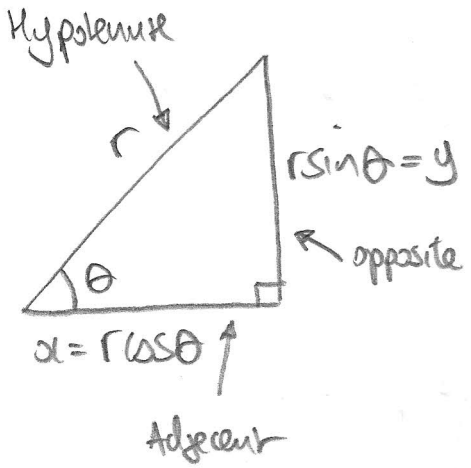
GCSE Trigonometry results



Sine and cosine are what you multiply the hypotenuse by to obtain the opposite or adjacent sides of a right angled triangle

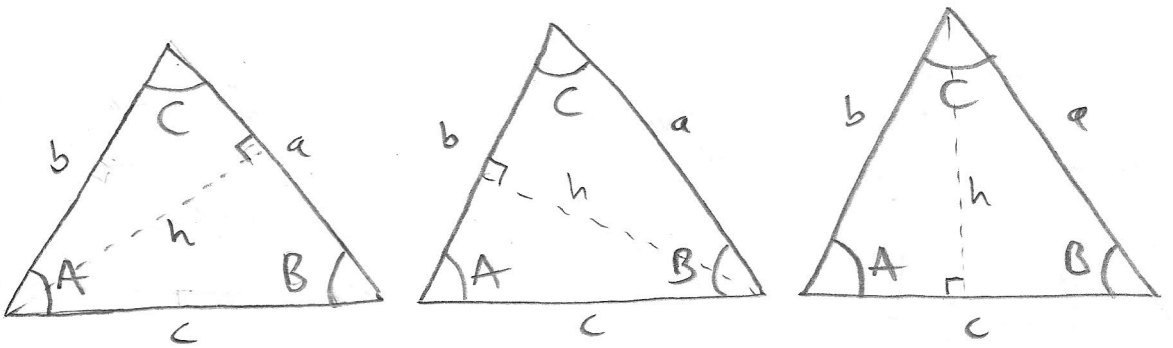
$$\tan\theta = \frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} \quad \text{so} \quad \boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

Pythagoras' Theorem $\boxed{r^2 = x^2 + y^2}$

So $r^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$$\Rightarrow \boxed{\cos^2\theta + \sin^2\theta = 1}$$

Sine rule

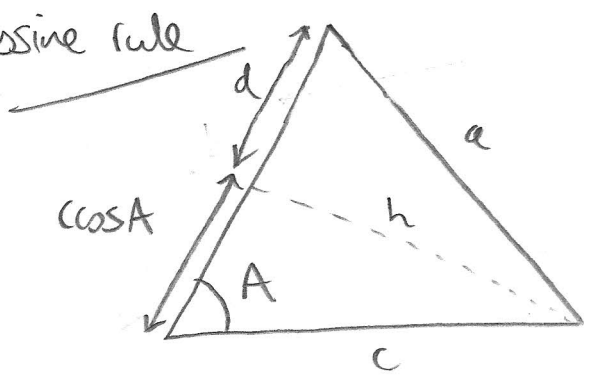


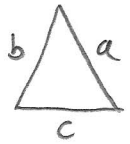
$h = b\sin C = c\sin B$ $\quad h = c\sin A = a\sin C$ $\quad h = b\sin A = a\sin B$

$$\Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C} \quad \Rightarrow \frac{c}{\sin C} = \frac{a}{\sin A} \quad \Rightarrow \frac{b}{\sin B} = \frac{a}{\sin A}$$

Hence $\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$

Cosine rule





So $d = b - c\cos A$

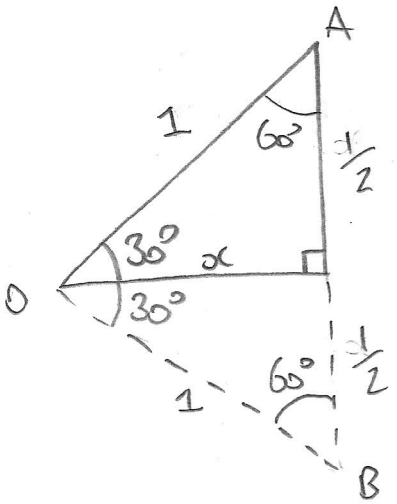
Pythagoras: $c^2 = c^2\cos^2 A + h^2$ ①

$a^2 = (b - c\cos A)^2 + h^2$ ②

② − ①: $a^2 - c^2 = b^2 - 2bc\cos A + c^2\cos^2 A - c^2\cos^2 A$

$$\Rightarrow \boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

Special triangles and trig results you need to know



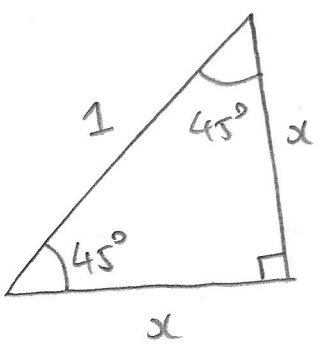
OAB is an equilateral triangle so all internal angles are 60°

$\therefore$ $\boxed{\sin 30° = \frac{1}{2}}$
$\boxed{\cos 60° = \frac{1}{2}}$

Pythagoras: $1^2 = \left(\frac{1}{2}\right)^2 + x^2$

$\Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$

So $\boxed{\cos 30° = \frac{\sqrt{3}}{2}}$
$\boxed{\sin 60° = \frac{\sqrt{3}}{2}}$
$\Rightarrow$ $\boxed{\tan 30° = \frac{1}{\sqrt{3}}}$ $\left(\frac{1/2}{x}\right)$
$\boxed{\tan 60° = \sqrt{3}}$ $\left(\frac{x}{1/2}\right)$



Pythagoras: $1^2 = 2x^2 \Rightarrow x = \frac{1}{\sqrt{2}}$

Hence $\boxed{\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}}$

$\boxed{\tan 45° = 1}$