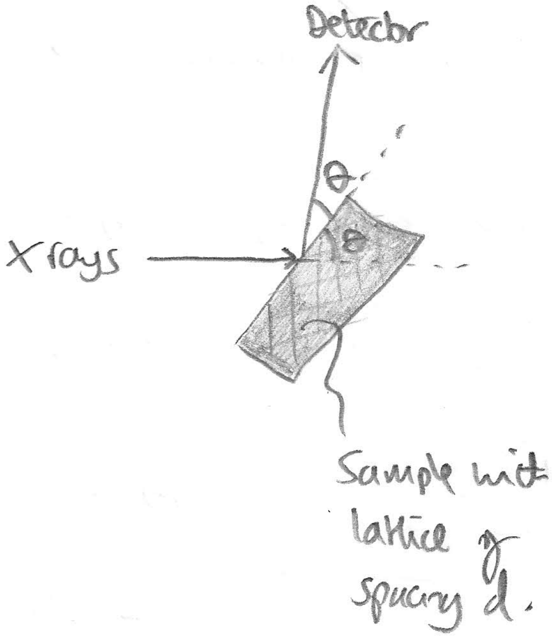
$$\therefore \theta_n = \sin^{-1}\left(\frac{n\lambda}{2d}\right) \rightarrow \theta_n = \sin^{-1}\left(n \times \frac{1}{7.32}\right)$$

let $d = 4.56\times10^{-10}$ m

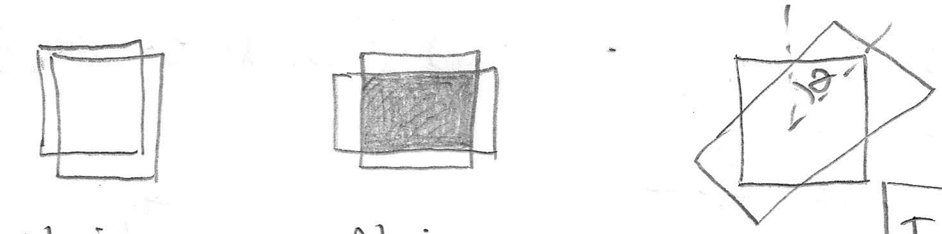
$\lambda = 0.123\times10^{-9}$ m

∴

n	θ_n
0	0
±1	7.75°
±2	15.6°
±3	23.9°
±4	32.6°
±5	42.4°
±6	54.0°
±7	70.7°



(vii)



polarizers aligned

polarizers 90° rotated

$$\boxed{I = I_{min} + (I_{max} - I_{min})\cos^2\theta}$$

Malus' law

$I_{max} = 100\,W/m^2$ $\qquad$ $I_{min} = 10\,W/m^2$

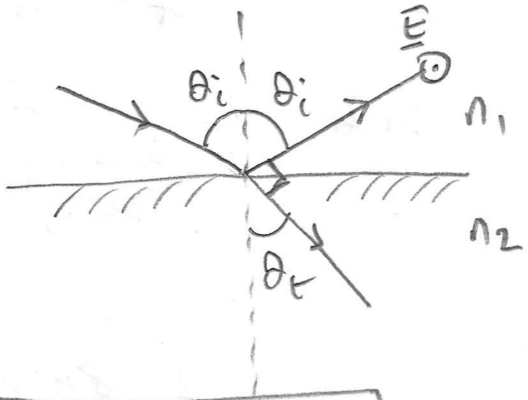
when $\theta = 30°$: $\quad I = 10 + (100-10)\cos^2 30°$

$$I = 10 + 90 \times 3/4 = \boxed{77.5\,W/m^2}$$

$$\sqrt{\frac{I - I_{min}}{I_{max} - I_{min}}} = \cos\theta \quad \therefore \quad \theta = \cos^{-1}\sqrt{\frac{42-10}{100-10}}$$

$\theta = \boxed{53.4°}$ when $I = 42\,W/m^2$

(viii)



when $\theta_i = \theta_B = \tan^{-1}(n_2/n_1)$ reflected $\underline{E}$ field is S polarized only

Brewster angle is:

$$\tan^{-1}\left(\frac{1.50}{1.00}\right) = \boxed{56.3°}$$

for an air : glass interface.

Proof of Brewster angle:

$\theta_i + 90° + \theta_t = 180°$

$\therefore \theta_i + \theta_t = 90°$

Snell: $n_1 \sin\theta_i = n_2 \sin\theta_t$

$n_1 \sin\theta_i = n_2 \sin(90° - \theta_i)$

$n_1 \sin\theta_i = n_2 \cos\theta_i$

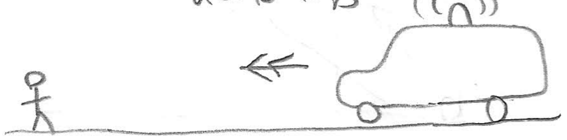
$\therefore \tan\theta_i = n_2/n_1$

$$\therefore \boxed{\theta_i = \tan^{-1}(n_2/n_1)}$$

5 $\quad [\sin(90° - \theta_i) = \sin 90° \cos\theta_i - \cos 90° \sin\theta_i = \cos\theta_i]$

(ii)

$u = 13.4\,m/s$ speed of sound $c = 340\,m/s$.



(A)

Doppler shift is
$$\Delta f = \frac{-\left(\frac{-13.4}{340}\right)}{1-\frac{13.4}{340}} \times 2{,}000 \quad (Hz)$$

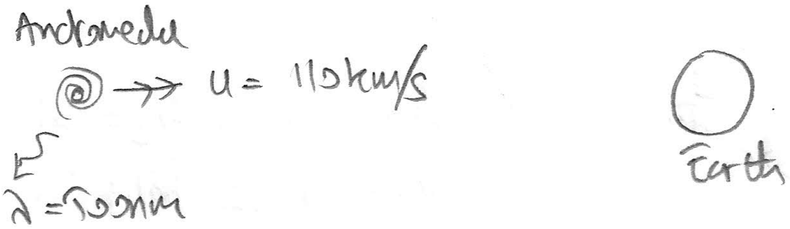
$$\boxed{\Delta f = 82.1\ Hz}$$

So wavelength emitted is $\lambda_e = \frac{340}{2000} = \boxed{0.17\,m}$

wavelength observed is $\lambda_o = \frac{340}{2082} = \boxed{0.163\ m}$

ie "blueshifted" (!) Siren whine is higher pitched than emitted signal.

(x)



$$\left[\Delta f = \frac{-\frac{u}{w}\cos\theta}{1+\frac{u}{w}\cos\theta} f\right]^*$$

u is recessional velocity.

Doppler shift of 500nm light is:
$$\Delta f = \frac{-\left(\frac{-110\times10^3}{2.998\times10^8}\right)}{1+\left(\frac{-110\times10^3}{2.998\times10^8}\right)} \times \frac{2.998\times10^8}{500\times10^{-9}} \quad (Hz)$$

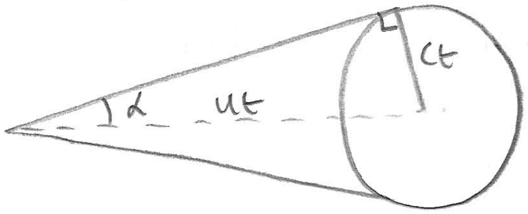
$$= +2.2008\ldots \times10^{11}\ Hz \leftarrow \text{calc memory}$$

$\therefore$ redshift $= \frac{-\Delta f}{f+\Delta f} = \boxed{-3.669\times10^{-4}}$
$\underbrace{}_{z}$

[So using $\frac{\lambda_o}{\lambda_e} - 1 = z \Rightarrow \lambda_o = \lambda_e(1+z) = \boxed{499.8\,nm}$]

$(\Delta\lambda = -0.183\,nm)$

(xi)



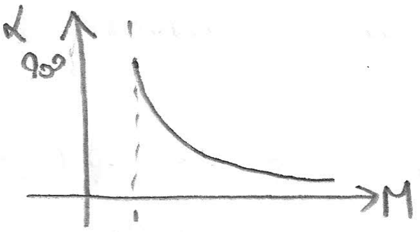
$ut \sin\alpha = ct$

$\therefore \quad \alpha = \sin^{-1}\left(\frac{1}{M}\right)$

if Mach # $M = u/c$

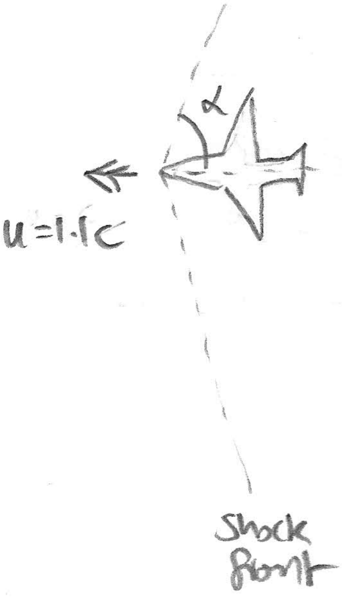
if $M = 6.70$ for the X-15

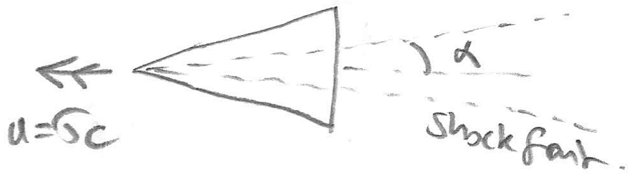
$\Rightarrow \quad \alpha = \sin^{-1}\left(\frac{1}{6.70}\right) = \boxed{8.6^\circ}$



As $M \to 1, \quad \alpha \to 90^\circ$

so shock front "follows aircraft" and $\therefore$ 'sonic boom' can be heard below, rather than in an increasingly tight angle behind the aircraft. Beyond the shock front, the air will not be disturbed by the motion of the aircraft.





$M = 1.1$

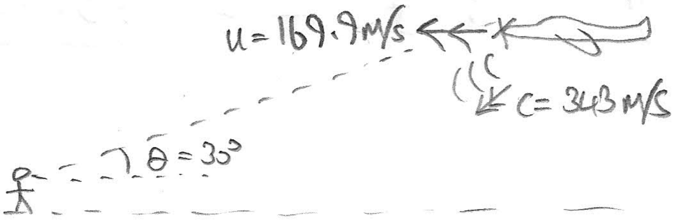
$\alpha = \sin^{-1}\left(\frac{1}{1.1}\right) = 65.4^\circ$

$M = 5$

$\alpha = \sin^{-1}\left(\frac{1}{5}\right) = 11.5^\circ$

(xii) Assume $f = 4 \times 1431/60$ Hz $= \boxed{95.4 \text{ Hz}}$ (propeller rotation frequency)

a)



Doppler shift is:

$$\boxed{\Delta f = \frac{-\frac{u}{c}\cos\theta}{1+\frac{u}{c}\cos\theta} f}$$

(-ve since approaching)

$$\therefore \quad \Delta f = \frac{169.9/343 \cos 30^\circ}{1 - \frac{169.9}{343}\cos 30^\circ} \times 95.4 \quad (\mathrm{Hz})$$

$$= \boxed{71.7\,\mathrm{Hz}} \qquad (\text{ie } u = -169.9 \text{ m/s})$$

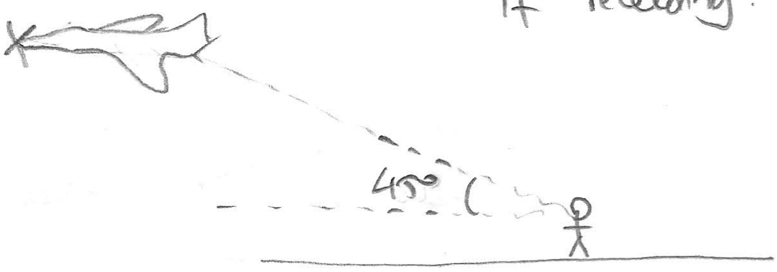
So observed frequency on ground is $f + \Delta f = \boxed{167.1\,\mathrm{Hz}}$

b) If the Spitfire is overhead, $\theta = 90^\circ \;\therefore\; \cos\theta = 0$

So $\Delta f = 0$. $\therefore$ observer hears $\boxed{f = 95.4\,\mathrm{Hz}}$

(You only get a transverse doppler effect at speeds close to the speed of light → in this case the Δf formula is slightly different).

c)



If receding: $\Delta f = \dfrac{-\frac{u}{c}\cos\theta}{1 + \frac{u}{c}\cos\theta}\, f$

and $\theta = 45^\circ$

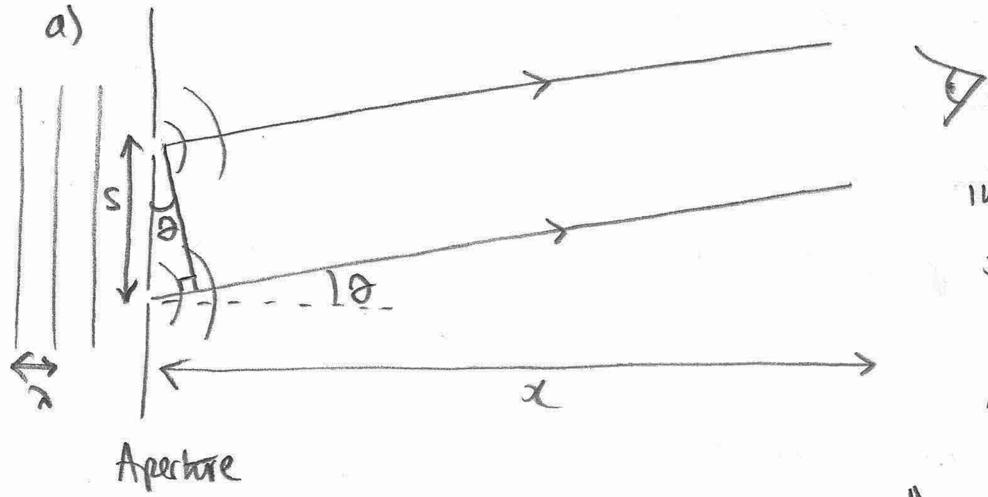
$u = 169.9$ m/s

$$\Delta f = \frac{-\frac{169.9}{343}\cos 45^\circ}{1 + \frac{169}{343}\cos 45^\circ} \times 95.4 \quad (\mathrm{Hz})$$

$\Delta f = \boxed{-24.7\,\mathrm{Hz}}$

So observed frequency on ground is $f + \Delta f = \boxed{70.7\,\mathrm{Hz}}$

2/ a)

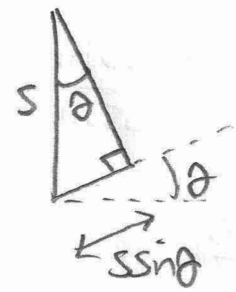


in $\boxed{\text{far field}}$

$x \gg s^2/\lambda$

rays from slits are ≈ parallel.

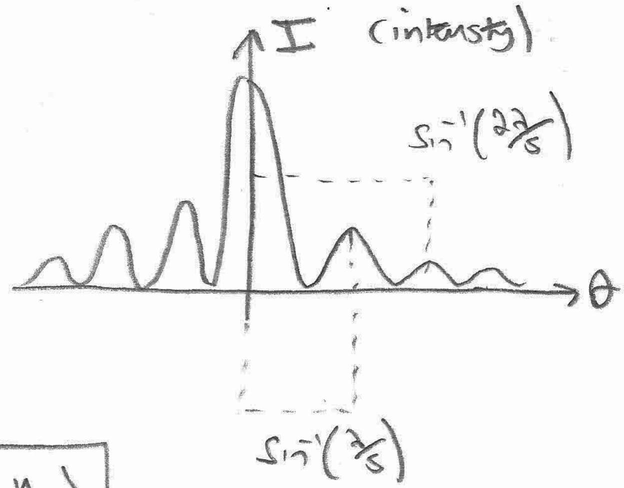
path difference is $\boxed{s\sin\theta}$



so for constructive interference:

$$s\sin\theta = n\lambda$$

$$\Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{n\lambda}{s}\right)}$$



b) $\lambda = 650\,nm$ (red)

$s = 3900\,nm$

$\therefore \frac{\lambda}{s} = \frac{1}{6}$ so $\boxed{\theta_n = \sin^{-1}\left(\frac{n}{6}\right)}$

$\Rightarrow$ Maxima at: (±)

$$\boxed{\begin{array}{l}\theta_0 = 0^\circ,\ \theta_1 = 9.6^\circ,\ \theta_2 = 19.5^\circ \\ \theta_3 = 30^\circ,\ \theta_4 = 41.8^\circ,\ \theta_5 = 56.4^\circ \\ \theta_6 = 90^\circ\end{array}}$$

Note slit width $w = 50\,nm$ is $\frac{w}{\lambda} = 0.077$

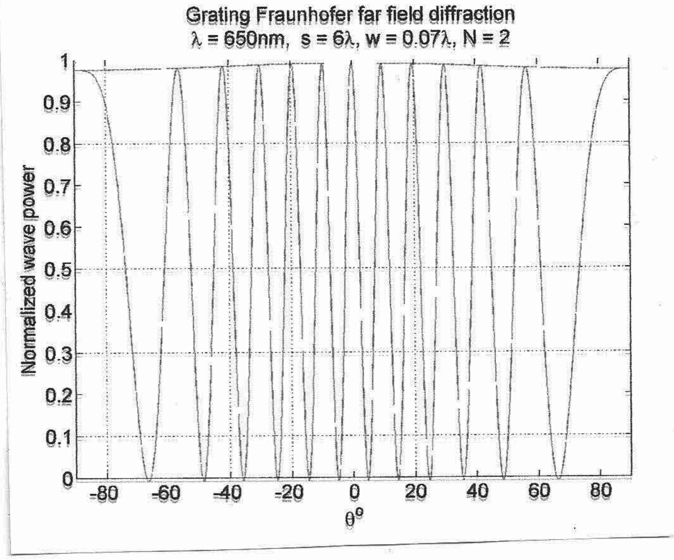
so we can ignore effect of slit width.

↓ PTO

(3)

So diffraction pattern is:

ie slit width is so small that very little 'envelope' ie attenuation of pattern at large angles.



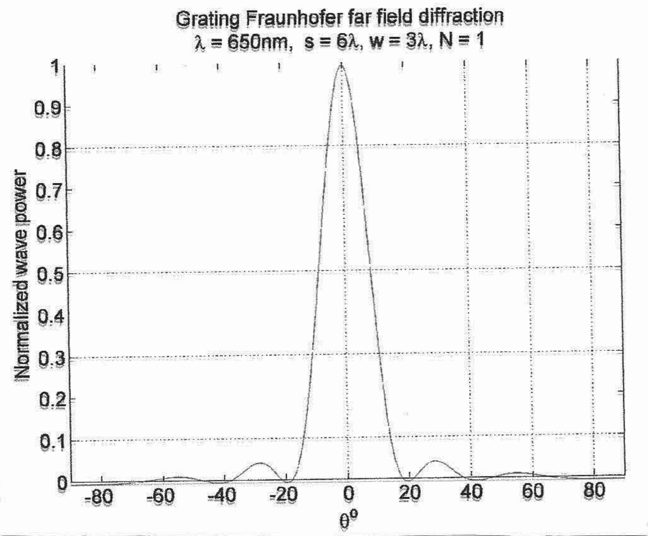
c) Slit width $w = 1950\text{nm}$

$$\frac{w}{\lambda} = \frac{1950\text{nm}}{650\text{nm}} = 3$$

So zeros of single slit pattern at

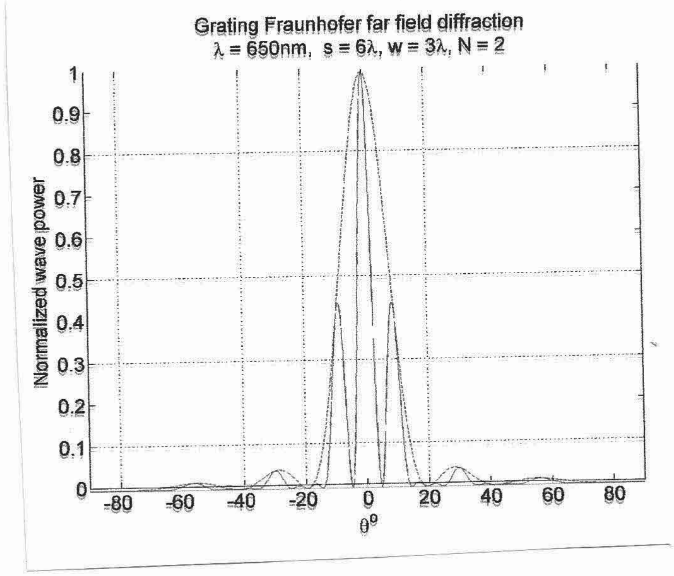
$$\theta_n = \sin^{-1}\left(\frac{n}{3}\right)$$

$$\boxed{19.5^\circ, \quad 41.8^\circ, \quad 90^\circ}$$



The 3λ slit width means you don't get a big → maxima at 19.5°, 41.8° or 90°.

d) So multiplying the grating lobe pattern via the single slit pattern ....



e) If a grating is used

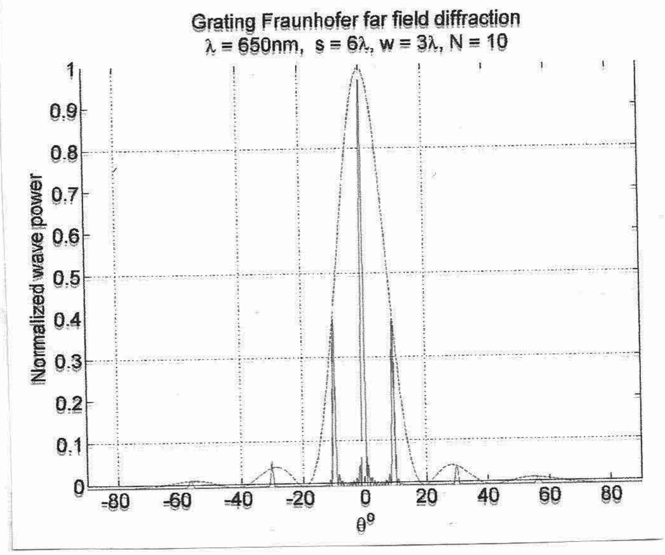
$$s = \frac{10^6\text{nm}}{256} \quad \leftarrow \text{"256 lines/mm"}$$

$= 3906.25 \text{ nm}$ ie $\approx 3900 \text{ nm}$

$\approx 6.0\lambda \quad (6.0096\lambda)$

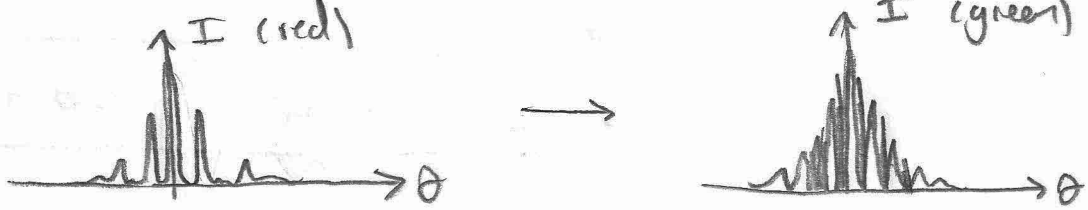
So very similar grating lobe pattern to that of (b), but much sharper maxima, since $N >> 2$ slits illuminated by the laser.

If the same $w = 3\lambda$ slits were used:



f)

Now if green light (495-570 nm) were used, both $\frac{s}{\lambda}$ and $\frac{w}{\lambda}$ are now going to be larger. So $\theta_n = \sin^{-1}\left(\frac{n\lambda}{s}\right)$ will be smaller angles. ie more maxima, more tightly spaced

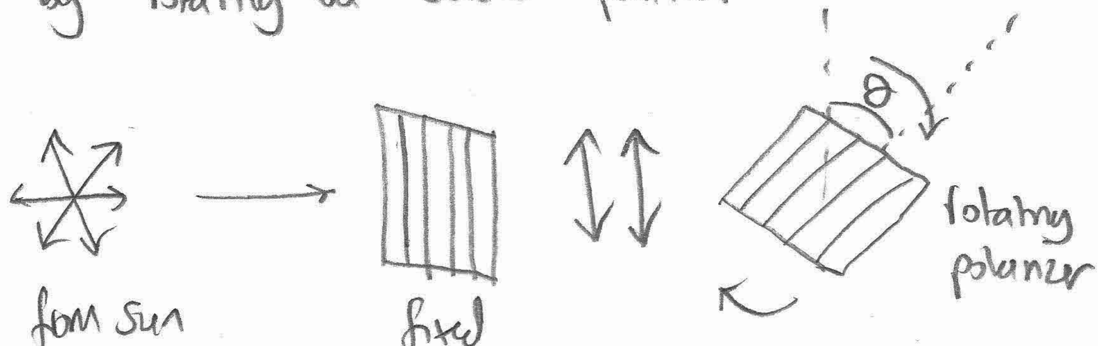


Grating lobes, if $\lambda = 533$ nm:

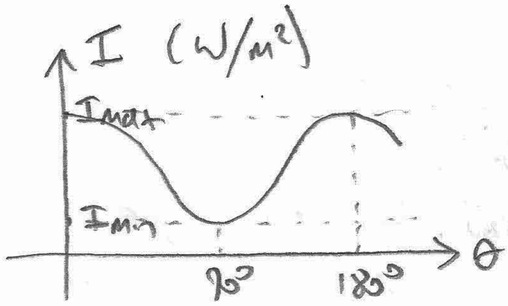
$$\theta_n = \sin^{-1}\left(n \times \frac{533}{3900}\right) = \boxed{7.9^\circ, 15.9^\circ, 24.2^\circ, 33.1^\circ, 43.1^\circ, 55.1^\circ, 73.1^\circ}$$

3/ [Culture → see Sci fi books by Iain M Banks!]

a) light from Sun is <u>unpolarized</u> so if you have the polarizer at any angle, some light will get through. If you polarize the light <u>first</u>, then you can attenuate by rotating the second polarizer.

b) 

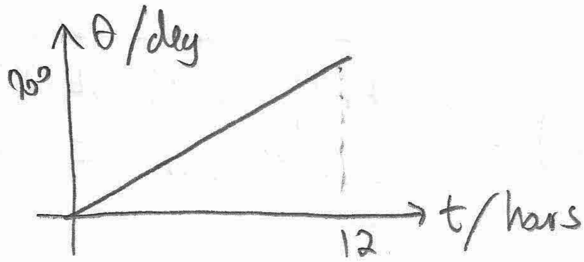
$\boxed{\text{Malus' law}}$



$$\frac{I - I_{min}}{I_{max} - I_{min}} = \cos^2\theta$$

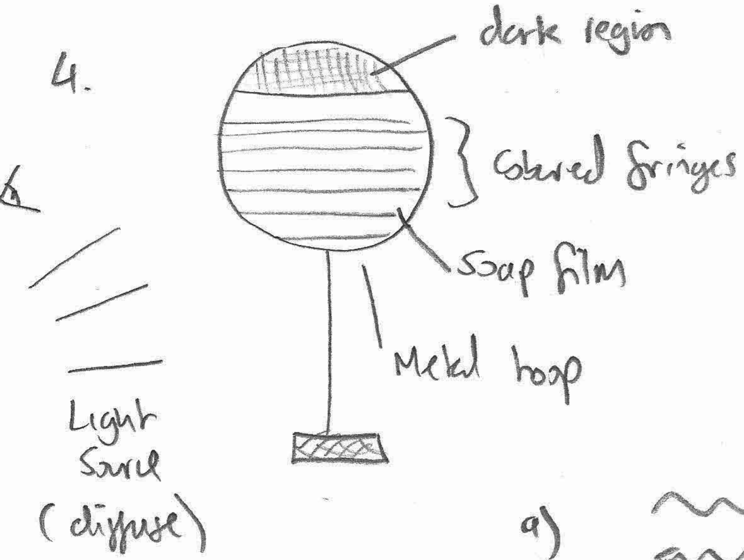
$$\Rightarrow \boxed{I = (I_{max} - I_{min})\cos^2\theta + I_{min}}$$

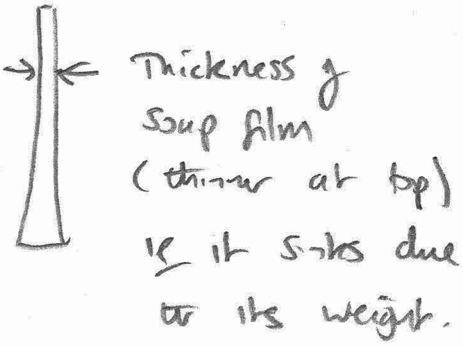
So you want I_{max} at 1200 and I_{min} at 0000

So 

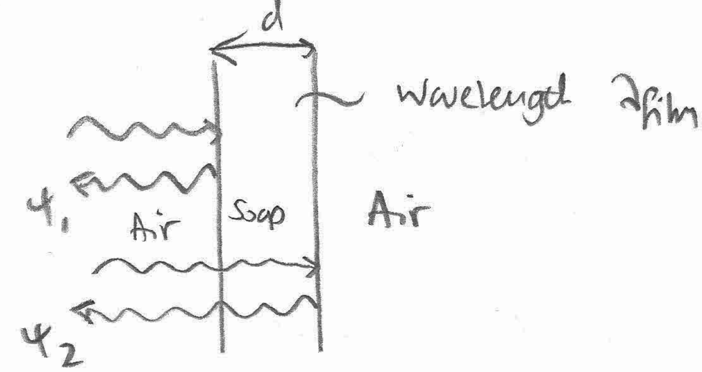
So rotation rate of windows is $90°/12$ hours or $\boxed{7.5°/\text{hour}}$

4.





a)



phase difference between waves ψ_1 and ψ_2 is

$$\Delta\phi = 2\pi \times \underbrace{\frac{2d}{\lambda_{film}}}_{\text{path difference / wavelength IN soap}} + \underbrace{\pi}_{\text{180° phase shift for } \psi_1 \text{ on reflection off air–soap boundary.}}$$

(2π radians is one whole cosine or sine oscillation)

For constructive interference, $\Delta\phi = 2\pi m$ (m integer)

$$\therefore \quad 2\pi m = \frac{4\pi d}{\lambda_{film}} + \pi$$

$$\Rightarrow \quad m = \frac{2d}{\lambda_{film}} + \frac{1}{2}$$

$$\Rightarrow \quad \boxed{d_m = \left(m - \tfrac{1}{2}\right)\lambda_{film}/2}$$

This explains why we see different fringes as soap film gets thicker. i.e. for each λ, m = 1, 2, 3 . . .

b) Air : $c = f\lambda$
Soap : $\frac{c}{n} = f\lambda_{film}$ } Note f the <u>same</u>

$$\therefore \quad \frac{c}{f} = \lambda \Rightarrow \boxed{\lambda_{film} = \frac{\lambda}{n}}$$

c) From above: $\boxed{d_m = \left(m-\frac{1}{2}\right)\frac{\lambda}{2n}}$

So minimum thickness is for $m=1$

$$\boxed{d(\lambda)_{min} = \frac{\lambda}{4n}}$$

$\therefore$ if $n = 1.33$ (ie very similar to water)

$\lambda = 680\,nm$ (red) $\Rightarrow d_{min} = \boxed{128\,nm}$

$\lambda = 380\,nm$ (violet) $\Rightarrow d_{min} = \boxed{71.4\,nm}$

d) If glass ($n = 1.50$) looks purple-blue

$\Rightarrow d_{min} < \frac{500\,nm}{4 \times 1.5}$

$\boxed{d_{min} < 83\,nm}$

ie for light with $\lambda > 500\,nm$ (green, yellow, red) glass isn't thick enough to result in constructive interference between inner and outer surface reflections.

e) $\boxed{d_m = \left(m-\frac{1}{2}\right)\lambda/2n}$

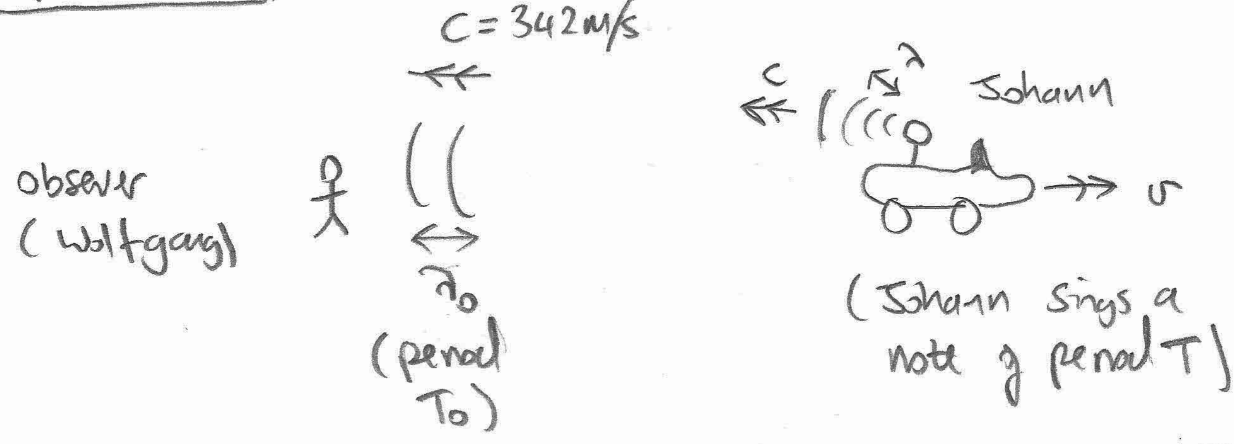
if $m = 5$, $\lambda = 680\,nm$, $n = 1.33$

$\Rightarrow d_5 = \left(5-\frac{1}{2}\right) \times \frac{680}{2 \times 1.33}\,nm$

$d_5 = \boxed{1150\,nm}$ (ie $1.1\,\mu m$).

8

5/ $\boxed{\text{Doppler shift}}$



[Frequency $f = \frac{1}{T}$]

$$\underbrace{T_0}_{\substack{\text{period of}\\ \text{waves}\\ \text{received}\\ \text{by Wolfgang}}} = \underbrace{T}_{\substack{\text{period of}\\ \text{waves}\\ \text{sung by}\\ \text{Johann}}} + \underbrace{\frac{vT}{c}}_{\substack{\text{extra time}\\ \text{between wave crests}\\ \text{emitted by Johann since}\\ \text{his car moves } vT \text{ during}\\ \text{one period}}}$$

$$\Rightarrow \quad \frac{1}{f_0} = \frac{1}{f}\left(1 + \frac{v}{c}\right) \quad \Rightarrow \quad \boxed{f_0 = \frac{f}{1 + v/c}}$$

Now when $v = +v$; $f_0 = 440 \times 2^{3/12}$ (C)

$v = -v$; $f_0 = 440 \times 2^{9/12}$ (F)

(Hz). $c = 342$ m/s

So $440 \times 2^{1/4} = \dfrac{f}{1 + v/c}$ ①

$440 \times 2^{3/4} = \dfrac{f}{1 - v/c}$ ②

②/① $\quad 2^{3/4 - 1/4} = \dfrac{1 + v/c}{1 - v/c}$

$\sqrt{2} = \dfrac{1 + v/c}{1 - v/c} \quad \therefore \quad (1 - \frac{v}{c})\sqrt{2} = 1 + \frac{v}{c}$

$\sqrt{2} - 1 = \frac{v}{c}(1 + \sqrt{2})$

$\therefore \quad v = c \times \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$\therefore \quad v = 342 \text{ m/s} \times \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$= \boxed{58.7 \text{ m/s}}$

1 mile = 1609.34m

So $\quad 1 \text{ mph} = \frac{1609.34\text{m}}{3600\text{s}} = 0.447 \text{ m/s}$

$\therefore \quad v = \frac{58.7}{0.447} \text{ mph}$

$= \boxed{131 \text{ mph}}$

So I hope Johann is not on a public road!
Perhaps the note he sings is one of terror!

So in ①: $\quad f = 440 \times 2^{1/4} \times \left(1 + \frac{v}{c}\right)$

$f = 440 \times 2^{1/4} \times \left(1 + \frac{58.7}{342}\right)$

$= \boxed{613.1 \text{ Hz}}$

So $\quad 440 \times 2^{m/12} = 613.1$

$\frac{m}{12} \ln 2 = \ln\left(\frac{613.1}{440}\right)$

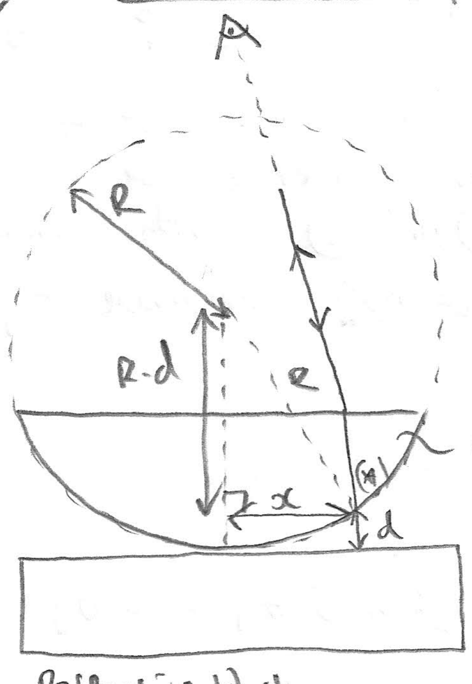
$m = 12 \ln\left(\frac{613.1}{440}\right) / \ln 2$

$m = 5.74$

So Johann sings a slightly flat Eb $(m=6)$.

[Eb is $440 \text{ Hz} \times 2^{6/12} = 622.3 \text{ Hz}$]

Q6/ $\boxed{\text{Newton's rings}}$



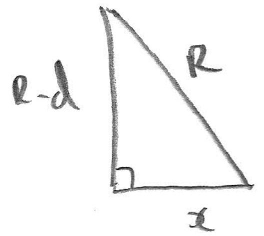
Phase difference between ray which reflects of lower lens surface and that which is transmitted and then reflected of the block is:

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda} \times 2d + \pi}$$

180° reflection of block.

gap is in air so don't need to reduce λ by n as in thin film

For constructive interference: $\boxed{\Delta\phi = 2\pi n}$ where n is integer.



Pythagoras:

$$R^2 = (R-d)^2 + x^2$$

$$\cancel{R^2} = \cancel{R^2} - 2Rd + d^2 + x^2$$

$$2Rd + d^2 = x^2$$

$$(2R + d)d = x^2$$

Now assume $R \gg d$, $\therefore$ $\boxed{x \simeq \sqrt{2Rd}}$ or $\boxed{d \simeq \frac{x^2}{2R}}$

so for constructive interference: $2\pi n = \frac{2\pi}{\lambda} \times \frac{x_n^2}{2R} \times 2 + \pi$

$$\therefore \quad n = \frac{x_n^2}{\lambda R} + \frac{1}{2}$$

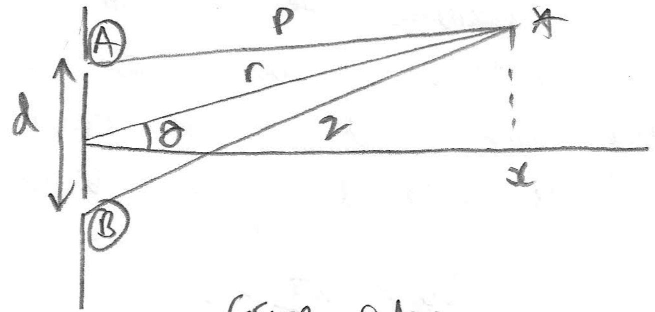
$$\boxed{\sqrt{\left(n - \frac{1}{2}\right)\lambda R} = x_n}$$

(so $n = 1, 2, 3, 4 \ldots$)

So $\boxed{R = \frac{x_n^2}{\left(n - \frac{1}{2}\right)\lambda}}$ $\therefore$ if $x = 3.14\,\text{mm}$, $\lambda = 400\,\text{nm}$, $n = 5$, $R = \frac{(3.14 \times 10^{-3})^2}{4.5 \times 400 \times 10^{-9}}$ (m)

$$\Rightarrow \boxed{R = 5.48\,\text{m}}$$

Q7



$$\psi(r,t) = A e^{-i\omega t}\left[\frac{e^{ikp}}{p} + \frac{e^{ikq}}{q}\right]$$

is wave disturbance at * due to spherical wave sources at (A) and (B) (both identical $k = 2\pi/\lambda$, amplitude A).

Cosine rule:

$$p^2 = r^2 + \left(\frac{d}{2}\right)^2 - 2r\left(\frac{d}{2}\right)\cos(90^\circ - \theta)$$

$$\therefore \boxed{p^2 = r^2 + d^2/4 - rd\sin\theta} \qquad [\cos(90^\circ - \theta) = \sin\theta]$$

$$q^2 = r^2 + \left(\frac{d}{2}\right)^2 - 2r\left(\frac{d}{2}\right)\cos(90^\circ + \theta)$$

$$\therefore \boxed{q^2 = r^2 + d^2/4 + rd\sin\theta} \qquad [\cos(90^\circ + \theta) = -\sin\theta]$$

$$\therefore p = r\left(1 + \left(\frac{d}{2r}\right)^2 - \frac{d\sin\theta}{r}\right)^{\frac{1}{2}}$$

$$\therefore q = r\left(1 + \left(\frac{d}{2r}\right)^2 + \frac{d\sin\theta}{r}\right)^{\frac{1}{2}}$$

if $r \gg d$:

$$p \simeq r - \frac{d\sin\theta}{2}$$

$$q \simeq r + \frac{d\sin\theta}{2}$$

(i.e. ignore terms $\left(\frac{d}{r}\right)^2$ and above)

Also in $\psi(r,t)$, $\frac{1}{p}$ and $\frac{1}{q} \approx \frac{1}{r}$ i.e. only consider deviation from r in phase term.

$$\therefore \psi(r,t) \simeq \frac{A}{r} e^{-i\omega t}\left(e^{ik\left(r - \frac{d\sin\theta}{2}\right)} + e^{ik\left(r + \frac{d\sin\theta}{2}\right)}\right)$$

$$\simeq \frac{A}{r} e^{i(kr - \omega t)}\left[e^{\frac{ikd\sin\theta}{2}} + e^{\frac{-ikd\sin\theta}{2}}\right]$$

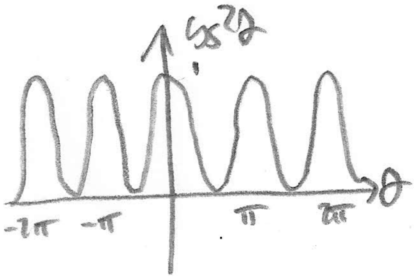
Now $\boxed{\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)}$ {prove from $e^{i\theta} = \cos\theta + i\sin\theta$}

So:

$$\psi(r,t) \simeq \frac{2A}{r} e^{i(kr-\omega t)} \cos\left(\frac{kd\sin\theta}{2}\right)$$

∴ wave power is $\propto |\psi|^2$ where:

$$\boxed{|\psi|^2 \simeq \frac{4A^2}{r^2} \cos^2\left(\tfrac{1}{2}kd\sin\theta\right)}$$



Maxima when $\frac{1}{2}kd\sin\theta = n\pi$

$$\therefore \quad \frac{1}{2} \times \frac{2\pi}{\lambda} d \sin\theta_n = n\pi$$

$$\sin\theta_n = \frac{n\lambda}{d}$$

$$\therefore \quad \boxed{\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right)}$$

AR 28/7/2020.