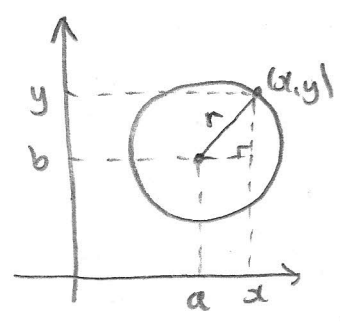


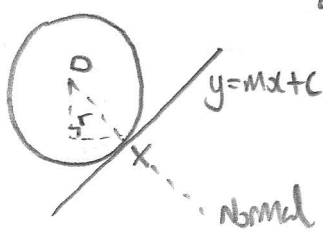
MP extra topics

Cartesian Equation of a circle

Pythagoras: $(x-a)^2 + (y-b)^2 = r^2$

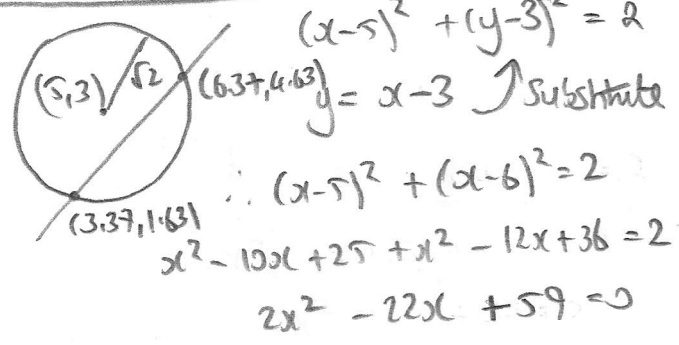


Tangent to a circle through X



- work out gradient of normal through OX
- Tangent gradient is $-\frac{1}{\text{normal gradient}} = m$
- if X has coordinates (x, y) find c using $y = mx + c$

Intersection of line with a circle

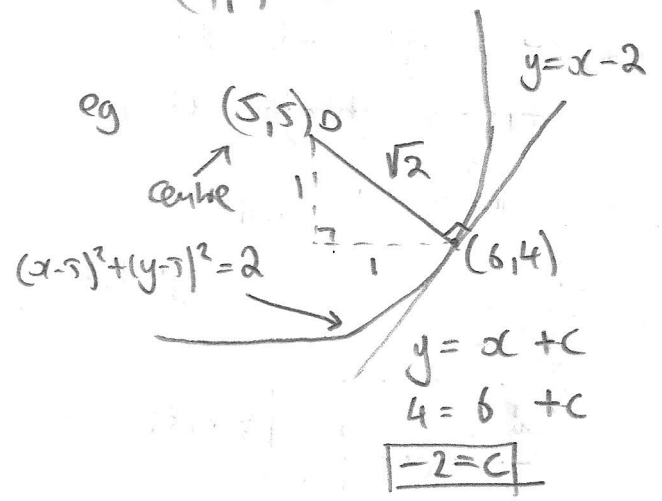


$(x-5)^2 + (y-3)^2 = 2$
 $y = x - 3$ substitute

$(x-5)^2 + (x-6)^2 = 2$
 $x^2 - 10x + 25 + x^2 - 12x + 36 = 2$
 $2x^2 - 22x + 59 = 0$

$x = \frac{22 \pm \sqrt{22^2 - 4(2)(59)}}{2(2)}$

$x = 6.37, 4.63$
 $y = 3.37, 1.63$



$(x-5)^2 + (y-5)^2 = 2$
 $y = x + c$
 $4 = 6 + c$
 $-2 = c$

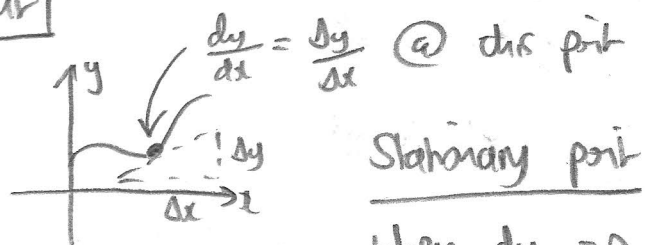
Surds

$\frac{1}{a+\sqrt{b}} = \frac{a-\sqrt{b}}{(a+\sqrt{b})(a-\sqrt{b})} = \frac{a-\sqrt{b}}{a^2-b}$
 eg $\frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2^2-3} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$

Differential Calculus

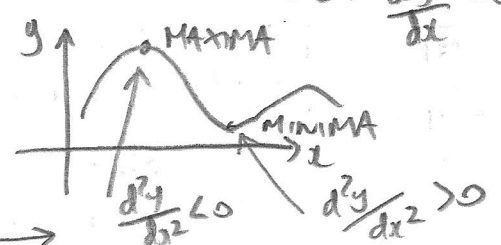
$y = ax^n$
 $\frac{dy}{dx} = nax^{n-1}$

$\frac{dy}{dx}$ is the gradient of y vs x.



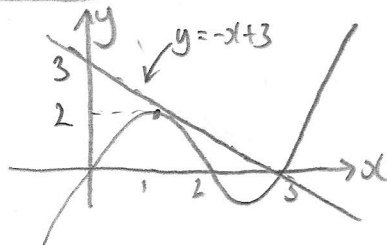
eg $y = x^3$
 $\frac{dy}{dx} = 3x^2$

$y = \frac{1}{\sqrt{x}} = x^{-1/2}$
 $\frac{dy}{dx} = -\frac{1}{2}x^{-3/2}$



Second derivative

Tangents and normals to curves using Calculus



$$y = x(x-2)(x-3)$$

$$y = x(x^2 - 5x + 6)$$

$$y = x^3 - 5x^2 + 6x$$

$$\frac{dy}{dx} = 3x^2 - 10x + 6$$

$$= 3\left\{x^2 - \frac{10}{3}x\right\} + 6$$

Complete the square

$$= 3\left\{\left(x - \frac{5}{3}\right)^2 - \frac{25}{9}\right\} + 6$$

$$= 3\left(x - \frac{5}{3}\right)^2 - \frac{25}{3} + 6$$

$$= \boxed{3\left(x - \frac{5}{3}\right)^2 - \frac{25}{3}}$$

So when $x = 1$,

$$\frac{dy}{dx} = 3\left(1 - \frac{5}{3}\right)^2 - \frac{25}{3}$$

$$= 3 \times \frac{4}{9} - \frac{25}{3}$$

$$= -1$$

\therefore Since $y = 1(1-2)(1-3) = 1(-1)(-2) = 2$ when $x = 1$

tangent through $(1, 2)$ is $y_T = -x + c$. Now $2 = -1 + c \therefore c = 3$

Normal is $y_N = x + d$. $\therefore 2 = 1 + d \therefore d = 1 \therefore y_N = x + 1$

$y_T = -x + 3$

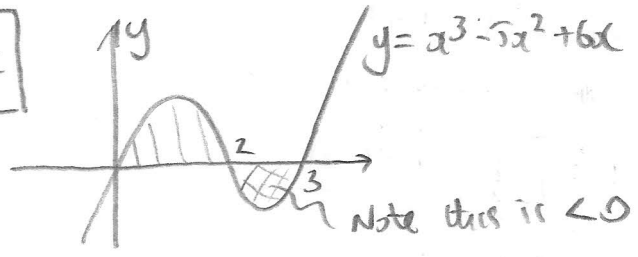
↑

gradient = $-\frac{1}{(-1)}$

Integral calculus $\int f(x) dx$

Use definite integrals to find areas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$



Anti differential

Differentiate

"Indefinite integral"

$$\frac{1}{3}x^3 + c = \int x^2 dx$$

Substn of integration

$$\frac{d}{dx} \left(\frac{1}{3}x^3 + c \right) = x^2$$

$$\frac{d}{dx} \left(\frac{1}{3}x^3 - c \right) = x^2$$

$$\int_0^2 (x^3 - 5x^2 + 6x) dx = A$$

is "Sum $\int dx f(x)$ "

$$A = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 \right]_0^2$$

$$A = \left(\frac{1}{4}2^4 - \frac{5}{3}2^3 + 3 \times 2^2 \right) - (0)$$

$$A = 4 - 13\frac{1}{3} + 12$$

$$A = \boxed{2\frac{2}{3}}$$

Arithmetic sequence

$$u_n = a + (n-1)d$$

$$S_n = a + a+d + a+2d + \dots + a + (n-1)d$$

$$S_n = a + (n-1)d + \dots + a$$

$$2S_n = n(2a + (n-1)d)$$

$$S_n = \frac{n}{2} (2a + (n-1)d) = \frac{n}{2} (u_1 + u_n)$$

a is the first term

d is the common difference

