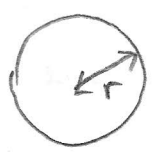


IDEAL GASES, HEAT ENGINES, ENTROPY AND RADIATION

Q1 / (i)



$P = 22,632 \text{ Pa}$
 $T = (273 - 56.5) \text{ K}$
 $h = 11 \text{ km}$ (top of troposphere ← start of the 'tropopause')

Weather balloon →



$P_0 = 101325 \text{ Pa}$
 $T_0 = (15 + 273) \text{ K}$
 $h = 0 \text{ m}$ (sea level)
 $r_0 = 0.600 \text{ m}$

Assume an ideal gas in the balloon, and no leaks.

$$P_0 V_0 = n R T_0 \quad (1)$$

$$P V = n R T \quad (2)$$

$$\frac{P V}{P_0 V_0} = \frac{T}{T_0}$$

$$V = \left(\frac{P_0}{P} \right) \left(\frac{T}{T_0} \right) V_0$$

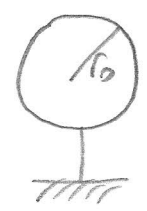
∴ if $V = \frac{4}{3} \pi r^3$ and $V_0 = \frac{4}{3} \pi r_0^3$

$$r = \left(\frac{P_0}{P} \right)^{\frac{1}{3}} \left(\frac{T}{T_0} \right)^{\frac{1}{3}} r_0$$

$$\Rightarrow r = \left(\frac{101325}{22632} \right)^{\frac{1}{3}} \left(\frac{273 - 56.5}{273 + 15} \right)^{\frac{1}{3}} \times 0.600 \text{ (m)}$$

$$= 1.499 \times 0.600 = \boxed{0.899 \text{ m}}$$

(ii) $P_0 V_0 = n R T_0 \quad \therefore n = \frac{P_0 V_0}{R T_0}$

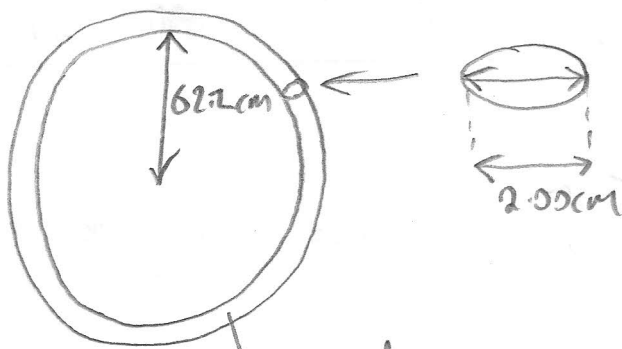


$P_0 = 101500 \text{ Pa}$
 $T_0 = (30 + 273) \text{ K}$
 $r_0 = 0.800 \text{ m}$

$$\therefore n = \frac{101500 \times \frac{4}{3} \pi (0.800)^3}{8.314 \times (30 + 273)}$$

$$= \boxed{86.4 \text{ moles}}$$

(iii)



Volume of inflated inner tube is:

$$V_T = \pi \times (1.00 \text{ cm})^2 \times 2\pi \times 63.2 \text{ cm}$$

* add radius of cross section *

$$V_T = \frac{632}{5} \pi^2 \text{ cm}^3$$

$$\approx 1248 \text{ cm}^3$$

Assume constant temp T_0

Inflated bike tyre inner tube @ 120 PSI

$$= 827,400 \text{ Pa} = P_T$$

Pump volume is $V_p = 317 \text{ cm}^3$ (air drawn in at $P_0 = 101325 \text{ Pa}$)

So # moles for one pump volume is:

$$n_p = \frac{P_0 V_p}{RT_0}$$

Assume ideal gas.

moles in inflated tyre is:

$$n_T = \frac{P_T V_T}{RT_0}$$

\therefore # of strokes of pump to inflate tyre is

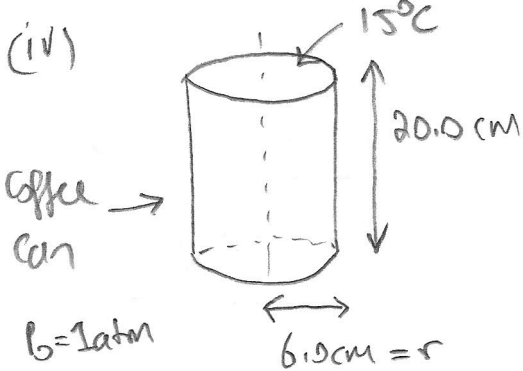
$$N = \frac{n_T}{n_p} = \frac{P_T V_T}{RT_0} / \frac{P_0 V_p}{RT_0} = \frac{P_T V_T}{P_0 V_p}$$

$$= \frac{827400}{101325} \times \frac{632\pi^2}{5} \frac{1}{317}$$

$$= 32.14$$

So since it is best not to over-inflate

$$\Rightarrow N = 32 \text{ strokes}$$



If ideal gas in can.

$$n = \frac{P_0 V_0}{RT_0}$$

$$n = \frac{101325 \times \pi \times 6.0^2 \times 20.0 \times 10^{-6}}{8.314 \times (15 + 273)}$$

$$= \boxed{9.57 \times 10^{-2} \text{ moles in can}}$$

$$[1 \text{ cm} = 10^{-2} \text{ m} \quad \therefore 1 \text{ cm}^3 = 10^{-6} \text{ m}^3]$$

If can heated to 80°C in an isochoric (constant volume) process and does not leak

$$P V_0 = n R T$$

where $T = (273 + 80) \text{ K}$.

\therefore pressure difference between inside and outside of the can is $\Delta P = P - P_0$

$$\Delta P = \frac{n R T}{V_0} - P_0$$

Now $n = \frac{P_0 V_0}{R T_0}$ so $\Delta P = \frac{P_0 V_0}{R T_0} \times \frac{R T}{V_0} - P_0$

$$\Rightarrow \boxed{\Delta P = P_0 \left(\frac{T}{T_0} - 1 \right)}$$

\therefore Force (upwards) on the lid of the can is

$$F = \Delta P \times \pi r^2$$

$$F = 101325 \left(\frac{273 + 80}{273 + 15} - 1 \right) \times \pi \times (6.0 \times 10^{-2})^2 \quad (\text{N})$$

$$= \boxed{259 \text{ N}}$$

(is equivalent to 26.4 kg using $g = 9.81 \text{ N/kg}$)

Heat transferred is $\boxed{Q = C_V \Delta T}$

Heat capacity is $C_V = \frac{1}{2} n R$

$$\therefore Q = \frac{1}{2} \times 3 \times 9.57 \times 10^{-2} \times 8.314 \times (80 - 15) \quad (\text{J})$$

$$= \boxed{77.6 \text{ J}}$$

(3)

