Mandlebrot transformations of **complex numbers**

$$i^2 = -1$$

$$z = x + iy$$

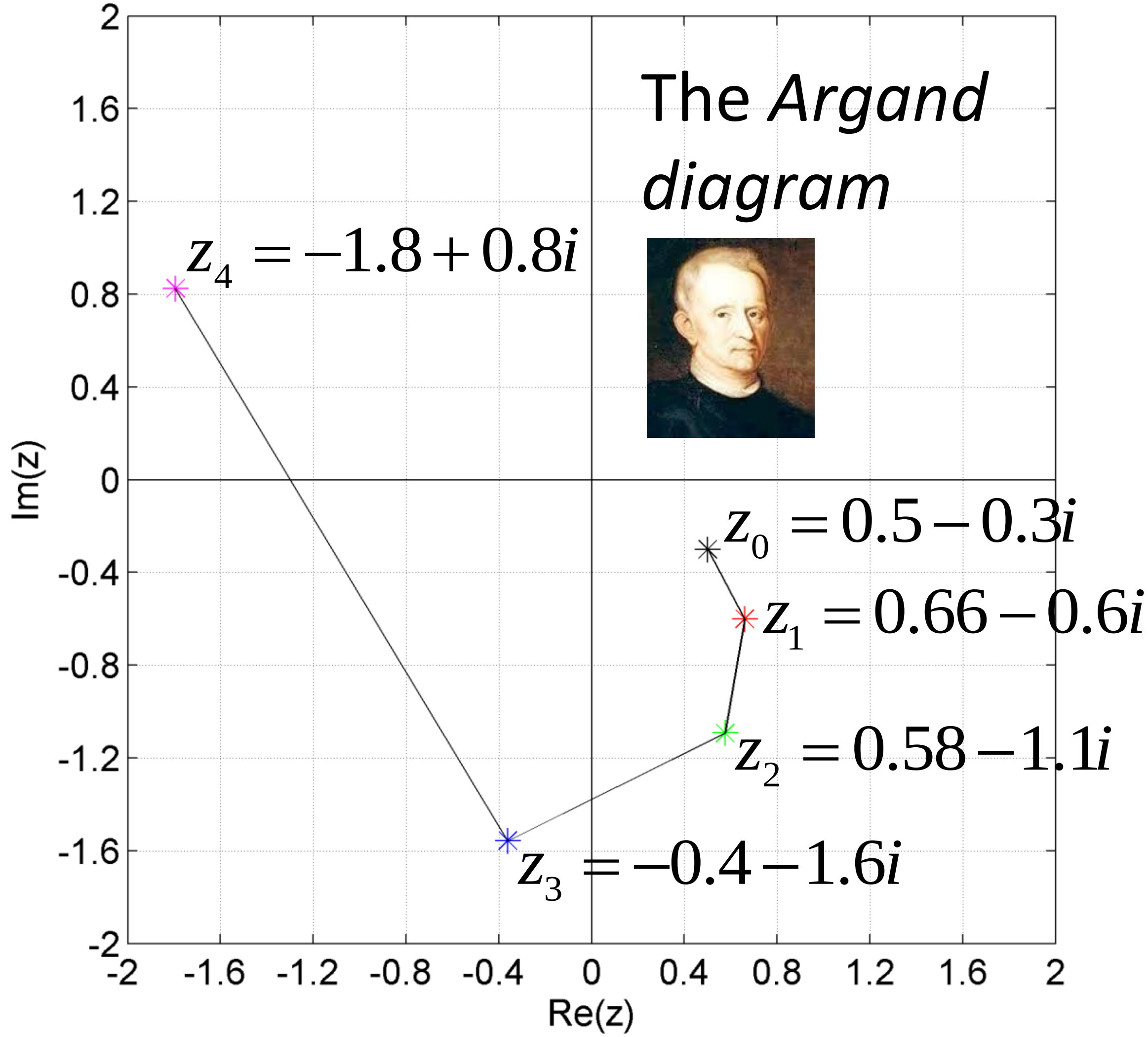
$$x = \mathrm{Re}(z)$$

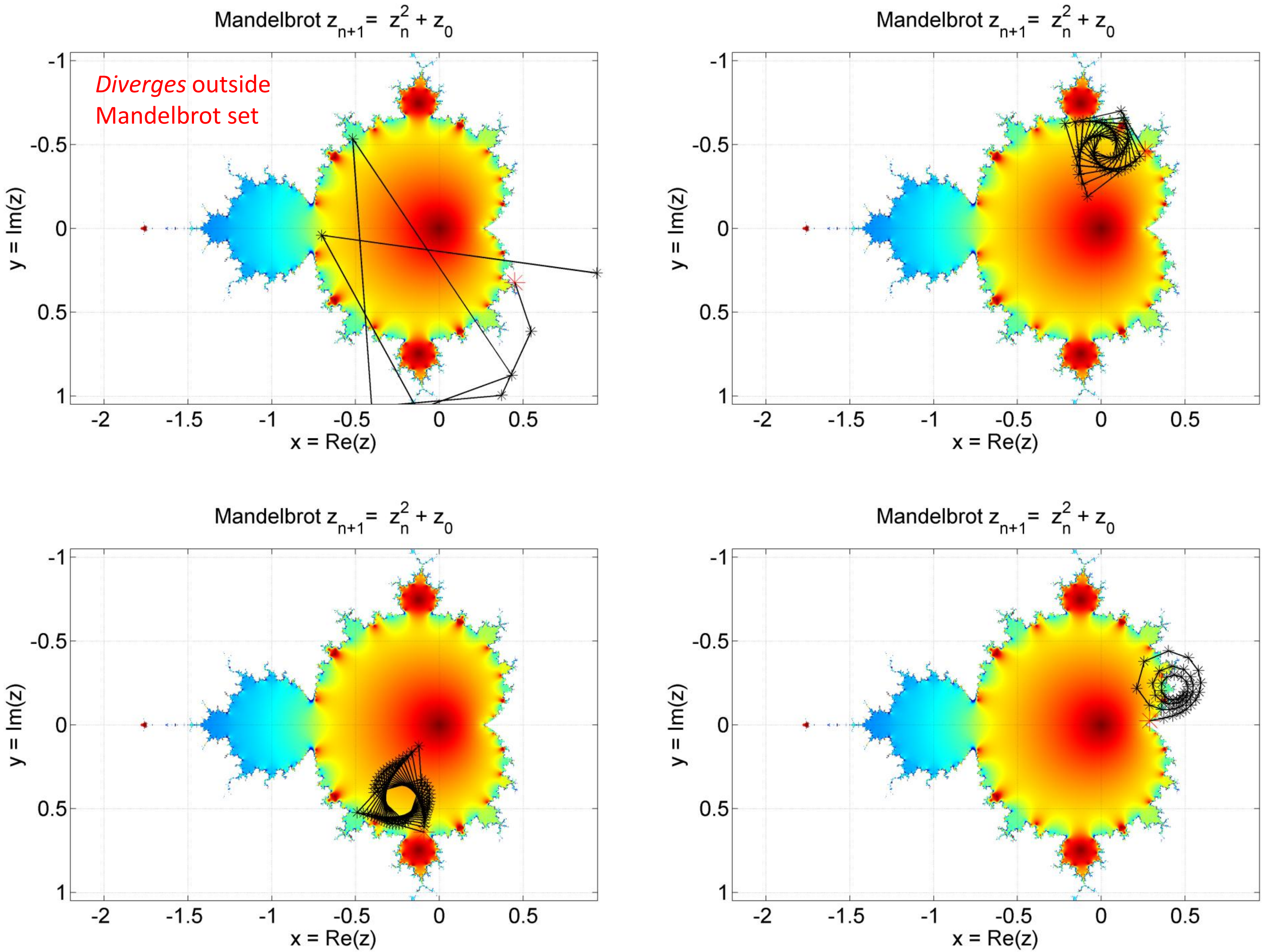
$$y = \mathrm{Im}(z)$$

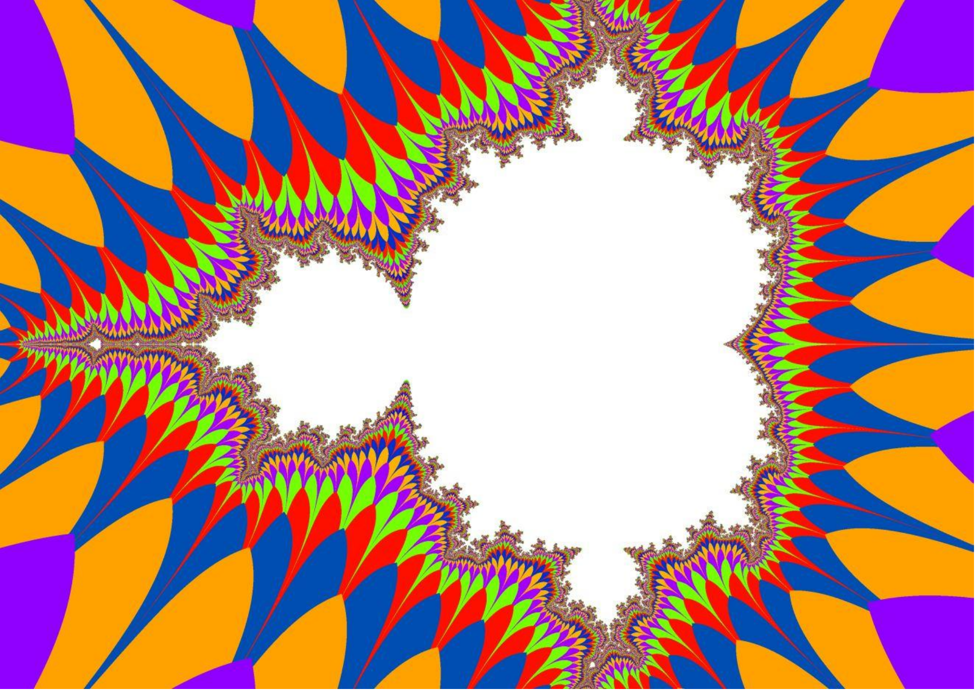
$$|z| = \sqrt{x^2 + y^2}$$

$$(1+i)(1+i)$$
$$= 1 + 2i + i^2$$
$$= 1 + 2i - 1$$
$$= 2i$$

$$z_{n+1} = z_n^2 + z_0$$



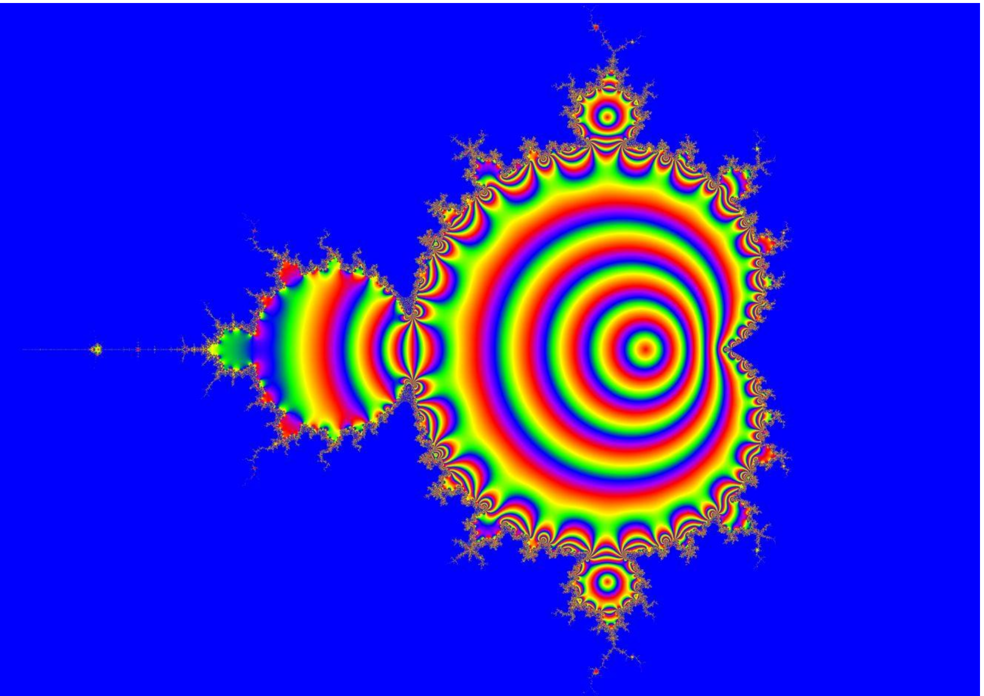

Mandelbrot $z_{n+1} = z_n^2 + z_0$
Diverges outside
Mandelbrot set
y = Im(z)
x = Re(z)
-1
-0.5
0
0.5
1
-2
-1.5
-1
-0.5
0
0.5
Mandelbrot $z_{n+1} = z_n^2 + z_0$
y = Im(z)
x = Re(z)
Mandelbrot $z_{n+1} = z_n^2 + z_0$
y = Im(z)
x = Re(z)
Mandelbrot $z_{n+1} = z_n^2 + z_0$
y = Im(z)
x = Re(z)



`julia.m` plot option `abs diverge`

Plot a surface with height $h(x,y)$. This is the *iteration number* when $|z|$ exceeds a certain value e.g. 4

In this case *colours* indicate height $h(x,y)$. It is a 'colour-map'.



`julia.m` plot option `plot z`

Plot a surface with height $h(x,y)$

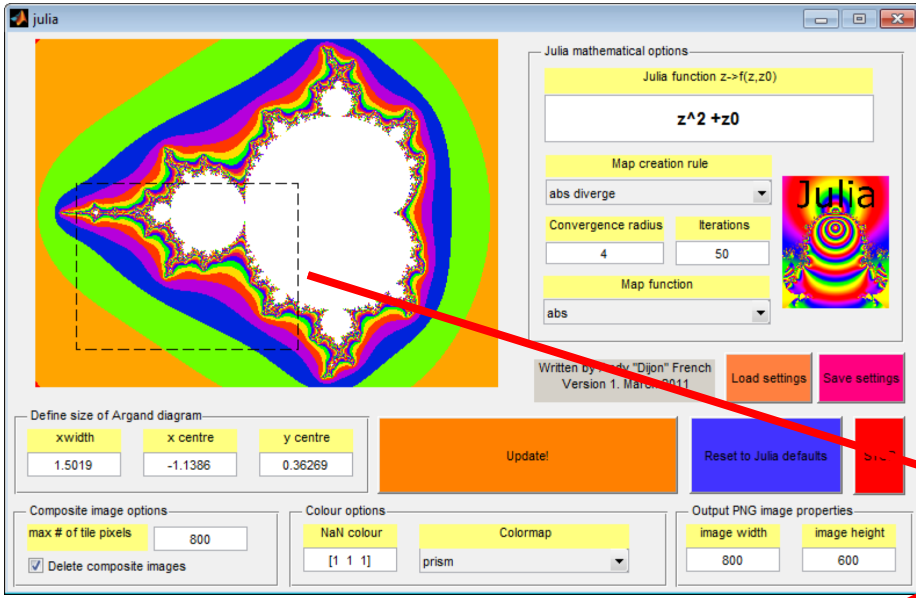
$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

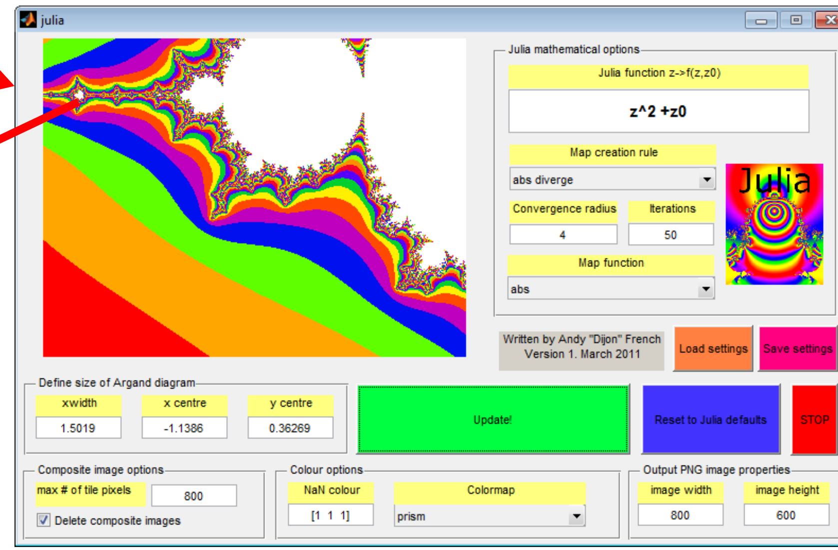
$$h(x,y) = e^{-\sqrt{x^2+y^2}}$$

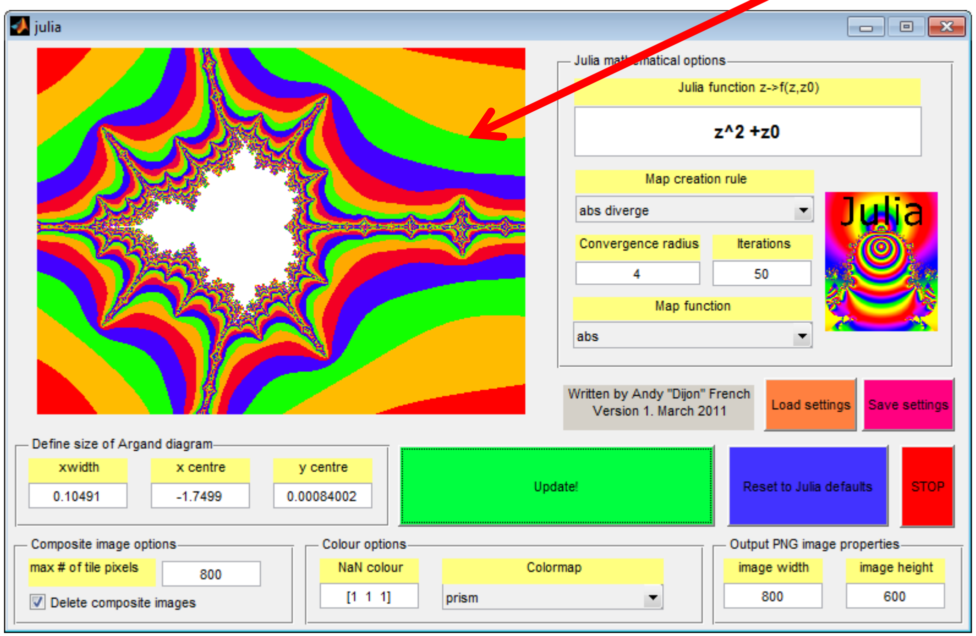
Mandlebrot, complex numbers and iteration

The *Mandlebrot Set* has infinite complexity!

... But a recursive *fractal* geometry

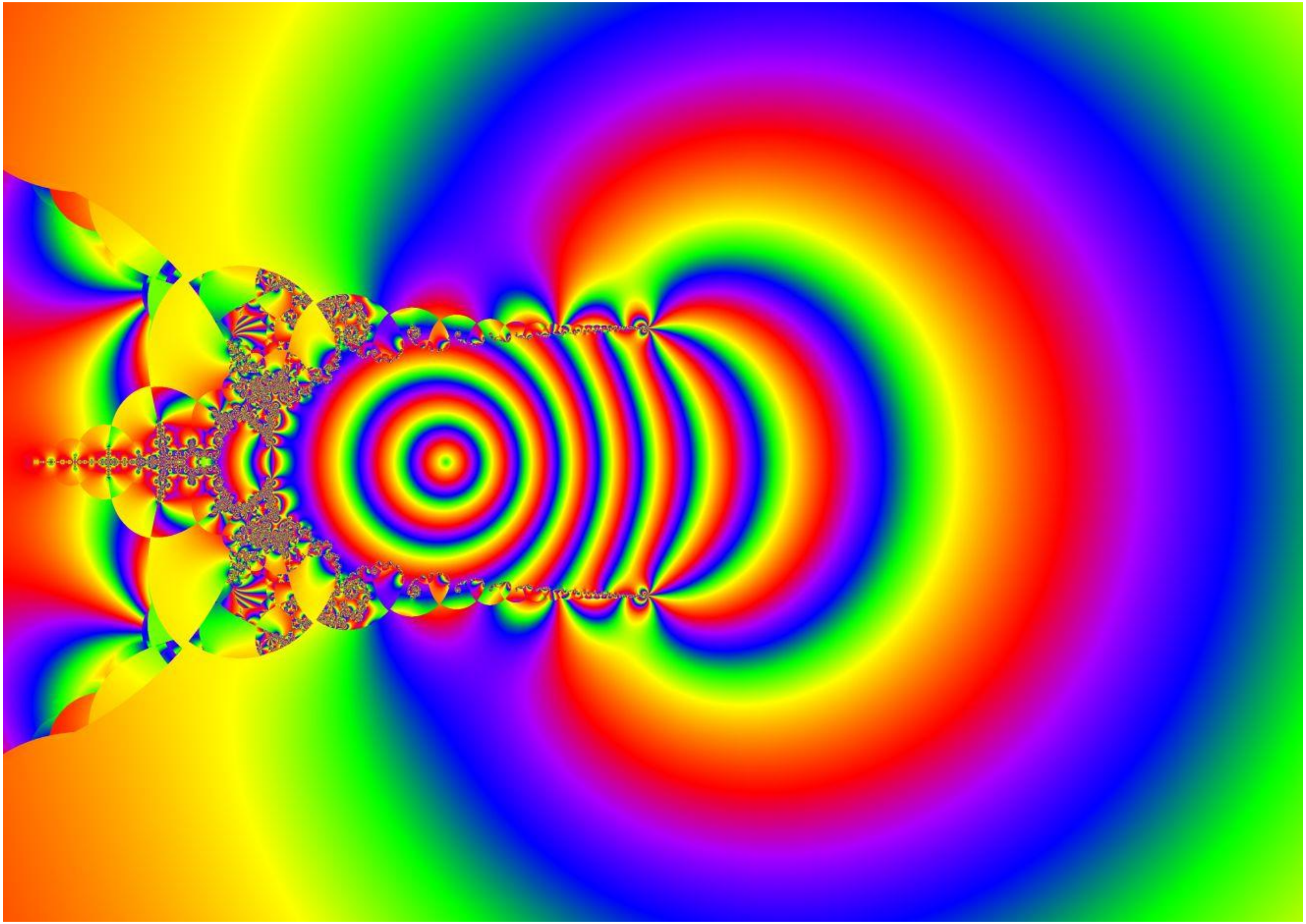






Benoit Mandlebrot (1924-2010)

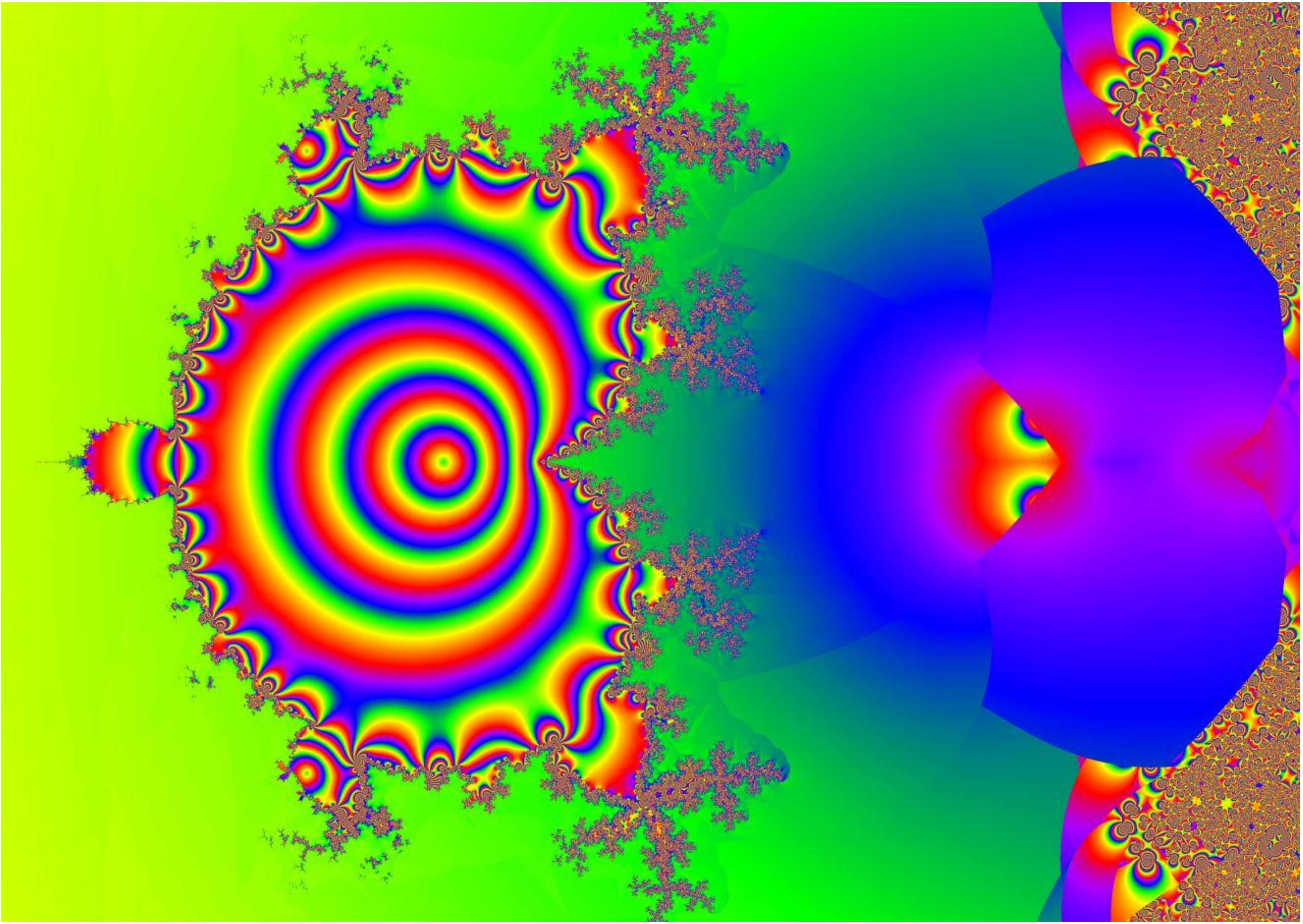
The Mandlebrot Variations



The light bulb

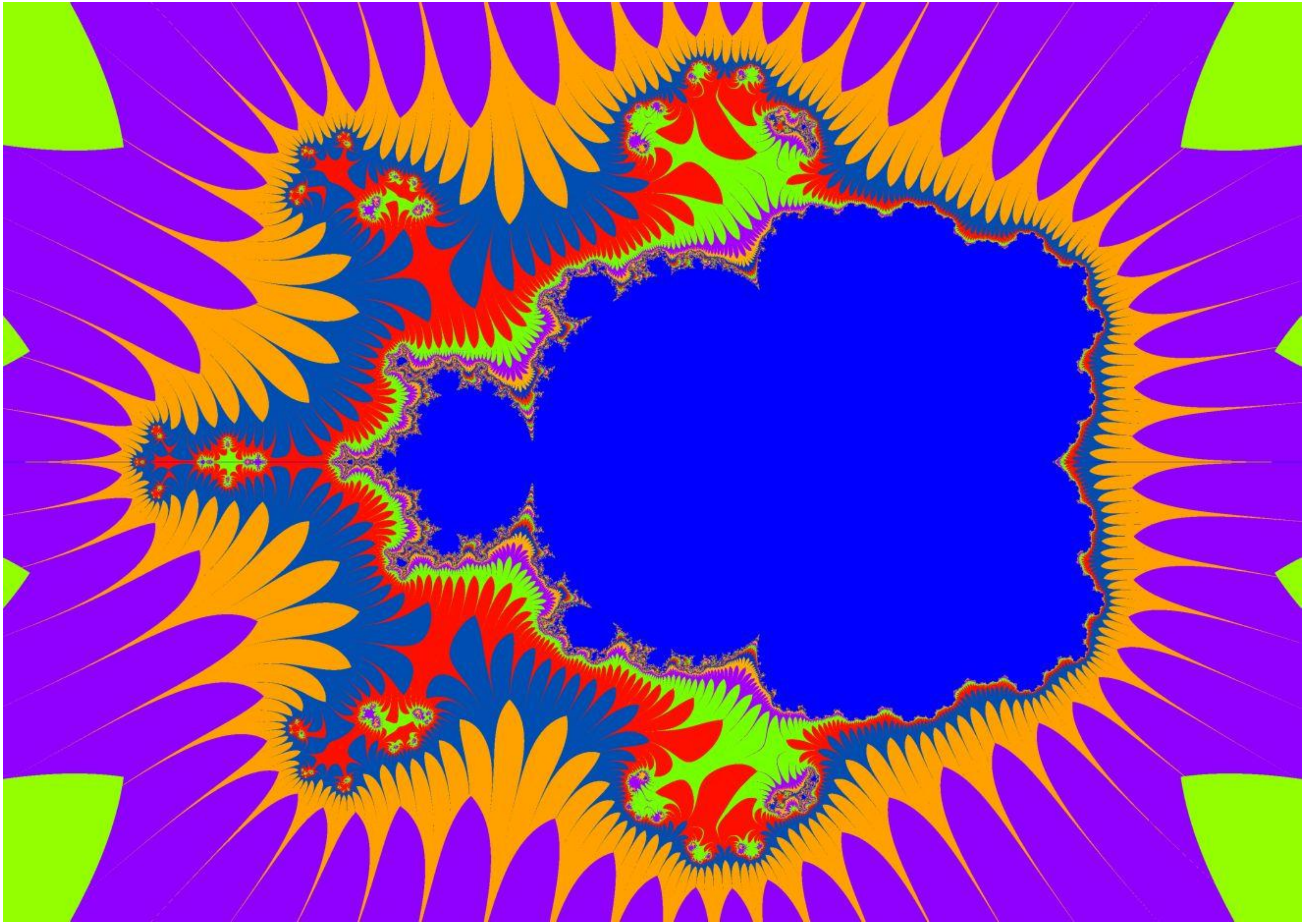
$$z_{n+1} = \log\left(z_n^2 + z_0\right)$$

7 steps to enlightenment $$z_{n+1} = \tan^{-1}\left(z_n^2 + z_0\right)$$



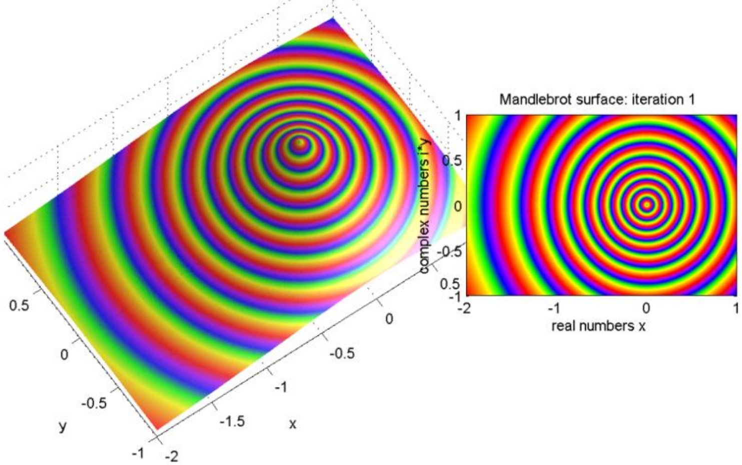
The Mandlerocket!

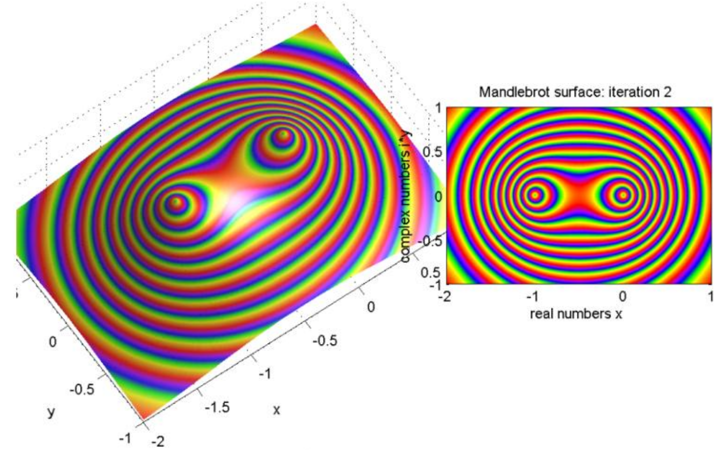
$$z_{n+1} = \sin^{-1}\left(z_n^2 + z_0\right)$$

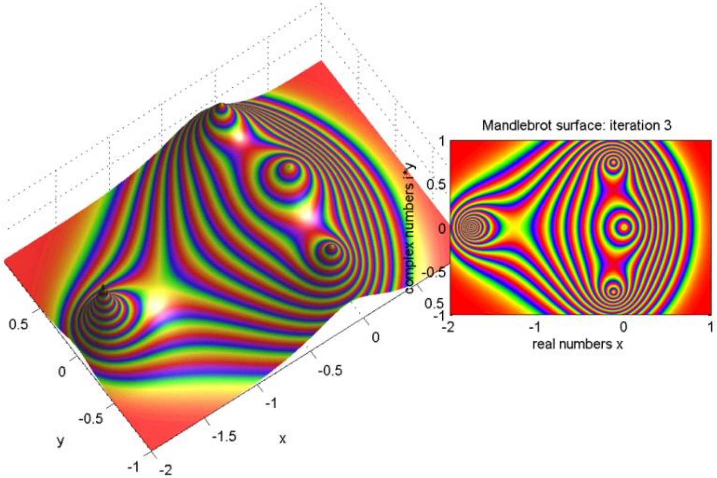


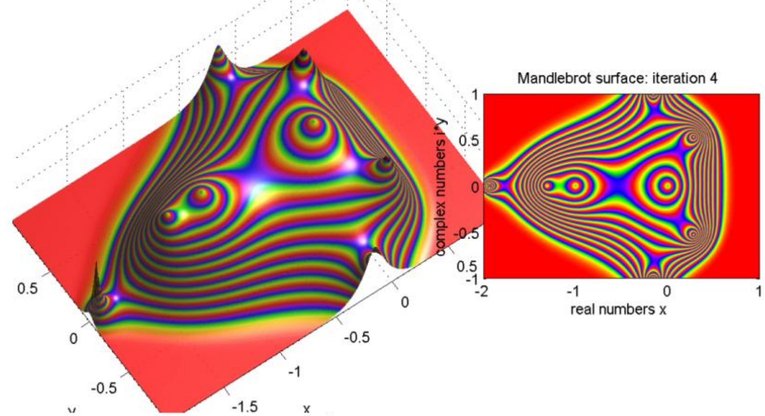
Micro mandlebeast

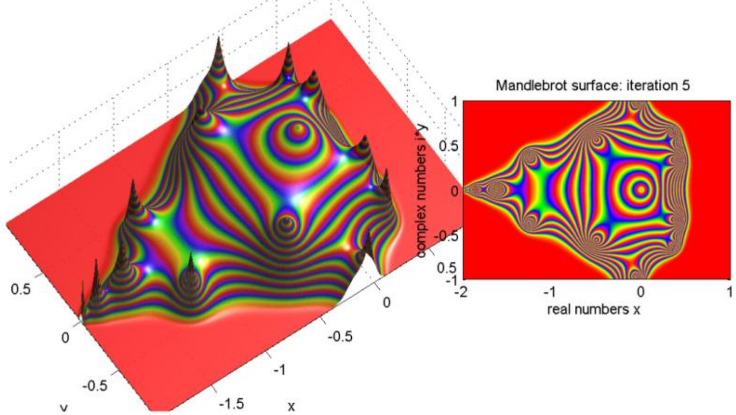
$$z_{n+1} = \left(z_n^2 + z_0 \right)^2$$

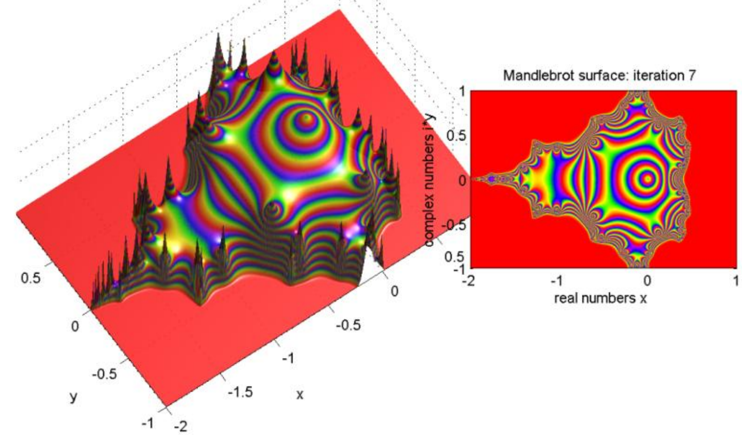

Mandlebrot surface: iteration 1
complex numbers i*y
real numbers x
x
y

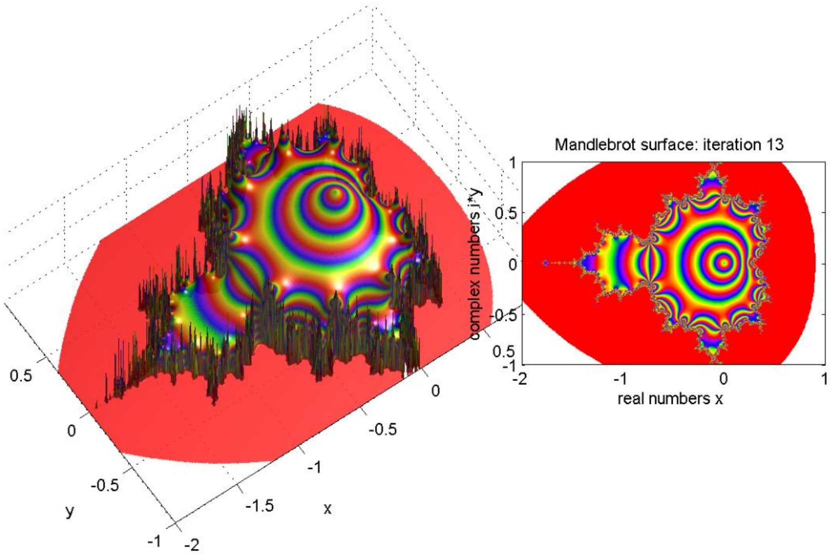

Mandlebrot surface: iteration 2
complex numbers i*y
real numbers x
x
y

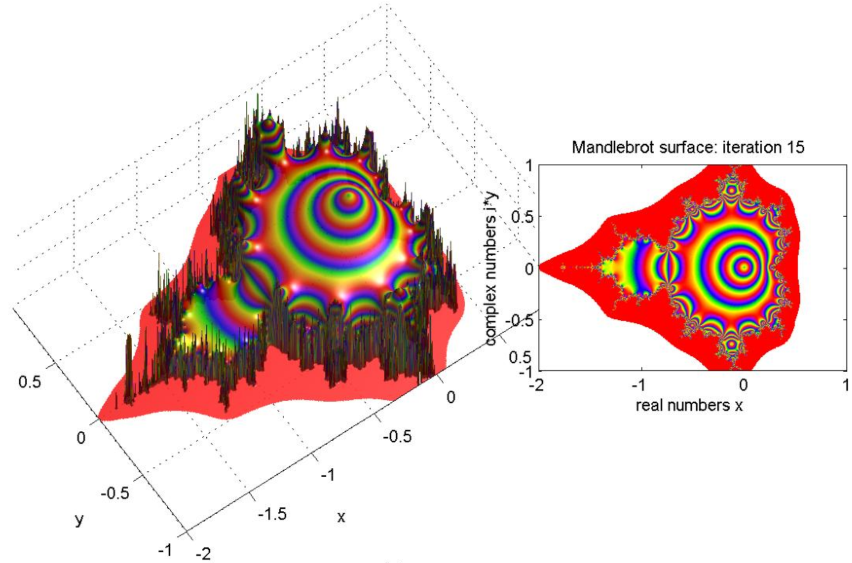

Mandlebrot surface: iteration 3
complex numbers i*y
real numbers x
x
y

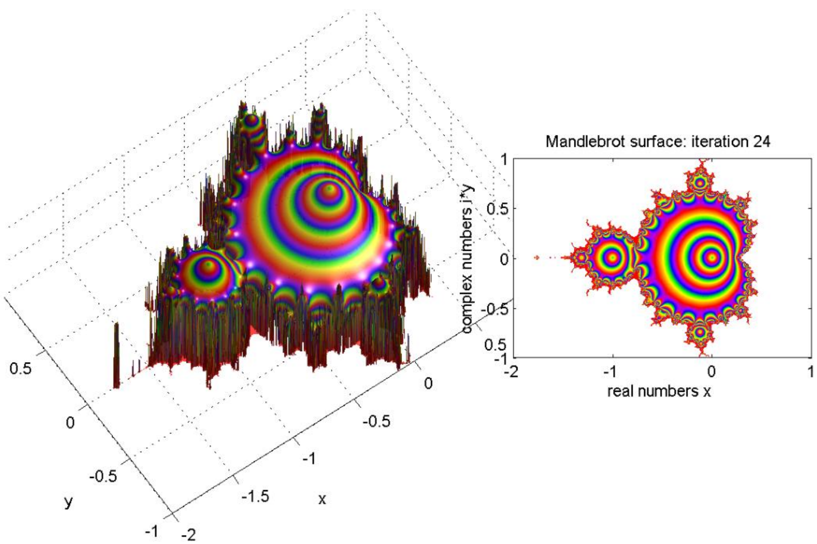

Mandlebrot surface: iteration 4
complex numbers i*y
real numbers x
x

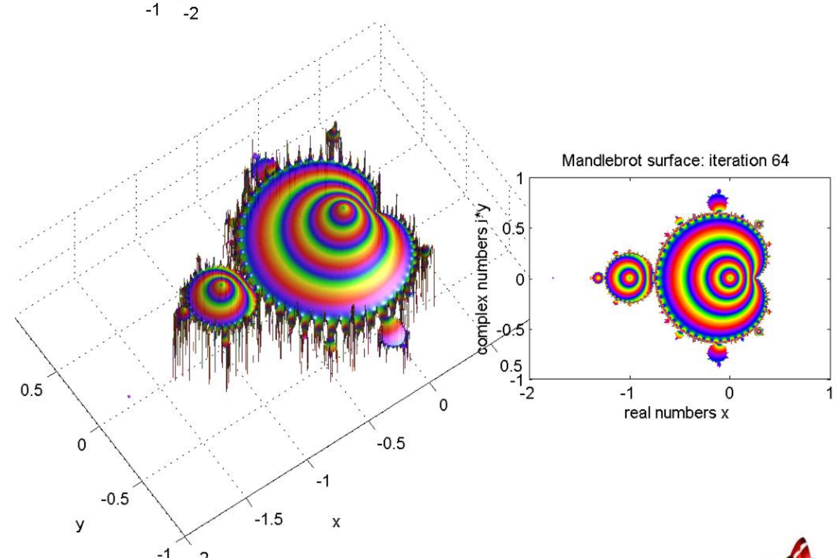

Mandlebrot surface: iteration 5
complex numbers i*y
real numbers x
x


Mandlebrot surface: iteration 7
complex numbers i*y
real numbers x
x
y





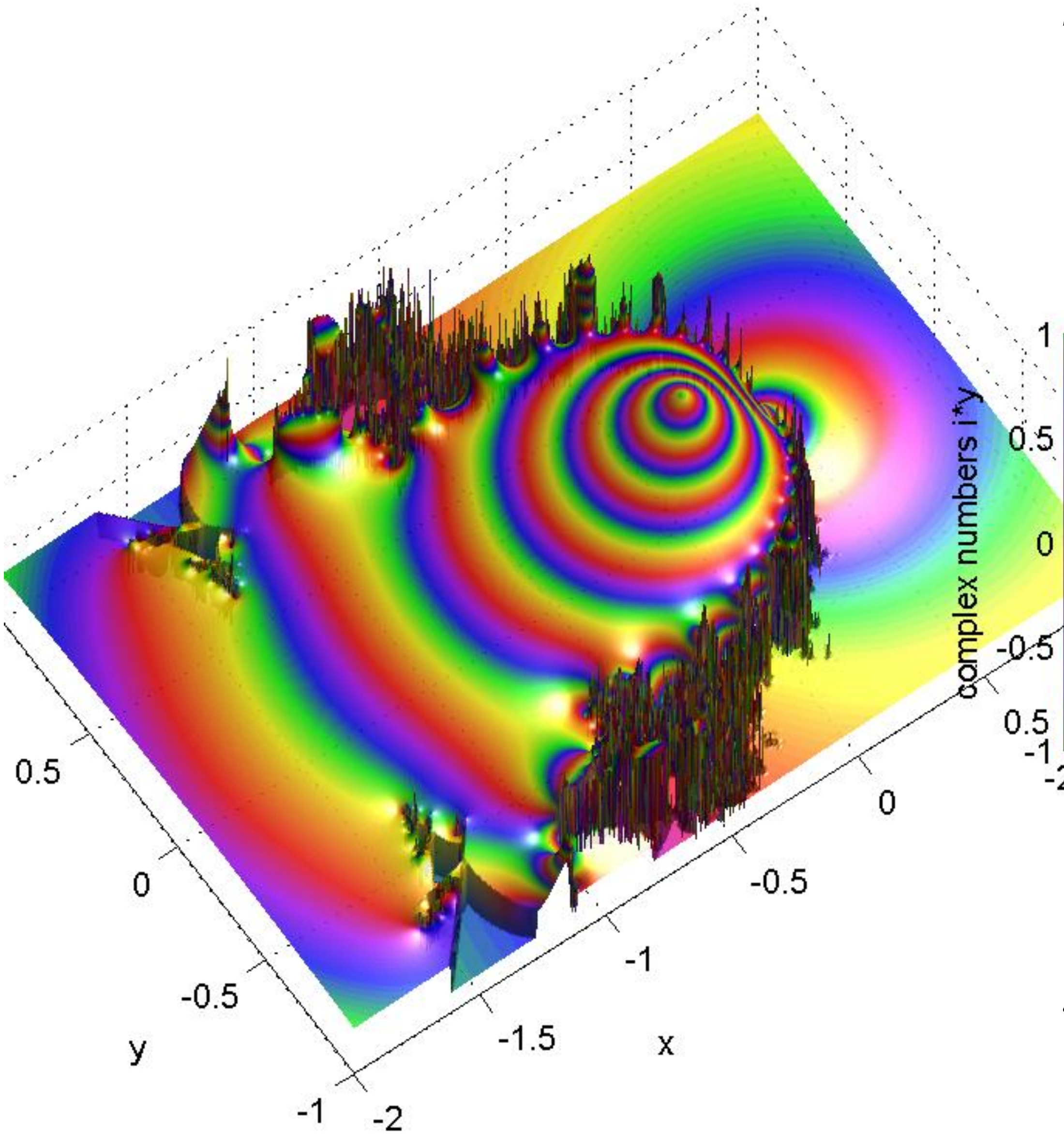


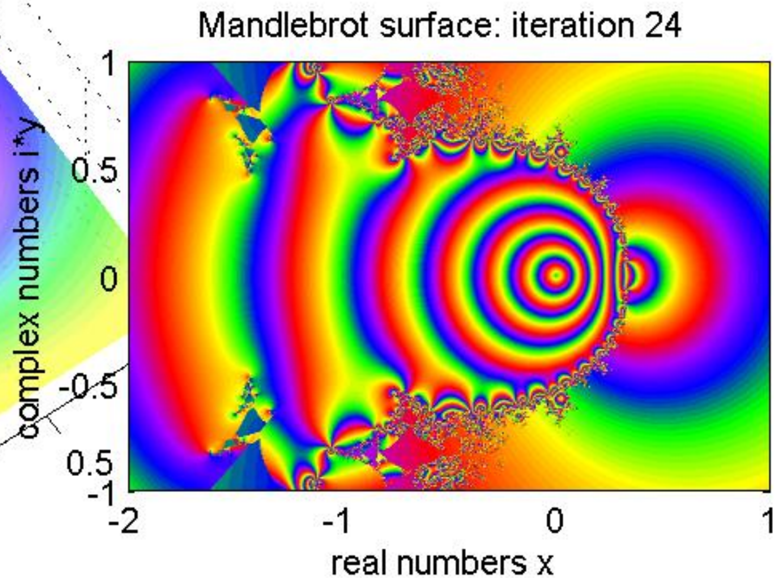


Selection from *Day of Julia.*
Mathematicon Exhibition, 2014

7 steps to enlightenment

$$z_{n+1} = \tan^{-1}\left(z_n^2 + z_0\right)$$



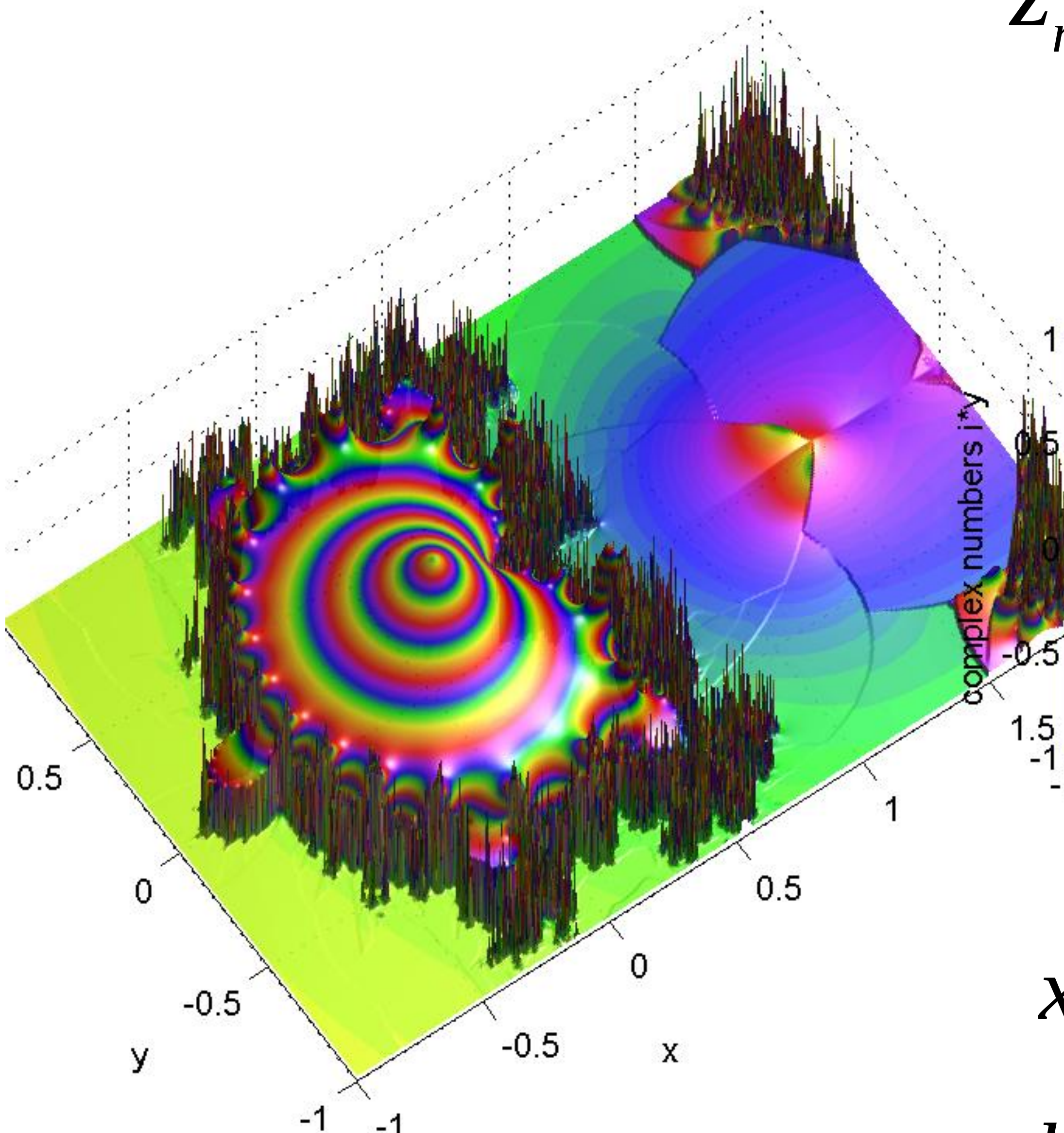


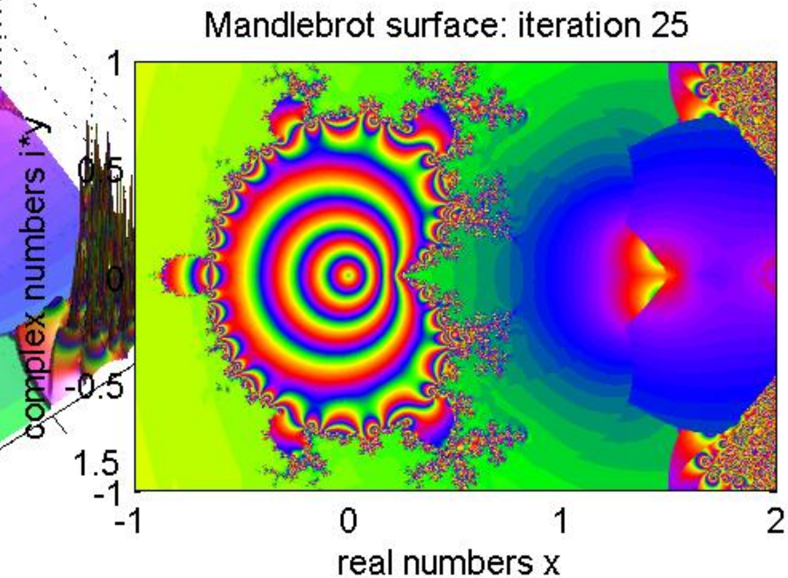
$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x,y) = e^{-\sqrt{x^2+y^2}}$$

The Mandlerocket

$$z_{n+1} = \sin^{-1}\left(z_n^2 + z_0\right)$$





$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x, y) = e^{-\sqrt{x^2 + y^2}}$$

$$e^{i\pi} = -1$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$