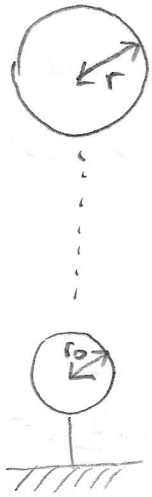
IDEAL GASES, HEAT ENGINES, ENTROPY AND RADIATION

Q1/ (i)



$P = 22{,}632$ Pa
$T = (273 - 56.5)$ K
$h = 11$ km (top of troposphere ← start of the 'tropopause')

Weather balloon →

$P_0 = 101325$ Pa $r_0 = 0.600$ m
$T_0 = (15 + 273)$ K
$h = 0$ m (sea level)

Assume an $\boxed{\text{ideal gas}}$ in the balloon, and no leaks.

$P_0 V_0 = n R T_0$ ①
$pV = nRT$ ②

$\therefore \dfrac{pV}{P_0 V_0} = \dfrac{T}{T_0}$

$\therefore \boxed{V = \left(\dfrac{P_0}{P}\right)\left(\dfrac{T}{T_0}\right) V_0}$

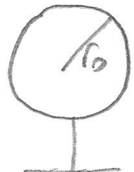
$\therefore$ if $V = \frac{4}{3}\pi r^3$ and $V_0 = \frac{4}{3}\pi r_0^3$

$$\boxed{r = \left(\frac{P_0}{P}\right)^{\frac{1}{3}} \left(\frac{T}{T_0}\right)^{\frac{1}{3}} r_0}$$

$$\Rightarrow \quad r = \left(\frac{101325}{22632}\right)^{\frac{1}{3}} \left(\frac{273 - 56.5}{273 + 15}\right)^{\frac{1}{3}} \times 0.600 \quad \text{(m)}$$

$$= 1.499 \times 0.600 = \boxed{0.899 \text{ m}}$$

(ii) $P_0 V_0 = n R T_0$ $\therefore n = \dfrac{P_0 V_0}{R T_0}$

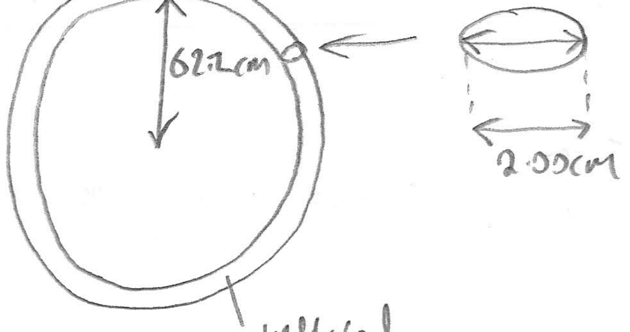


$P_0 = 101500$ Pa
$T_0 = (30 + 273)$ K
$r_0 = 0.800$ m

$$\therefore n = \frac{101500 \times \frac{4}{3}\pi (0.800)^3}{8.314 \times (30 + 273)}$$

$$= \boxed{86.4 \text{ Moles}}$$

(iii)



inflated bike tyre inner tube @ 120 PSI $= \boxed{827{,}400\ \text{Pa} = P_T}$

{ Assume constant temp T_0 }

Volume of inflated inner tube is:

$V_T = \pi \times (1.00\,\text{cm})^2 \times 2\pi \times \boxed{63.2\,\text{cm}}$

* add radius of cross section *

$$\boxed{V_T = \frac{632}{5}\pi^2 \ \text{cm}^3}$$

$$\simeq \boxed{1248\ \text{cm}^3}$$

Pump volume is $\boxed{V_p = 317\,\text{cm}^3}$ (air drawn in at $P_0 = 101325\,\text{Pa}$)

So # moles for one pump volume is:

$$\boxed{n_p = \frac{P_0 V_p}{R T_0}}$$

} Assume ideal gas.

moles in inflated tyre is: $\boxed{n_T = \frac{P_T V_T}{R T_0}}$

$\therefore$ # of strokes of pump to inflate tyre is

$$N = \frac{n_T}{n_p} = \frac{P_T V_T}{R T_0} \Big/ \frac{P_0 V_p}{R T_0} = \frac{P_T V_T}{P_0 V_p}$$

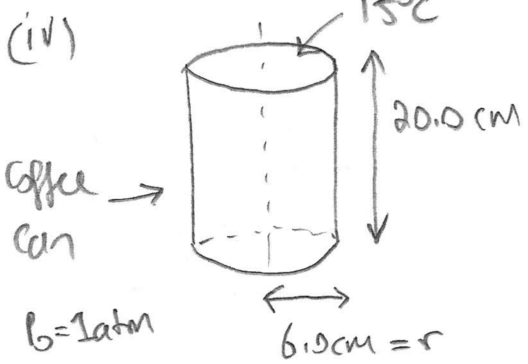
$$= \frac{827400}{101325} \times \frac{632\pi^2}{5}\ \frac{1}{317}$$

$$= \boxed{32.14}$$

So since it is best not to over-inflate

$$\Rightarrow \boxed{N = 32 \ \text{Strokes}}$$

(iv)



$n = \frac{P_0 V_0}{R T_0}$ if ideal gas in can.

$$n = \frac{101325 \times \pi \times 6.0^2 \times 20.0 \times 10^{-6}}{8.314 \times (15+273)}$$

$$= \boxed{9.57 \times 10^{-2} \text{ moles in can}}.$$

[$1 \text{ cm} = 10^{-2} \text{ m}$ $\therefore$ $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$]

If can heated to 80°C in an isochoric (constant volume) process and does not leak

$$p V_0 = nRT \qquad \text{where } T = (273+80)\text{ K}.$$

$\therefore$ pressure difference between inside and outside of the can is $\Delta p = p - P_0$

$$\Delta p = \frac{nRT}{V_0} - P_0$$

Now $n = \frac{P_0 V_0}{R T_0}$ so $\Delta p = \frac{P_0 \cancel{V_0}}{\cancel{R} T_0} \times \frac{\cancel{R} T}{\cancel{V_0}} - P_0$

$$\Rightarrow \boxed{\Delta p = P_0 \left(T/T_0 - 1 \right)}$$

$\therefore$ Force (upwards) on the lid of the can is

$$F = \Delta p \times \pi r^2$$

$$F = 101325 \left(\frac{273+80}{273+15} - 1 \right) \times \pi \times (6.0 \times 10^{-2})^2 \quad \text{(N)}$$

$$= \boxed{259 \text{ N}}$$

(is equivalent to 26.4 kg using $g = 9.81$ N/kg)

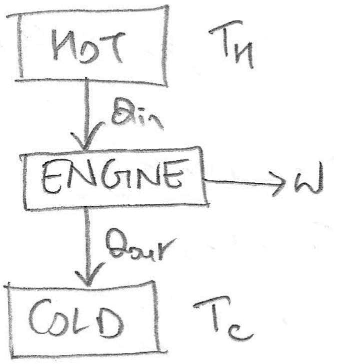
Heat transferred is $\boxed{Q = C_V \Delta T}$

Heat capacity is $C_V = \frac{1}{2} \alpha n R$

$$\therefore Q = \tfrac{1}{2} \times 3 \times 9.57 \times 10^{-2} \times 8.314 \times (80-15) \quad \text{(J)}$$

$$= \boxed{77.6 \text{ J}}$$

v)



(W is work done by heat engine)

$$\eta \leq 1 - \frac{T_c}{T_H}$$

η is efficiency

$$\boxed{\eta = W/Q_{in}}$$

$$T_c = (20 + 273)\,\text{K}$$

so $\frac{T_c}{T_H} \leq 1 - \eta$

$$\boxed{\frac{T_c}{1-\eta} \leq T_H}$$

$\therefore$ if $\eta = 0.66$ $\therefore$ $T_H \geq \frac{293}{1 - 0.66}$

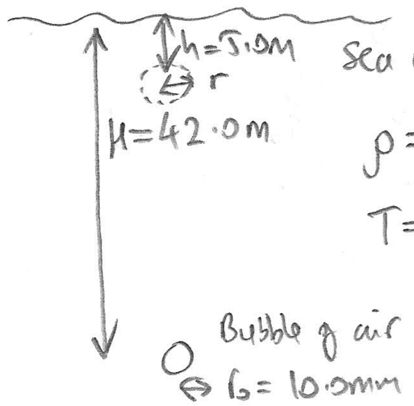
$$T_H \geq 862\,\text{K}$$

$$\boxed{T_H \geq 589°\text{C}}$$

$g = 9.81\,\text{N/kg}$

$P_0 = 101325\,\text{Pa}$

(vi)



Sea water

$\rho = 1025\,\text{kg/m}^3$

$T = 10°\text{C}$

Uniform temperature so Boyle's law. $\boxed{pV = \text{constant}}$

$$\therefore (P_0 + \rho g H)\tfrac{4}{3}\pi r_0^3$$

$$= (P_0 + \rho g h)\tfrac{4}{3}\pi r^3$$

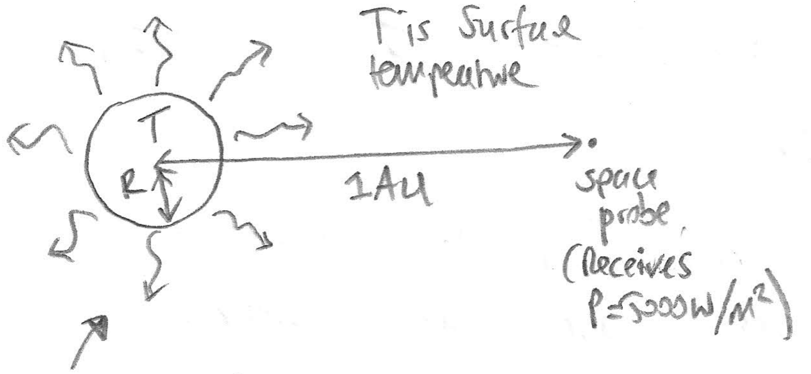
$$\therefore \boxed{r = \left(\frac{P_0 + \rho g H}{P_0 + \rho g h}\right)^{1/3} r_0}$$

ie bubble radius at $h = 5.0\,\text{m}$.

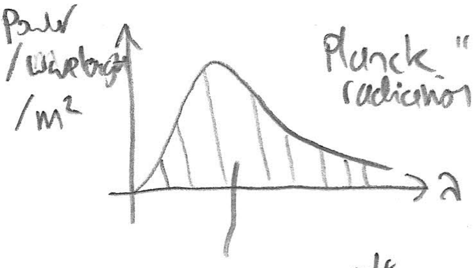
$$\therefore r = \left(\frac{101325 + 1025 \times 9.81 \times 42}{101325 + 1025 \times 9.81 \times 5}\right)^{1/3} \times 10.0\,\text{mm}$$

$$= \boxed{15.1\,\text{mm}}$$

(vii)



Blue star, uniform radiation @ $\lambda_{max} = 470\,nm$



$$\text{Area} = \sigma T^4$$

$$\lambda_{max} T = 2.90 \times 10^{-3}$$

$$\therefore\ T = \frac{2.90 \times 10^{-3}}{470 \times 10^{-9}} \quad (K)$$

$$\boxed{T = 6170\ K}$$

$\therefore$ Luminosity of star (is radiated power) is

$$L = 4\pi R^2 \sigma T^4$$

Now at radius r from star centre, power received /m² is:

$$P = \frac{L}{4\pi r^2} = \left(\frac{R}{r}\right)^2 \sigma T^4$$

So $\boxed{\sqrt{\frac{P}{\sigma T^4}}\, r = R}$

$\therefore$ radius of the star is:

$$R = \sqrt{\frac{5000}{\underbrace{5.67 \times 10^{-8}}_{\sigma} \times 6170^4}} \times 1.496 \times 10^{11}\ (m)$$

$$\boxed{R = 1.167 \times 10^{9}\ m}$$

$(R_{\odot} = 696{,}340\ km)$

$$\boxed{R = 1.68\ R_{\odot}}$$

(viii) Isothermal compression : $P_0 V_0 = nRT$ where T = constant

Work done on the gas is $W = \int_{V_0}^{0.2V_0} \underbrace{(-p\,dV)}_{dW}$

$pV = nRT$ So $p = \frac{nRT}{V}$

$$\therefore\ W = -nRT \int_{V_0}^{0.2V_0} \frac{1}{V}\, dV = -nRT \left[\ln V\right]_{V_0}^{0.2V_0}$$

$$W = nRT \ln\left(\frac{1}{0.2}\right) = \boxed{nRT \ln 5}$$

$\therefore \quad W = 3.00 \times 8.314 \times (273+42) \ln 5 \qquad (J)$

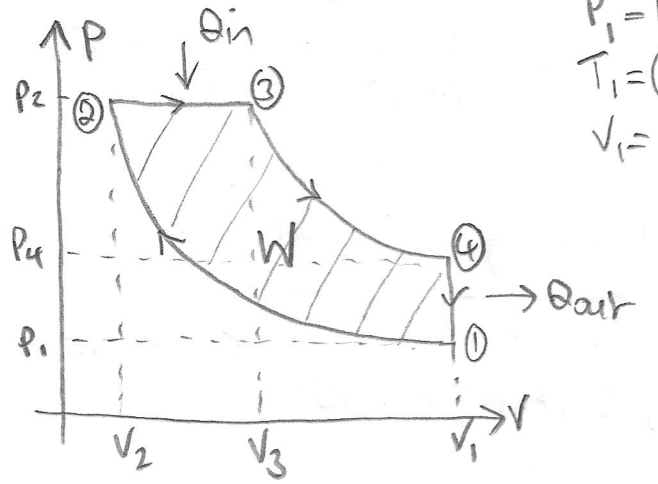
$= \boxed{12.6 \text{ kJ}}$

Now if process is isothermal, this means the internal energy of the system must remain constant.

$[\, U = \frac{1}{2} \alpha n R T \,] \quad \therefore$ Work done on the gas W must equal the heat Q released.

So $\boxed{Q = 12.6 \text{ kJ}}$

(ix) Diesel cycle



$P_1 = 101325$ Pa
$T_1 = (12+273)$ K
$V_1 = 1000 \text{ cm}^3$

$\Big\} \; n = \dfrac{P_1 V_1}{R T_1}$

$n = \dfrac{101325 \times 1000 \times 10^{-6}}{8.314 \times (12+273)}$

$= \boxed{4.28 \times 10^{-2} \text{ moles}}$

Work done by the expanding gas:

② → ③ is

$$W = \int_{V_2}^{V_3} p \, dV.$$

Since $p = P_2$ during this process (ie isobaric: constant pressure):

$$\Rightarrow \boxed{W = P_2 (V_3 - V_2)}$$

Heat input is $\boxed{Q_{in} = C_p (T_3 - T_2)}$ $\quad \boxed{C_p = (\tfrac{1}{2}\alpha + 1) n R}$

$V_2 = 320 \text{ cm}^3$
$V_3 = 500 \text{ cm}^3$

① → ② is Adiabatic compression $\therefore \; pV^{\gamma} = \text{constant}$.

$\gamma = 1 + 2/3 = 5/3$

$\therefore \quad P_1 V_1^{5/3} = P_2 V_2^{5/3} \quad \therefore \quad P_2 = P_1 \left(\dfrac{V_1}{V_2} \right)^{5/3}$

$\therefore \quad P_2 = 101325 \times \left(\dfrac{1000}{320} \right)^{5/3} = \boxed{676{,}810 \text{ Pa}}$

$\therefore \quad W = 676{,}810 \times (500 - 320) \times 10^{-6} \quad (J)$

$= \boxed{121\,J}$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{676{,}810 \times 320 \times 10^{-6}}{4.28 \times 10^{-2} \times 8.314} \quad (K)$$

$$= \boxed{609\,K}$$

$$T_3 = \frac{P_2 V_3}{nR} = \frac{676\,810 \times 500 \times 10^{-6}}{4.28 \times 10^{-2} \times 8.314} \quad (K)$$

$$= \boxed{951\,K}$$

$$\therefore \quad Q_{in} = \underbrace{\left(\tfrac{1}{2} \times 3 + 1\right) \times 4.28 \times 10^{-2} \times 8.314}_{C_p} \times (951 - 609)$$

$$= \boxed{203\,J}$$

(x) $$\Delta S = \int \frac{dQ}{T} = \int_{T_2}^{T_3} \frac{C_p\,dT}{T} = C_p \ln\left(\tfrac{T_3}{T_2}\right)$$

$$\Delta S = \tfrac{5}{3} \times 4.28 \times 10^{-2} \times 8.314 \times \ln\left(\tfrac{951}{609}\right)$$

$$= \boxed{0.264\,J\,K^{-1}}$$

(x) Adiabatic expansion : $pV^{\gamma} = \text{constant}.$

$V \to 3V$ $U_0 = \tfrac{1}{2}\alpha nRT_0 = \tfrac{1}{2}\alpha P_0 V_0$

V_0 V_1 since $P_0 V_0 = nRT_0$. (initial internal energy)

Work done <u>by</u> the gas is: $W = \int_{V_1}^{V_2} p\,dV$

$pV^{\gamma} = P_0 V_0^{\gamma}$ so $W = \int_{V_0}^{V_1} \frac{P_0 V_0^{\gamma}}{V^{\gamma}}\,dV$

$$W = P_0 V_0^{\gamma} \left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_0}^{V_1}$$

$$W = \frac{P_0 V_0^{\gamma}}{1-\gamma}\left(V_1^{-\gamma+1} - V_0^{-\gamma+1}\right)$$

Now $\quad V_1 = k V_0 \qquad (k=3)$

$$W = \frac{P_0 V_0^{\gamma}}{1-\gamma}\left(k^{-\gamma+1} V_0^{-\gamma+1} - V_0^{-\gamma+1}\right)$$

$$W = \frac{P_0 V_0}{1-\gamma}\left(k^{1-\gamma} - 1\right)$$

$$\therefore \frac{W}{U_0} = \frac{\frac{P_0 V_0}{1-\gamma}\left(k^{1-\gamma}-1\right)}{\frac{1}{2}\alpha P_0 V_0}$$

$$\frac{W}{U_0} = \frac{k^{1-\gamma}-1}{\frac{1}{2}\alpha(1-\gamma)}$$

$\gamma = 1 + 2/\alpha \quad \therefore \quad \alpha\gamma = \alpha + 2$ and $1-\gamma = -2/\alpha$

So $\alpha(1-\gamma)$

$= \alpha - \alpha - 2$

$$\therefore \frac{W}{U_0} = \frac{k^{-2/\alpha}-1}{\frac{1}{2}(\alpha-\alpha-2)}$$

$$\therefore \boxed{\frac{W}{U_0} = 1 - k^{-2/\alpha}}$$

$$\therefore \frac{W}{U_0} = 1 - 3^{-2/3} = \boxed{0.519}$$