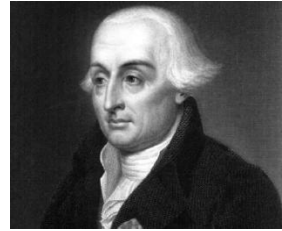


**Lagrange points** are where the gravitational potential energy (*in a rotating frame of reference where two orbiting bodies are stationary*) is a **local minimum, maximum or a saddle point**. These are places where objects are in 'gravitational balance' i.e. a good place for a satellite. Note the balance is *unstable* at a local maxima.

$$\Omega = \frac{\sqrt{G(M_1 + M_2)}}{a^{\frac{3}{2}}} \quad \text{Orbital angular speed from Kepler III}$$



Joseph Lagrange  
1736-1813

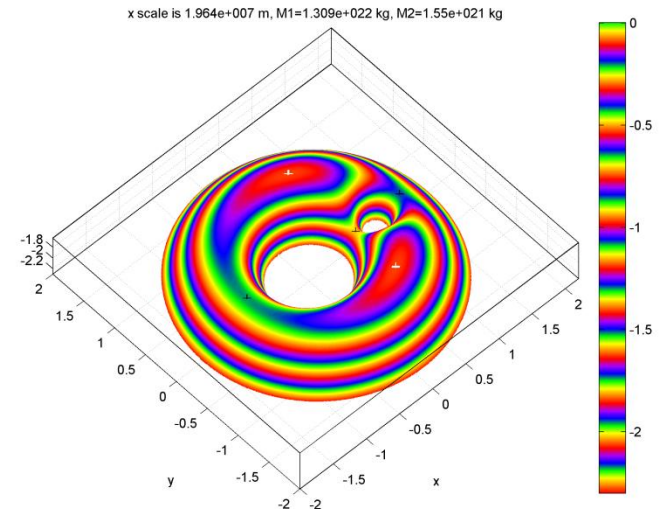
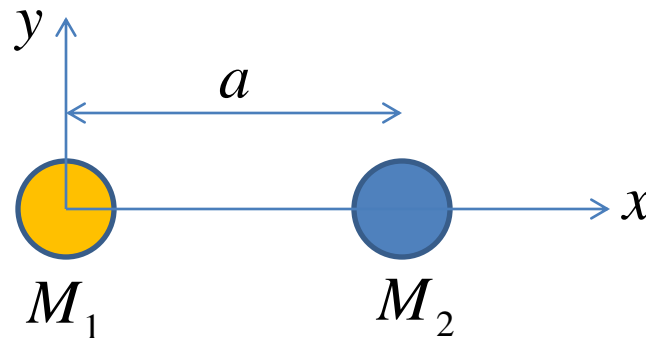
Potential energy at location  $x, y$  from  $M_1$

$$\phi = -\frac{GM_1}{\sqrt{x^2 + y^2}} - \frac{GM_2}{\sqrt{(x-a)^2 + y^2}} - \frac{1}{2}\Omega^2 \left( \left( x - \frac{a}{1 + \frac{M_1}{M_2}} \right)^2 + y^2 \right)$$

$$\mathbf{g} = -\nabla \phi$$

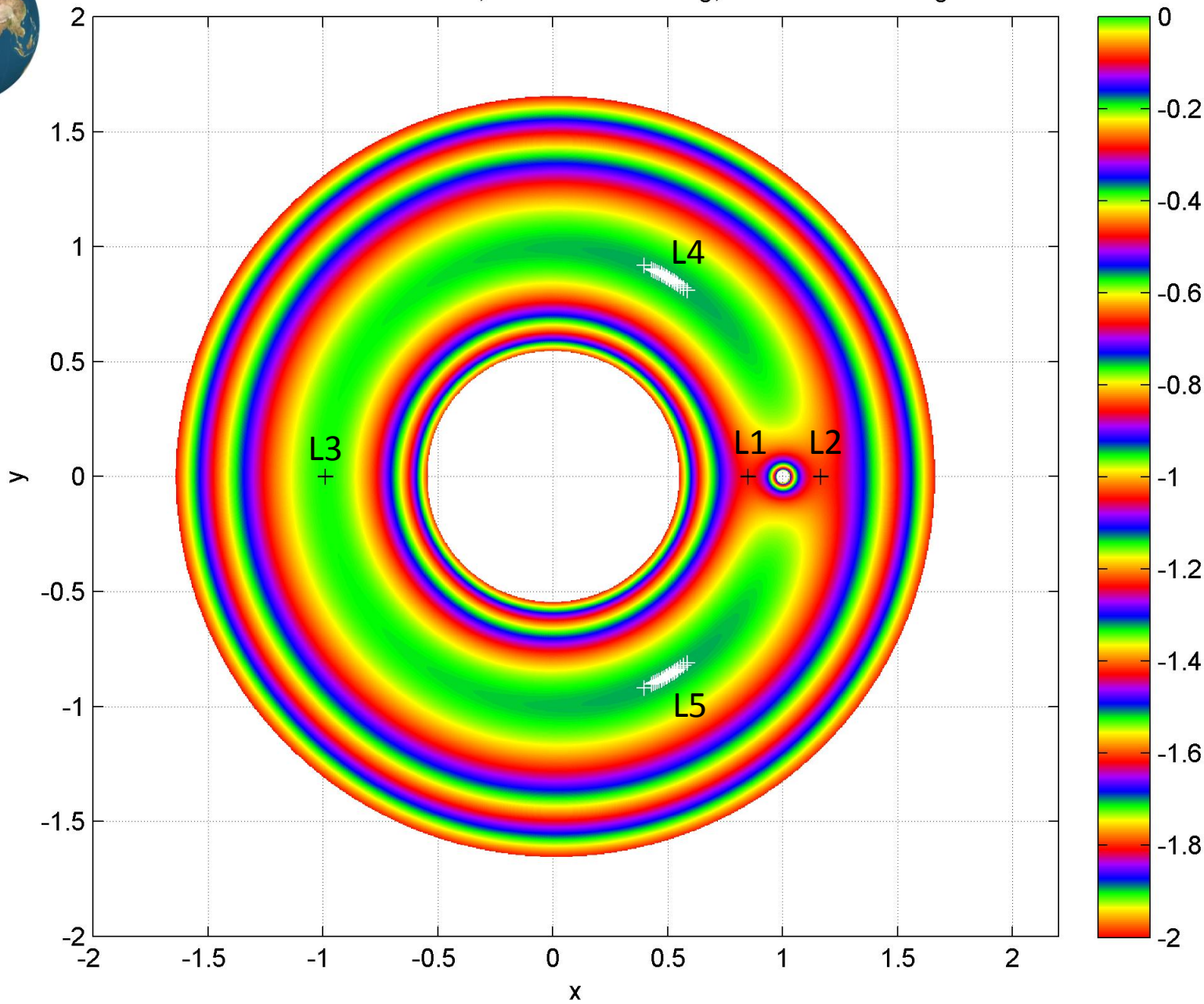
$$\phi_{scale} = \frac{GM_1}{a}$$

$$x_{scale} = a$$





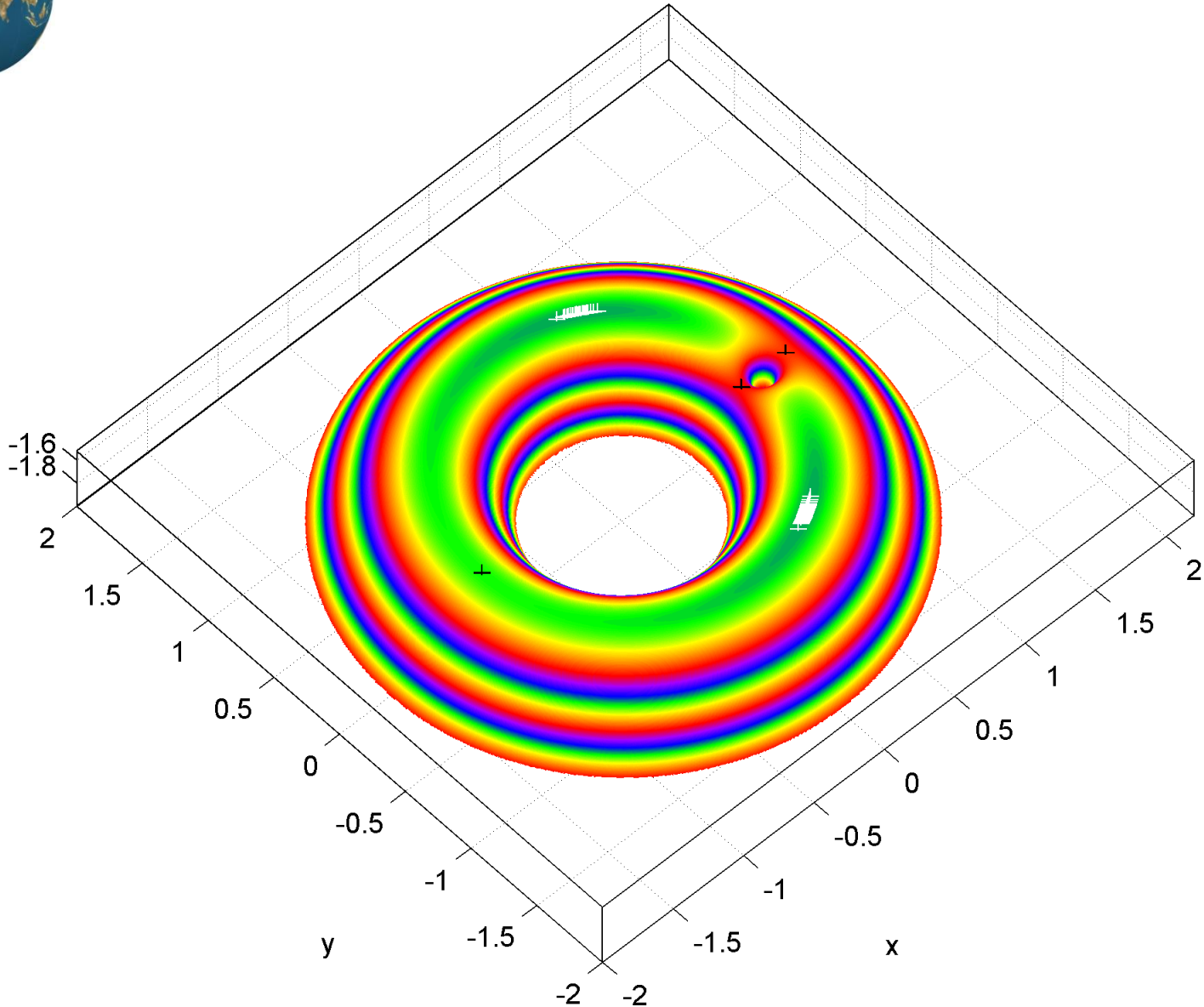
x scale is  $3.844\text{e}+008$  m,  $M1=5.972\text{e}+024$  kg,  $M2=7.348\text{e}+022$  kg



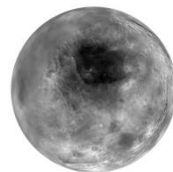
Earth & Moon



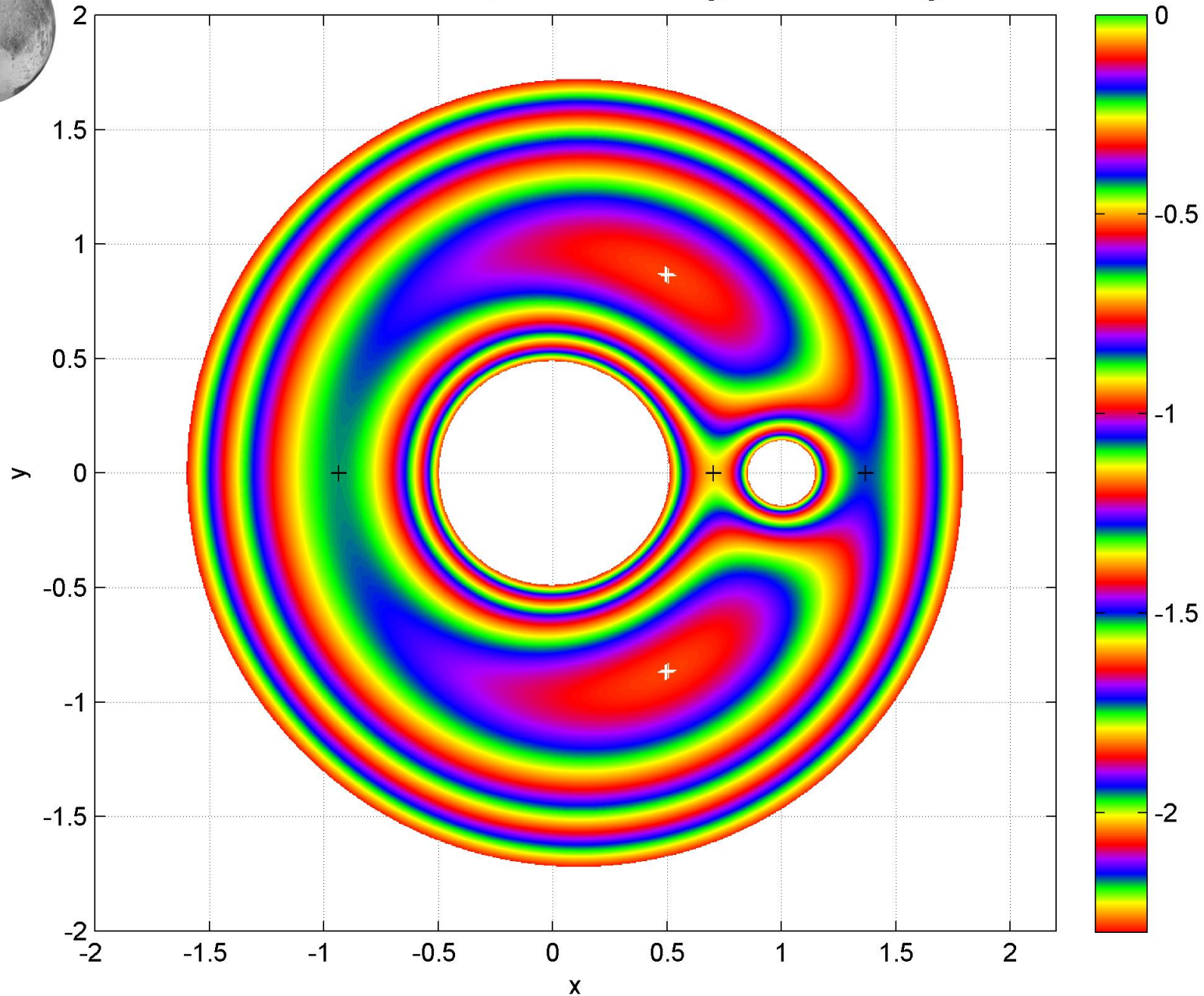
x scale is 3.844e+008 m, M1=5.972e+024 kg, M2=7.348e+022 kg



Earth & Moon



x scale is  $1.964 \times 10^7$  m,  $M_1 = 1.309 \times 10^{22}$  kg,  $M_2 = 1.55 \times 10^{21}$  kg

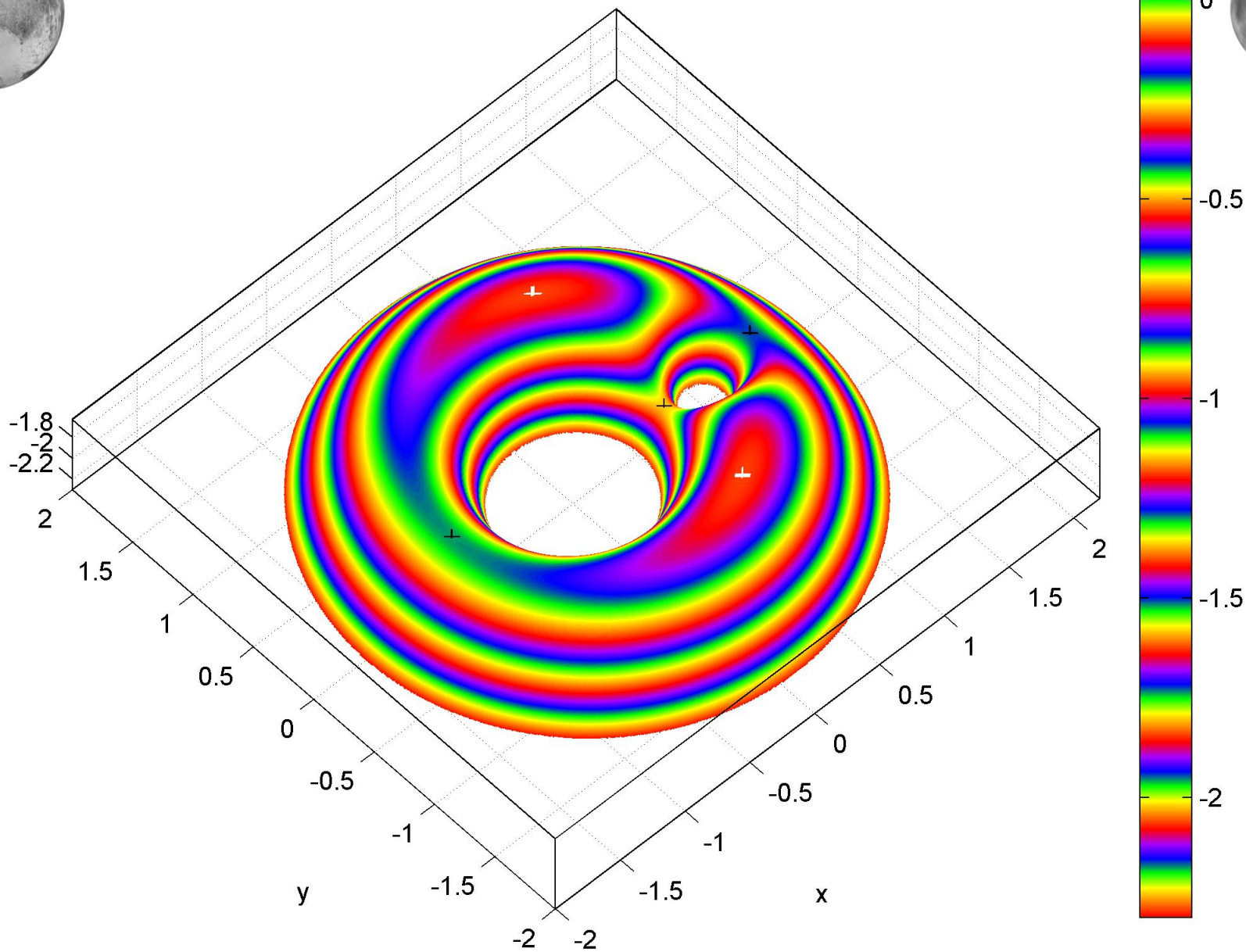
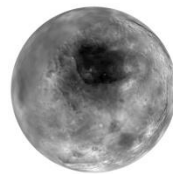


**Pluto & Charon**





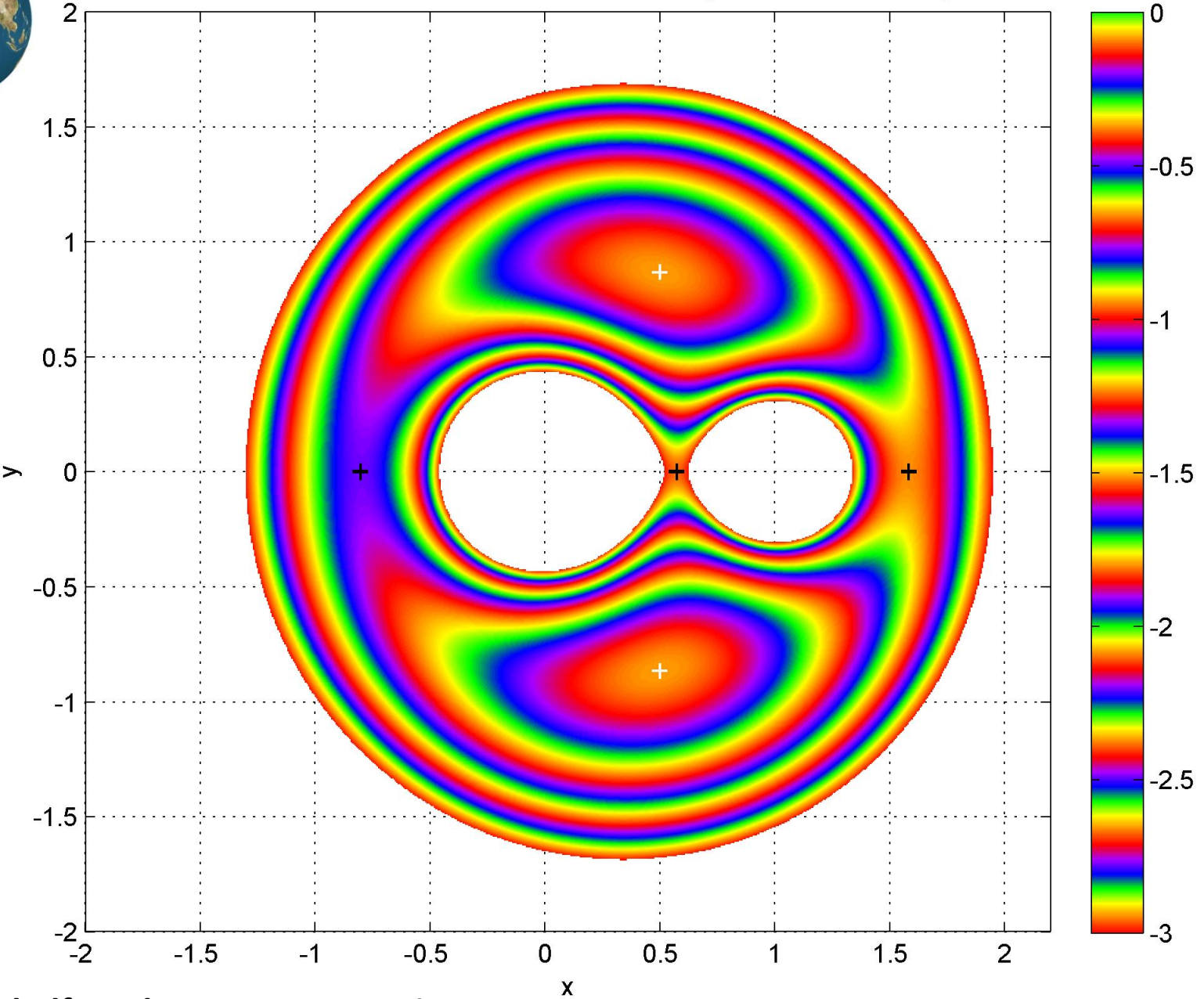
x scale is  $1.964\text{e}+007$  m,  $M_1=1.309\text{e}+022$  kg,  $M_2=1.55\text{e}+021$  kg



**Pluto & Charon**



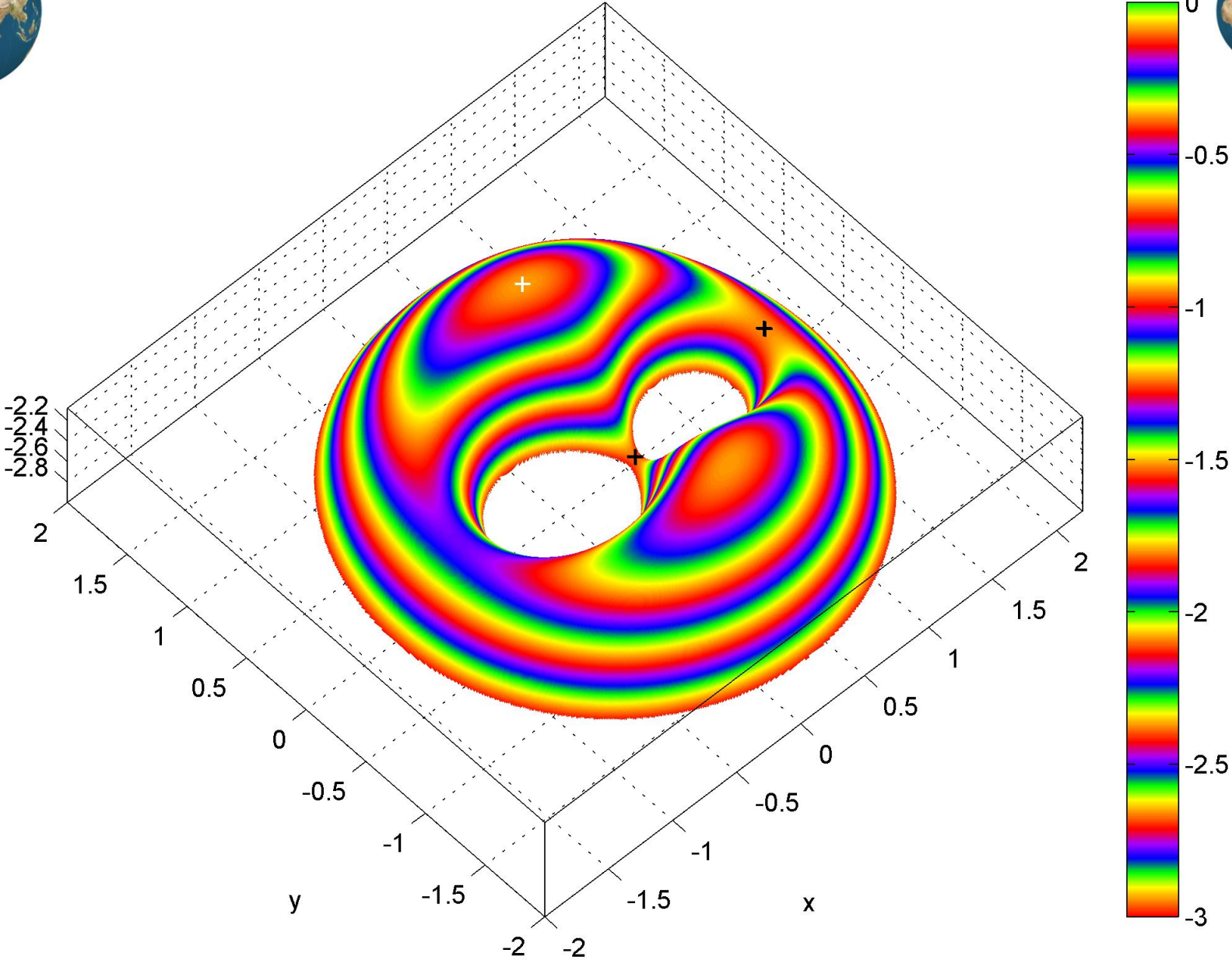
x scale is  $3.844\text{e}+008\text{ m}$ ,  $M1=5.972\text{e}+024\text{ kg}$ ,  $M2=2.986\text{e}+024\text{ kg}$



Earth & half Earth – moon separation

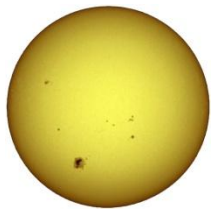


x scale is  $3.844\text{e}+008\text{ m}$ ,  $M1=5.972\text{e}+024\text{ kg}$ ,  $M2=2.986\text{e}+024\text{ kg}$

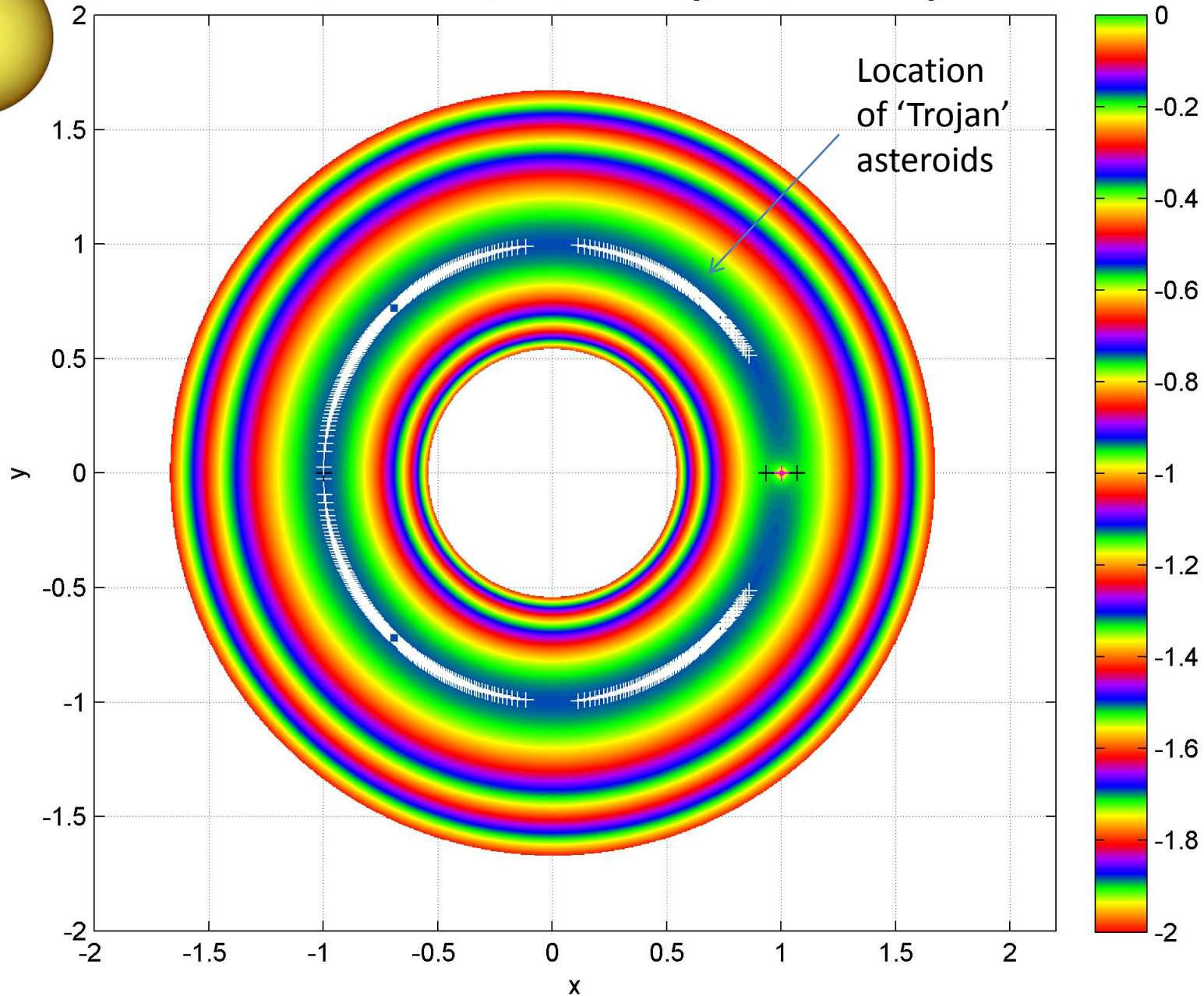


Earth & half Earth – moon separation



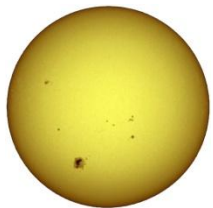


x scale is  $7.782 \times 10^{11}$  m,  $M_1 = 1.99 \times 10^{30}$  kg,  $M_2 = 1.898 \times 10^{27}$  kg

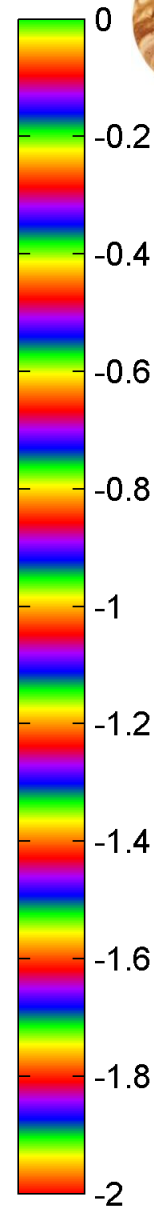
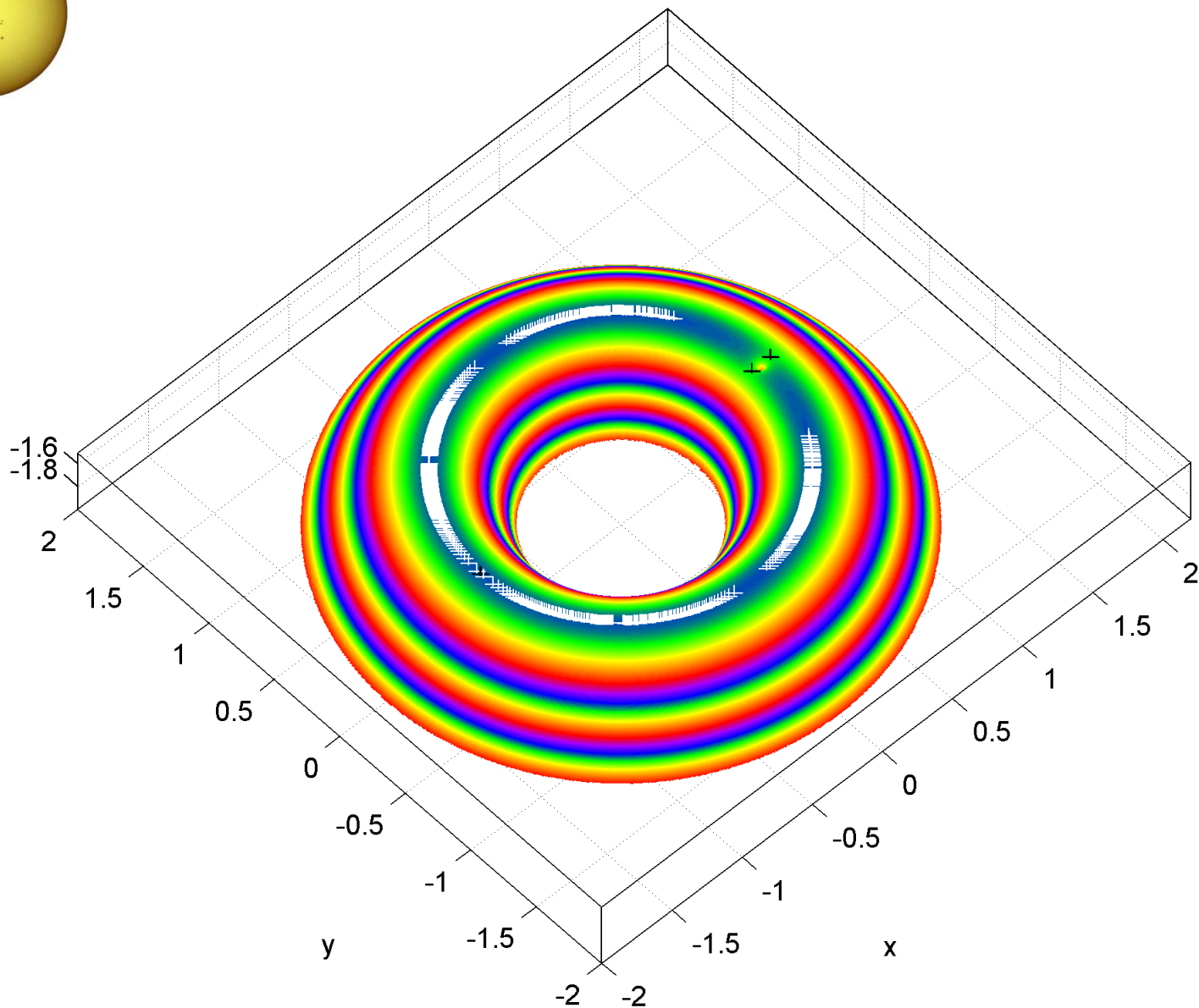


Sun & Jupiter





x scale is  $7.782 \times 10^{11}$  m,  $M_1 = 1.99 \times 10^{30}$  kg,  $M_2 = 1.898 \times 10^{27}$  kg



Sun & Jupiter