Q2

i) $pV = nRT$

Ideal gas equation

P pressure /Pa
V volume /m^3
n # moles of gas
R Molar gas constant $8.314 \ J \ mol^{-1} k^{-1}$
T Absolute temperature / k

ii)
* Ignore intermolecular forces
* molecules collide randomly and elastically
* large # of molecules ie statistical / average behaviour
* Molecular size << mean free path (ie average distance between collisions)

ii)   a) | Charles' law | P, n constant | $V \propto T$ |
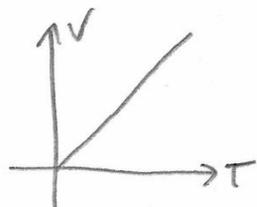
b) | Boyle's law | T, n constant | $p \propto \frac{1}{V}$ |
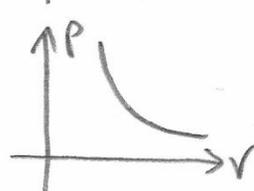
c) | Pressure law | V, n constant | $P \propto T$ |

iii)

$T_0 = 298 k$

← Syringe

$V_0 = 100 \ cm^3$

$P_0 = 1 \ atm$
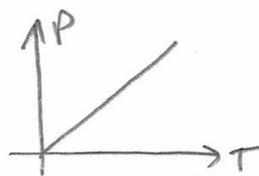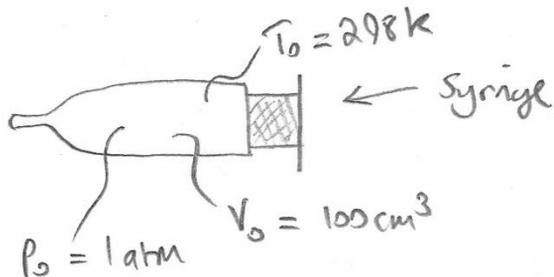
| $P_0 V_0 = n_0 R T_0$ |

a) $V = 2V_0$ $n = 1.2 n_0$ $T = T_0$ ← � Maintain thermal equilibrium

$P = nRT/V = \dfrac{1.2 n_0 R T_0}{2 V_0}$

Now $\dfrac{n_0 R T_0}{V_0} = P_0$

So $P = 0.6 P_0$ or $0.6 \ atm$

b) $\qquad P = 3P_0$, $V = ?$, $n = 0.9n_0$, $T = 308$

$\therefore T = \dfrac{308}{298} T_0$

$$\boxed{PV = nRT}$$

$$\therefore V = \frac{nRT}{P}$$

$$V = \frac{0.9n_0 R \left(\dfrac{308}{298}\right) T_0}{3P_0}$$

$$V = \frac{0.9 \times 308}{3 \times 298} \quad \underbrace{\frac{n_0 R T_0}{P_0}}_{} \leftarrow V_0$$

$$V = 0.310 \times 100 \ cm^3$$

$$\boxed{V = 31.0 \ cm^3}$$

c) $\qquad \dfrac{V_0 P_0}{RT_0} = n_0 \qquad\qquad \# \ \text{molecules} = n_0 N_A$

$$= \underbrace{\frac{100 \times (10^{-2} m)^3 \times 101325 \ Pa}{8.314 \times 298}}_{} \times 6.02 \times 10^{23}$$

$$\overset{\nwarrow}{} \ 4.09 \times 10^{-3} \ \text{moles.}$$

$$= \boxed{2.46 \times 10^{21} \ \text{molecules}}$$

Q3

Heat ↑ ↑ $T_a = 20°C$

20cm

$T_0 = 42°C$ surface

29cm

(i)

$T/°C$

42

20

t/s
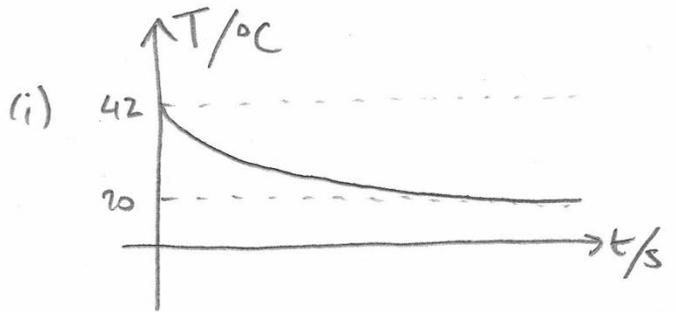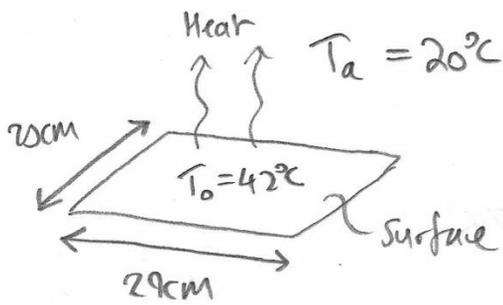
If heat flow via conduction only, and ambient temp. remains at 20°C, expect **Newton's law** of cooling (= Fourier's law of conduction) ⇒ **exponential** decay of temperature with time

T_0

T_a

t
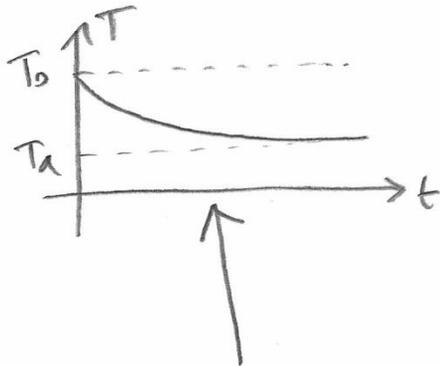
(ii)

$$T = (T_0 - T_a)e^{-kt} + T_a$$

Now

$$\frac{T_0 - T_a}{T - T_a} = e^{kt} \quad \therefore \quad k = \frac{1}{t}\ln\left(\frac{T_0 - T_a}{T - T_a}\right)$$

So

$$k = \frac{1}{123}\ln\left(\frac{42 - 20}{30 - 20}\right) = 6.41 \times 10^{-3} \; s^{-1}$$

$$\left(\text{or} \quad \frac{\ln 2.2}{123}\right)$$

$$\therefore \quad t = \frac{1}{k}\ln\left(\frac{T_0 - T_a}{T - T_a}\right)$$

$$\therefore \quad t = \frac{123}{\ln 2.2}\ln\left(\frac{42 - 20}{22 - 20}\right) = \frac{123 \ln 11}{\ln 2.2} = 374 \, s$$

∴ about 6 mins 14s

(iii)   Max radiative power is $\boxed{P_{max} = \sigma A T_0^4}$

$\therefore P_{max} = 5.67 \times 10^{-8} \times (29 \times 15^2 \times 20 \times 15^{-2}) \times (42 + 273)^4$

$= \boxed{32.4 \text{ W}}$

(iv)   Average power loss due to conduction (if radiation ignored)

$$\boxed{P_{av} \simeq \frac{cm\Delta T}{t}} \simeq \frac{910 \times 0.77 \times (42-30)}{123}$$

$$= \boxed{68.4 \text{ W}}$$

So   radiative power loss   is   not   negligible. (It is at maximum about half the average power loss due to conduction). $\therefore$ expect the cooling to be more rapid particularly at earlier times (since $P_{rad} \propto T^4$).

So perhaps more like 5mins to cool to 22°C?

$$\left[\text{To model we would need to solve numerically} \quad mc \frac{dT}{dt} = -\sigma A T^4 - mck(T-T_a) \right]$$

$\underbrace{\phantom{mc \frac{dT}{dt}}}$
Heat change of Surface

$\underbrace{}$
radiation

$\underbrace{}$
conduction

↖ e.g via Euler method if fixed timestep Δt ...

4 (i) when input power/m^2 = radiated power/m^2

$$1388 = \sigma T^4$$

$$\therefore \quad T = \left(\frac{1388}{5.67 \times 6^{-8}}\right)^{\frac{1}{4}}$$

$$\boxed{T = 396 \, k} \qquad (\text{i} \approx 122.6\,^\circ c)$$

(ii)

$$\boxed{\underbrace{cm\frac{dT}{dt}}_{\substack{\text{Rate of} \\ \text{change of} \\ \text{internal} \\ \text{energy}}} = \underbrace{-\sigma T^4 A}_{\substack{\text{Power loss} \\ \text{due to} \\ \text{radiation}}}}$$

$$\therefore \quad \int_{T_0}^{T} \frac{dT}{T^4} = -\frac{\sigma A}{mc}\int_{0}^{t} dt$$

$$\left[-\frac{1}{3T^3}\right]_{T_0}^{T} = -\frac{\sigma A}{mc}t$$

$$\frac{1}{T_0^3} - \frac{1}{T^3} = -\frac{3\sigma A t}{mc}$$

$$\frac{1}{T_0^3} + \frac{3\sigma A t}{mc} = T^{-3}$$

$$\boxed{T = \left(\frac{1}{T_0^3} + \frac{3\sigma A t}{mc}\right)^{-\frac{1}{3}}}$$

$$\therefore T = \left(\frac{1}{396^3} + \right)^{-\frac{1}{3}}$$

$$\frac{\left(3 + 5.67 \times 5^{-8} + 25 \times \ldots \times 12 + 3600\right)}{1234 \times 900}$$

$$= \boxed{189 \, k}$$

$$(-84.1\,^\circ c)$$

Carnot Cycle P–V diagram with points ①②③④, T_H, T_C, Q_{in}, Q_{out}, V_1, V_2.
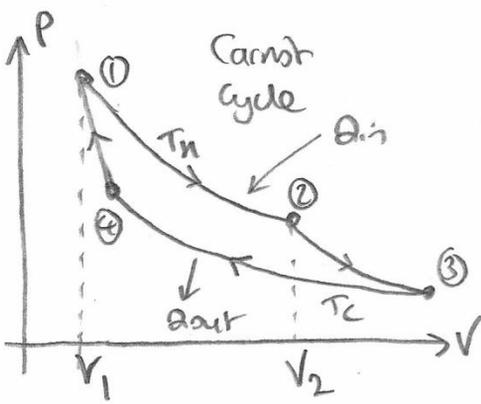
(i)

$$\boxed{M = M \, n}$$

mass of gas | molar mass | # moles

$$\therefore n = \frac{M}{M}$$

$$= \frac{1000 g}{28.97 \, g/mol}$$

$$= \boxed{34.15 \text{ moles}}$$

(ii) ① → ② $\boxed{\text{Isothermal expansion}}$

i.e gas takes on heat Q_{in} from hot reservoir (@ $T_H = 100°C$) and expands from V_1 to V_2 at constant temperature T_H. i.e internal energy $U = $ constant since $U = U(T)$. All of heat converted to work *

[* what about the 2nd law! well over the whole cycle
$\Delta S_{TOT} \geq 0$, and in isothermal stage
$\Delta S_{TOT} = 0$ i.e reservoir loses same entropy that gas gains]

in fact since Carnot $\eta = 1 - T_C/T_H \Rightarrow \boxed{\Delta S_{TOT} = 0}$

② → ③ $\boxed{\text{Adiabatic (Isentropic) expansion}}$

constant entropy change i.e no heat exchange.
change in internal energy of gas = work done on it.

③ → ④ $\boxed{\text{Isothermal Compression}}$ Gas loses Q_{out}

to a cold reservoir $T_C = 25°C$, and shrinks from V_3 to V_4.
Gas temp. is T_C, and internal energy constant.

④ → ① $\boxed{\text{Adiabatic compression}}$

compression from $V_4 \to V_1$ (i.e returns to original
(P_1, V_1) without heat exchange.   i.e work done on gas
                        = internal energy change.

(iii)   1st law $dU = dQ - pdV$

[Work done __on__ gas is $-pdV = dW$]

a)

gas
volume
being compressed

If dx
is -ve

$dW = pA \times -dx$
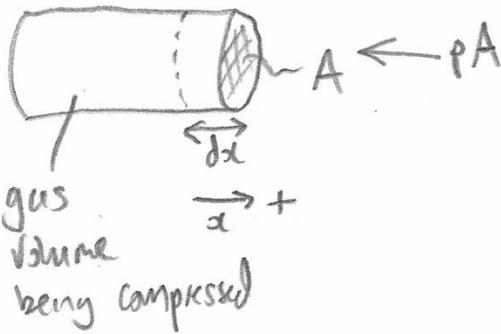
force   -displacement
          (compression)

Now $dV = dx\, A$

$\therefore \boxed{dW = -pdV}$

$\therefore$ Around cycle   ① → ② → ③ → ④ → ①

$$\oint dU = \oint dQ - \oint pdV$$

Since start at
T_H, end at T_H → $0 = Q_{in} - Q_{out} - \oint pdV$

$\therefore$ $\oint pdV = Q_{in} - Q_{out}$

Now

$W = \oint pdV$

$\oint pdV$ is area enclosed by p,v curve
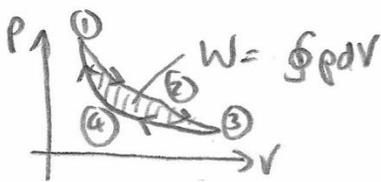
and $W = Q_{in} - Q_{out}$

$\therefore$ Work done __by__ gas

$$\boxed{W = \oint pdV = \text{area enclosed}}$$

b) $dQ = TdS$ $\therefore$ $\oint dU = \oint TdS - \oint pdV$

$\Rightarrow$ $\oint TdS = \oint pdV$ $\therefore$ $\boxed{\oint TdS = W}$

So
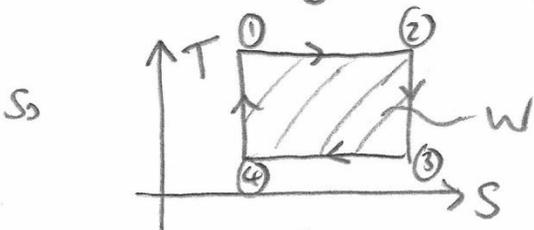
So area enclosed by T,S
graph is also W.

(iv) Now in isentropic stages ②→③ and ④→①
$\Delta S = 0$ for the gas ie vertical lines in T,S graph

Hence:
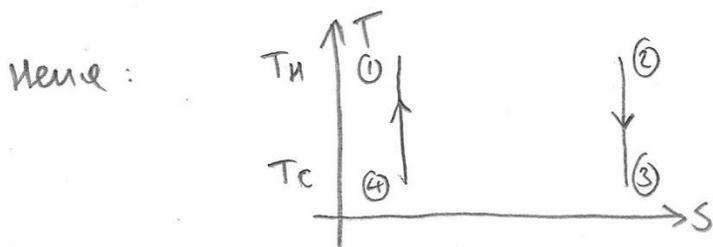
Since isothermal ①→② and ③→④

∴ T vs S must be a rectangle.
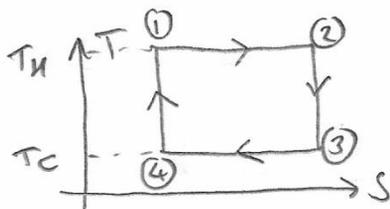
(v) | Isothermal change | $dU = 0$ ∴ 1st law: $0 = dQ - pdV$ ← dU

∴ $dQ = pdV$ Now | $pV = nRT$ | ∴ $p = \dfrac{nRT}{V}$

∴ $\int dQ = nRT \int_{V_1}^{V_2} \dfrac{dV}{V}$ ↑ ideal gas at constant temp T

$$\boxed{Q = nRT \ln\left(\dfrac{V_2}{V_1}\right)}$$

(vi) So $\Delta S_{12} = \dfrac{Q_{in}}{T_H} = nR \ln\left(\dfrac{V_2}{V_1}\right)$

$V_2 = 3V_1$ ∴ $\Delta S_{12} = nR \ln 3$

$\Delta S_{12} \approx 34.5 \times 8.314 \times \ln 3$

$\approx \boxed{315 \text{ J/k}}$

(This tallys with graph)

| NOTE THIS IS ALSO $-\Delta S_{34}$ | $= -\dfrac{Q_{out}}{T_C}$
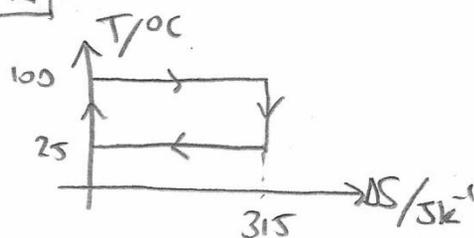
(vii) $Q_{in} = nRT_H \ln(V_2/V_1)$

$= 34.5 \times 8.314 \times (100+273) \times \ln 3$

$= \boxed{117.6 \text{ kJ}}$ (117.5 if use rounded n)

From T,S graph enclosed area:

$W = (T_H - T_C)\Delta S_{12}$

$\therefore \boxed{W = (T_H - T_C) nR \ln V_2/V_1}$

$W = 75 \times 34.5 \times 8.314 \times \ln 3$

$= \boxed{23.6 \text{ kJ}}$

$\therefore \eta = \dfrac{W}{Q_{in}} = \dfrac{23.6}{117.6} = \boxed{20.1\%}$

[If you do it directly from P,V graph

$W = Q_{in} - Q_{out}$

$Q_{out} = nRT_C \ln V_3/V_4$ (iso thermal)

Now $P_2 V_2^{5/3} = P_3 V_3^{5/3}$ } Isentropic stages

$P_1 V_1^{5/3} = P_4 V_4^{5/3}$

And $P_2 V_2 = nRT_H$ So $P_2 V_2^{5/3} = P_3 V_3^{5/3}$

$P_3 V_3 = nRT_C$ $\Rightarrow \dfrac{nRT_H}{V_2} V_2^{5/3} = \dfrac{nRT_C}{V_3} V_3^{5/3}$

$P_4 V_4 = nRT_C$ $\dfrac{T_H}{T_C} V_2^{2/3} = V_3^{2/3}$

$\boxed{V_2 \left(\dfrac{T_H}{T_C}\right)^{3/2} = V_3}$

Similarly:
$$P_4 V_4^{5/3} = P_1 V_1^{5/3}$$
$$nRT_C V_4^{2/3} = nRT_M V_1^{2/3}$$
$$\Rightarrow \quad \boxed{V_4 = V_1 \left(\frac{T_H}{T_C}\right)^{3/2}}$$

So $\boxed{\dfrac{V_3}{V_4} = \dfrac{V_2}{V_1}}$

$\therefore \quad Q_{out} = nRT_C \ln \frac{V_3}{V_4}$

$Q_{out} = nRT_C \ln \frac{V_2}{V_1}$

$\therefore \quad \eta = \dfrac{W}{Q_{in}} = 1 - \dfrac{Q_{out}}{Q_{in}}$

$$= 1 - \frac{nRT_C \ln \frac{V_2}{V_1}}{nRT_H \ln \frac{V_2}{V_1}}$$

$$= 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{25 + 273}{100 + 273}$$

$$= \boxed{20.1\%} \qquad]$$

(viii) $\eta = 1 - \dfrac{T_C}{T_H} = 1 - \dfrac{25 + 273}{100 + 273} = \boxed{20.1\%}$

This is the **maximum** efficiency of a heat engine. Since this equals the answer to (viii) (the efficiency of a carnot engine) $\Rightarrow$ Carnot engine is optimally efficient ie 'as efficient as possible'.