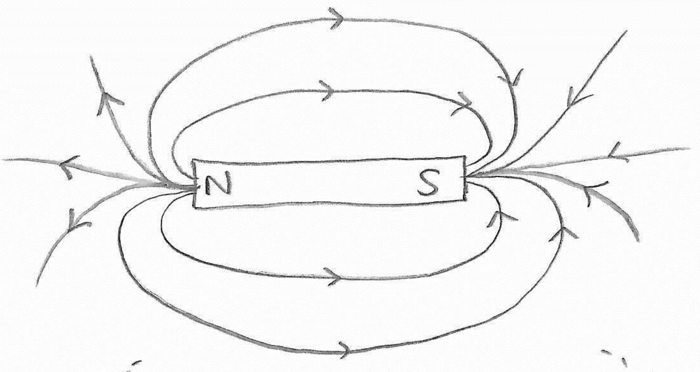
# MAGNETISM

1/ (i) a)

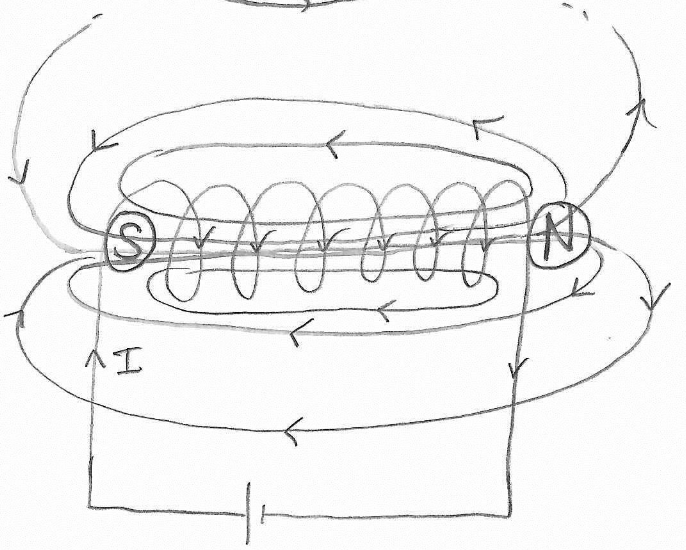



BAR MAGNET

n is the # of coil turns/unit length.

Field strength B inside solenoid is $\boxed{B = \mu_0 n I}$

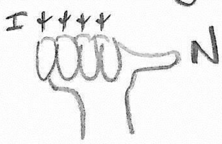
b)



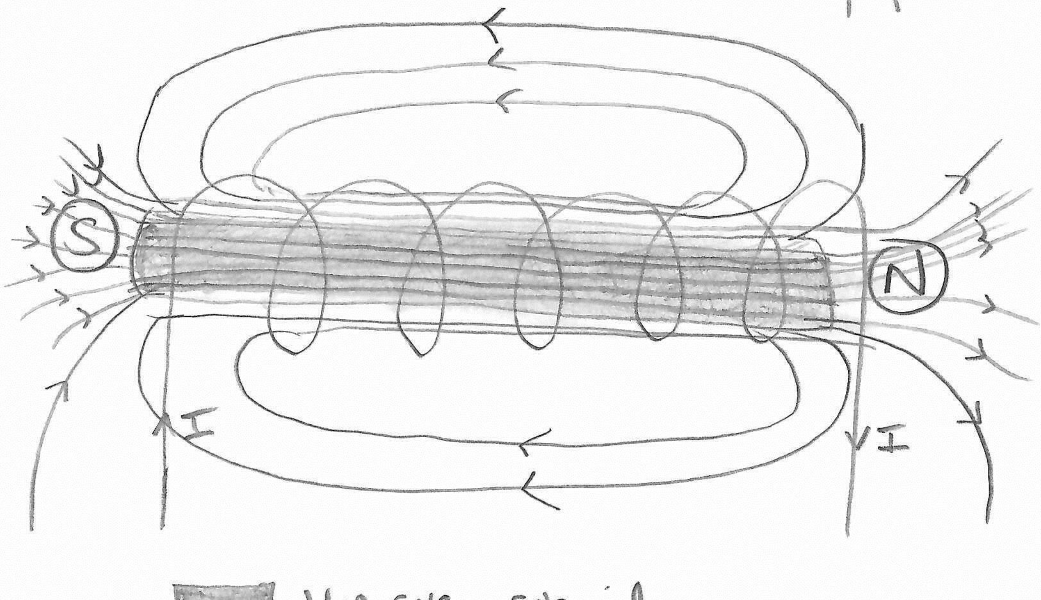

AIR CORED SOLENOID

i.e like a bar magnet, but you can control the field strength by altering n, I

(RIGHT HAND) Solenoid grip rule




c)



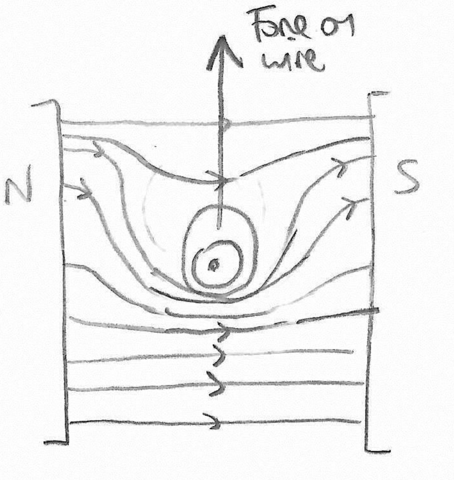

$B = \mu \mu_0 n I$

$\mu >> 1$ for Iron ($\mu \approx 800$)

Iron core solenoid

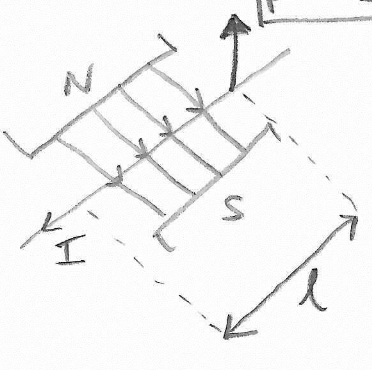
significantly intensified field inside iron core.

(ii)




Current I ⊙ out of the plane.

$\boxed{F = BIl}$





LEFT HAND RULE

$\underline{F} = q\, \underline{v} \times \underline{B}$

b) $F = BIl$ so $B = \frac{F}{Il}$

{ originally $F = 0.2N$ }

$B = 0.2N / 1.23A \times 0.1M$

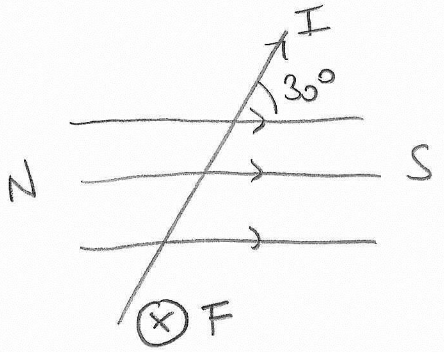
$\boxed{B = 1.63\,T}$

This is a very strong magnet!

More realistic: $F = 2\times10^{-4}\,N$ ← This is in the updated questions.

$\Rightarrow \boxed{B = 1.63\times10^{-3}\,T}$

c)



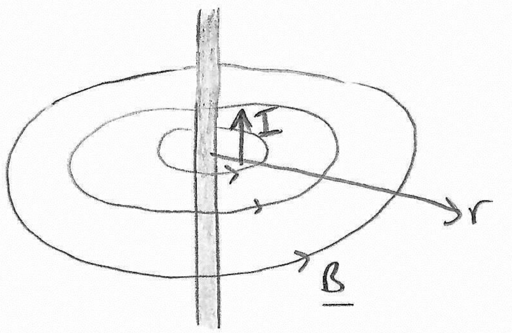

$\boxed{F = BIl\sin\theta}$

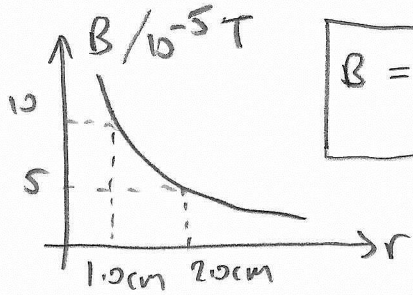
(ie component of current direction ⊥ $\underline{B}$ is $I\sin\theta$)

so force differs by a factor of $\sin 30° = \boxed{\frac{1}{2}}$

ie 50% less ($1.0\times10^{-4}N$)

iii)




$\boxed{B = \frac{\mu_0 I}{2\pi r}}$

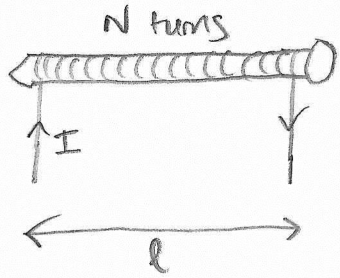
$$B = \frac{4\pi\times10^{-7}\times 5.0}{2\pi\times10^{-2}\,(r/cm)} \quad T$$

$$B = \frac{1.0\times10^{-4}}{r/cm} \quad T$$

So at

| | | |
|---|---|---|
| 1.0cm | : | $B = 1.0\times10^{-4}\,T$ |
| 2.0cm | : | $B = 5.0\times10^{-5}\,T$ |

(iv)

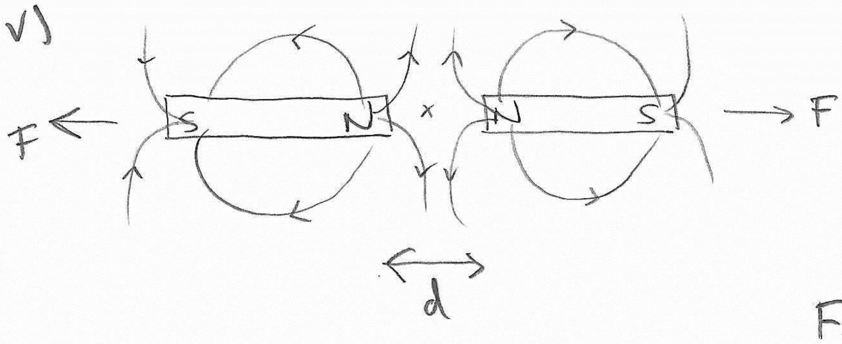



$\boxed{B = \mu\mu_0 \frac{N}{l} I}$ $\therefore N = \frac{lB}{\mu\mu_0 I}$

$$\therefore N = \frac{15\times10^{-2}\times 0.509}{800\times4\pi\times10^{-7}\times 2.0} = 37.97$$

So 38 turns to be sure of 0.50T.

v)

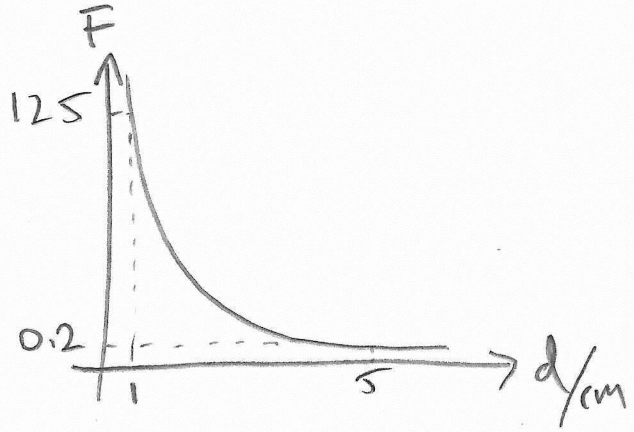



$$\boxed{F = \frac{k}{d^4}}$$ Repulsive force.

$$F_5 = \frac{k}{d_5^4}$$

$(d/\mathrm{cm})$

$d_5 = 5.0\,\mathrm{cm}$
$d_1 = 1.0\,\mathrm{cm}$
$F_5 = 0.2\,\mathrm{N}$

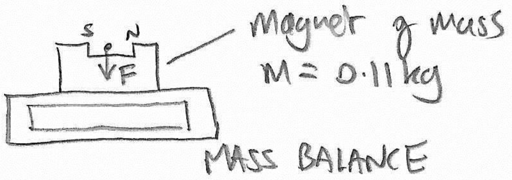



$$F_1 = \frac{k}{d_1^4}$$

$$\therefore \frac{F_1}{F_5} = \left(\frac{d_5}{d_1}\right)^4$$

$$\Rightarrow F_1 = 0.2\,\mathrm{N} \times \left(\frac{5.0\,\mathrm{cm}}{1.0\,\mathrm{cm}}\right)^4 = \boxed{125\,\mathrm{N}}$$

vi)




$g = 9.81\,\mathrm{N/kg}$

Force on mass balance is $Mg + BIl$.

Now if increase is $x\,\%$ $$1 + \frac{x}{100} = 1 + \frac{BIl}{Mg}$$

$$\Rightarrow B = \frac{Mgx}{100\,Il} = \frac{0.11 \times 9.81 \times 10}{100 \times 3.0 \times 6.0 \times 10^{-2}} \quad (\mathrm{T})$$

$$= \boxed{0.60\,\mathrm{T}}$$

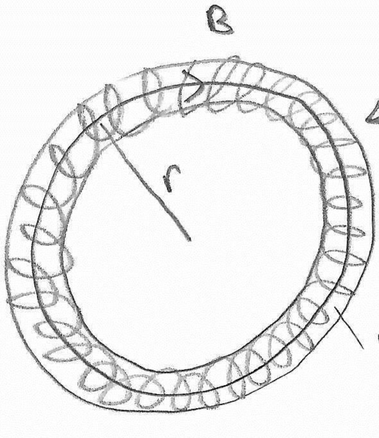
(ie quite high. So this means a thin slit if magnet is the standard steel type).

Now let a 4.0 A current pass in the opposite direction.

Mass balance value is now: $M - \frac{BIl}{g}$

$$= 0.11 - 0.60 \times 4.0 \times 6.0 \times 10^{-2} / 9.81 = \boxed{0.095\,\mathrm{kg}}$$

(vii)




← N = 100 turns

Fe₂O₃ → $Fe_2O_3$ (ferrite)

Iron core

$\mu = 800$

Ampère $\oint B \cdot dl = \mu_0 \mu I N$

$$2\pi r B = \mu \mu_0 I N$$

$$\boxed{B = \frac{\mu \mu_0 I N}{2\pi r}}$$

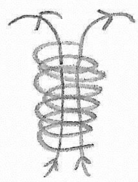
$r = 2.0\,cm$

$I = 2.3\,mA$

$$B = \frac{800 \times 4\pi \times 10^{-7} \times 2.3 \times 10^{-3} \times 100}{2\pi \times 2.0 \times 10^{-2}}$$

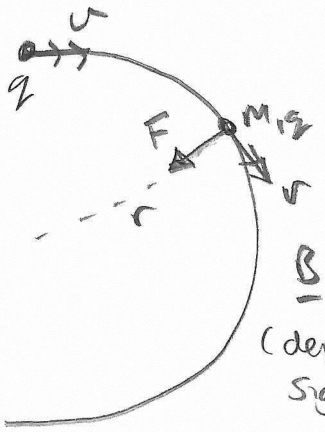
$$\boxed{B = 1.84 \times 10^{-3}\,T}$$

<u>Inductors</u> in a toroidal geometry are often more popular in electrical engineering applications than



coils because the magnetic fields are largely confined to the interior of the coil. In open coils, the magnetic fields will 'leak' which may adversely effect other devices. Also, the circular / toroidal geometry is typically very space efficient.

(viii)




<u>B</u> ⊙ or ⊗

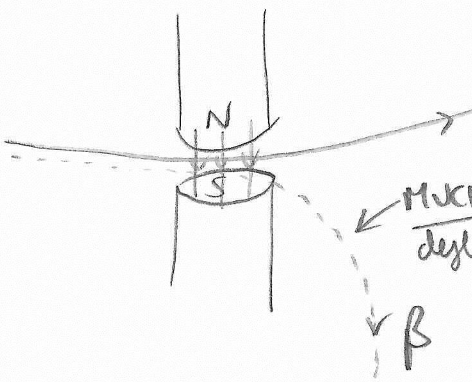
(depending on sign of q)

Centripetal force is $F = qvB$

∴ Newton II radially inwards

$$\frac{mv^2}{r} = qvB$$

$$\therefore \boxed{\frac{mv}{qB} = r}$$




← much more strongly deflected.

$\alpha$: $q = 2e = 2 \times 1.602 \times 10^{-19}\,C$

$m_\alpha = 6.6 \times 10^{-27}\,kg$

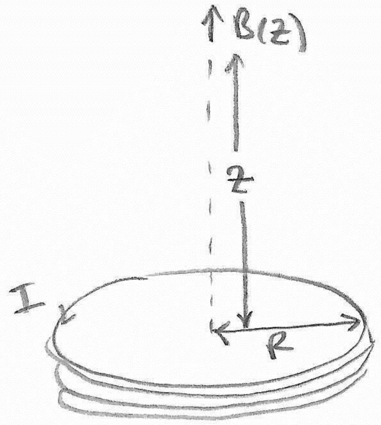
$\beta$: $q = -e = -1.602 \times 10^{-19}\,C$

$m_\beta = 9.1 \times 10^{-31}\,kg$

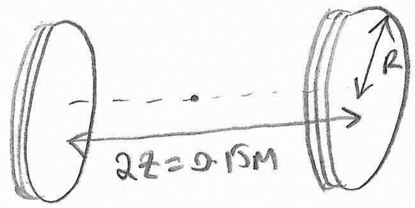
So deflection in <u>opposite directions</u> and

$$r_\beta / r_\alpha = \frac{m_\beta}{m_\alpha} \times 2 = \boxed{2.8 \times 10^{-4}}$$ i.e. a much tighter turning circle.

ix/




$$\boxed{B = \frac{\frac{1}{2}\mu_0 N I R^2}{(R^2+z^2)^{3/2}}}$$



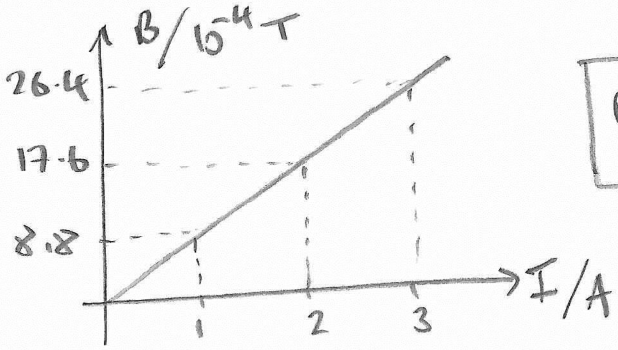

Helmholtz coils
R = 0.15m

At the centre point between the Helmholtz coils:

$$B = \frac{\mu_0 N I R^2}{(R^2+z^2)^{3/2}}$$

if $N = 130$, $R = 0.15\text{m}$, $z = \frac{0.15}{2}\text{m}$

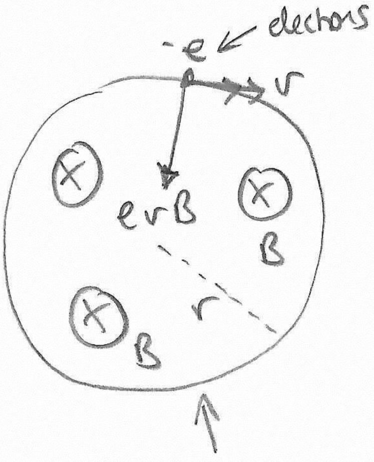
$$\therefore \quad B/I = \frac{4\pi\times10^{-7}\times 130\times 0.15^2}{\left(0.15^2 + \left(\frac{0.15}{2}\right)^2\right)^{3/2}} = \boxed{7.79\times10^{-4}\ \text{T/A}}$$




$$\boxed{B/T \simeq 7.79\times10^{-4}\ (I/A)}$$

(x)

a)




FINE BEAM TUBE

purple-blue electron beam (ionized $H_2$ gas)

NII radially inwards

$$\frac{M_e v^2}{r} = evB \quad \therefore \quad \frac{e}{M_e} = \frac{v}{rB}$$

Now $eV = \frac{1}{2}M_e v^2$ if electrons are accelerated using voltage V

$$\text{so} \quad \sqrt{\frac{2eV}{Me}} = v$$

$$\therefore \frac{e}{m_e} = \sqrt{\frac{2eV}{m_e}}\ \frac{1}{rB}$$

$$\frac{e^2}{m_e^2} = \frac{2eV}{m_e}\ \frac{1}{r^2B^2}$$

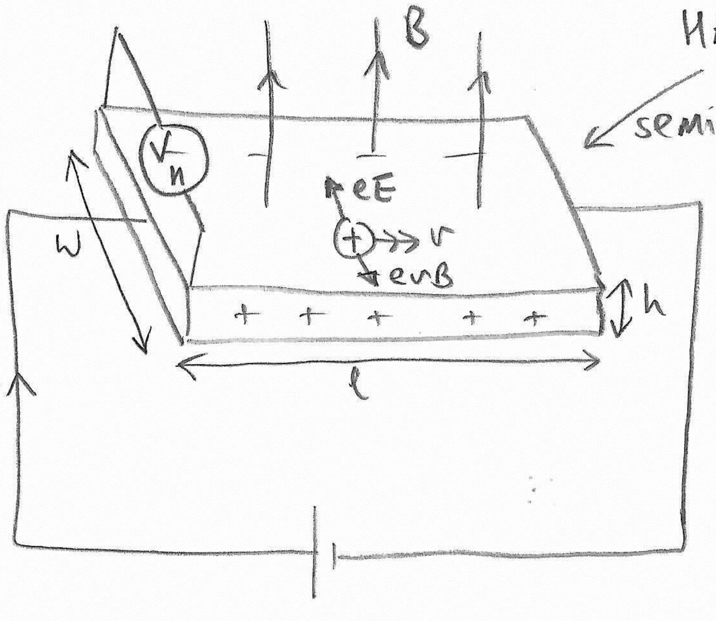
$$\boxed{\frac{e}{m_e} = \frac{2V}{r^2B^2}}$$

b) $\therefore r = \sqrt{\frac{2V}{B^2}\ \frac{m_e}{e}}$

$$\therefore r = \sqrt{\frac{2\times 170}{(0.8\times 10^{-3})^2}\ \frac{9.109\times 10^{-31}}{1.602\times 10^{-19}}}$$

$$\boxed{r = 0.055\ m} \quad (5.50\,cm)$$

2/




Charge / s moving through the semiconductor is I :

$\therefore I = e \times nwh \times v$ "Is of volume swept $\times$ n $\times$ e"
whv

# charge carriers / unit volume
charge of charge carrier

$$\therefore \boxed{v = \frac{I}{newh}}$$

(6)

b) In equilibrium forces balance on the electron (Note the '<u>holes</u>' are moving in the diagram above)

$$\text{So} \quad eE = evB \quad \therefore \quad v = E/B$$

$$\text{Now} \quad E = \frac{V_H}{w} \quad \text{So} \quad v = \frac{V_H}{wB}$$

$$\therefore \quad \frac{V_H}{wB} = \frac{I}{newh}$$

$$\therefore \quad \boxed{B = \frac{V_H neh}{I}}$$

c) $h = 0.10\text{ mm}$, $I = 0.200\text{A}$, $V_H = 1.78\text{ mV}$, $B = 1000\,\mu\text{T}$

$$\therefore \quad n = \frac{BI}{V_H eh} = \frac{1000\times10^{-6} \times 2.00\times10^{-1}}{1.78\times10^{-3} \times 1.602\times10^{-19} \times 0.10\times10^{-3}}$$

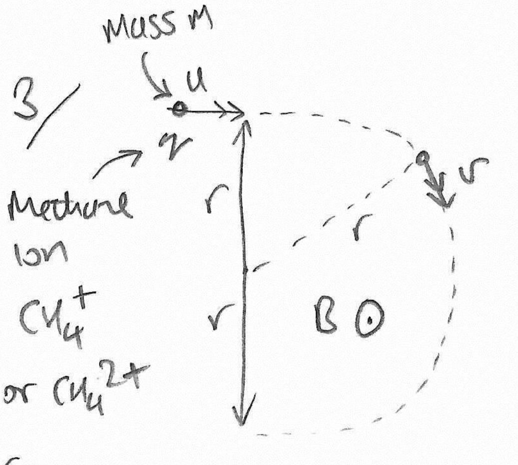
$$= \boxed{7.01\times10^{21}} \text{ charge carriers}/\text{m}^3$$

d) $\boxed{V_H = \frac{BI}{neh}}$

$$\text{So} \quad V_H = \frac{2.0 \times 0.200}{7.01\times10^{21} \times 1.6\times10^{-19} \times 0.10\times10^{-3}} \quad (\text{v})$$

$$= \boxed{3.56\text{ V}}$$

3/




Inside Mass Spectrometer in the Cassini-Huygens spacecraft.

$\frac{1}{2}Mv^2 = qV$ ① Acceleration of ion by voltage

$\frac{Mv^2}{r} = qvB$ ② NII, [Circular motion] radially inward.

$d = 2r$ ③

So $\quad v = \sqrt{2qV/M}$

$$\frac{Mv}{qr} = B \qquad \therefore \quad B = \frac{M}{qr}\sqrt{\frac{2qV}{M}}$$

$$B = \sqrt{\frac{M^2}{q^2r^2} \times \frac{2qV}{M}}$$

$$\boxed{B = \sqrt{\frac{2MV}{qr^2}}}$$

( Note result in Q1 (x) : $\frac{q}{M} = \frac{2V}{B^2r^2}$ $\therefore$ $B^2 = \frac{2VM}{qr^2}$

$\therefore$ $B = \sqrt{\frac{2mV}{qr^2}}$ ✓ )

let $M = 16.043 \times 1.661 \times 10^{-27}$ kg (mass of $CH_4$ molecule)

$V = 6{,}000\,V$ , $r = d/2 = 15.0 \times 10^{-2}\,m$

$q = n \times 1.602 \times 10^{-19}\,C$ $\qquad n = 1, 2$ for $CH_4^+$, $CH_4^{2+}$

$$\therefore \quad B = \frac{1}{\sqrt{n}} \times \sqrt{\frac{2 \times 16.043 \times 1.661 \times 10^{-27} \times 6000}{1.602 \times 10^{-19} \times (15.0 \times 10^{-2})^2}}$$

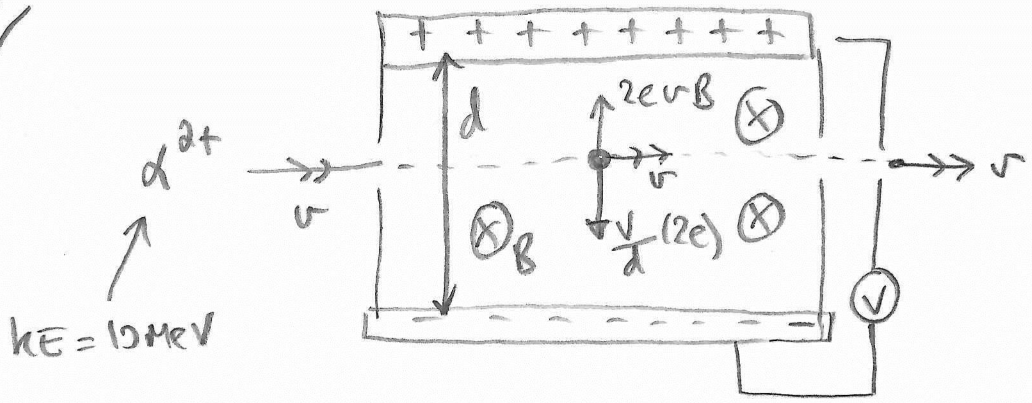
$$B = \frac{1}{\sqrt{n}} \times 0.298\ T.$$

So $\quad$ 0.298 T for $CH_4^+$ and 0.211 T for $CH_4^{2+}$.

[ Alternatively you fix B and have detectors at different $d = 2r$. $\quad d = \sqrt{\frac{2VM}{neB^2}}$ $\quad \boxed{d/\text{cm} = \frac{1}{\sqrt{n}} \times 14.09}$

(so $d = 6.53$ cm for $CH_4^{2+}$). $\qquad$ ($B = 0.300\,T$, $V = 600\,V$)

4/




$d = 2.5\,cm$
$B = 0.123\,T$

For the $\alpha$ particle to pass through the chamber illustrated above undeflected the electric force must balance the magnetic force.

$$\text{ie} \quad \frac{V}{d}(2e) = 2evB$$

$$\Rightarrow \quad V = vBd$$

{ note size of charge doesn't matter! }

Now $\alpha^{2+}$ energy is $E = 10\,MeV$

assuming classical dynamics $(v \ll c)$

$$\frac{1}{2}mv^2 = E \quad \therefore \quad v = \sqrt{\frac{2E}{m}}$$

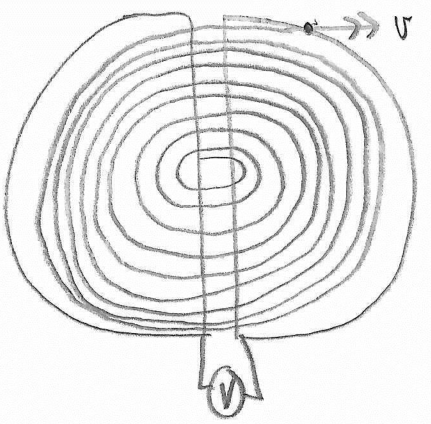
$$\therefore \quad \boxed{V = Bd\sqrt{2E/m}}$$

$$\therefore \quad V = 0.123 \times 2.5 \times 10^{-2}\sqrt{\frac{2 \times 10 \times 1.602 \times 10^{-19} \times 10^{6}}{6.6 \times 10^{-27}}}$$

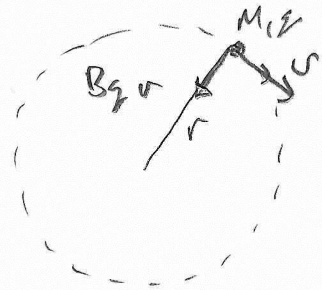
$$= \boxed{67,750 \text{ Volts}} \qquad (67.8\,kV)$$

5 a)

$B \odot$




$\boxed{\text{Cyclotron}}$




$\boxed{\text{Proton}}$
$M = 1.637 \times 10^{-27}$ kg
$q = 1.602 \times 10^{-19}$ C

NII radially inwards:

$$\frac{Mv^2}{r} = Bqv$$

$$Mv = rBq$$

T is 'orbit' period at radius r

$$M \times \frac{2\pi \cancel{r}}{T} = \cancel{r}Bq$$

$$\frac{2\pi M}{Bq} = T$$

"Cyclotron frequency"

$$\therefore \quad f_c = \frac{1}{T} = \boxed{\frac{Bq}{2\pi M}}$$

So if $V = V_0 \cos(2\pi t/T) = \boxed{V_0 \cos\left(\frac{Bq}{M} t\right)}$

then proton will get a boost in KE $\approx V_0 e$ every half a cycle. ie E field between 'Dees' aligns with proton velocity when it passes between them.

b) So $\frac{1}{2} M v_{n+1}^2 = \frac{1}{2} M v_n^2 + eV_0$

$$\therefore \quad \boxed{v_{n+1} = \sqrt{v_n^2 + \frac{2eV_0}{M}}}$$

let $V_0 = 100 \times 10^3 V$, $m = 1.637 \times 10^{-27} kg$

$e = 1.602 \times 10^{-19} C$ and let $v_0 = 0 \, m/s$

$v_0 = 0 \, m/s$

$v_1 = 0.0148c$

$v_2 = 0.0209c$

$v_3 = 0.0256c$

$v_4 = 0.0295c$

$v_5 = 0.0330c$

$v_6 = 0.0361c$

$v_7 = 0.0390c$

$v_8 = 0.0417c$

$v_9 = 0.0443c$

$v_{10} = 0.0467c$

$v_{11} = 0.0489c$

$v_{12} = 0.0511c$

So 12 boosts ie $\boxed{\text{Six orbits}}$ to accelerate a proton to $0.05c$ (using classical electrodynamics)

c) $$f_c = \frac{Bq}{2\pi m} = \frac{0.2 \times 1.602 \times 10^{-19}}{2\pi \times 1.637 \times 10^{-27}}$$

$$= 3.12 \times 10^6 \, Hz$$

ie $\boxed{3.12 \, MHz}$

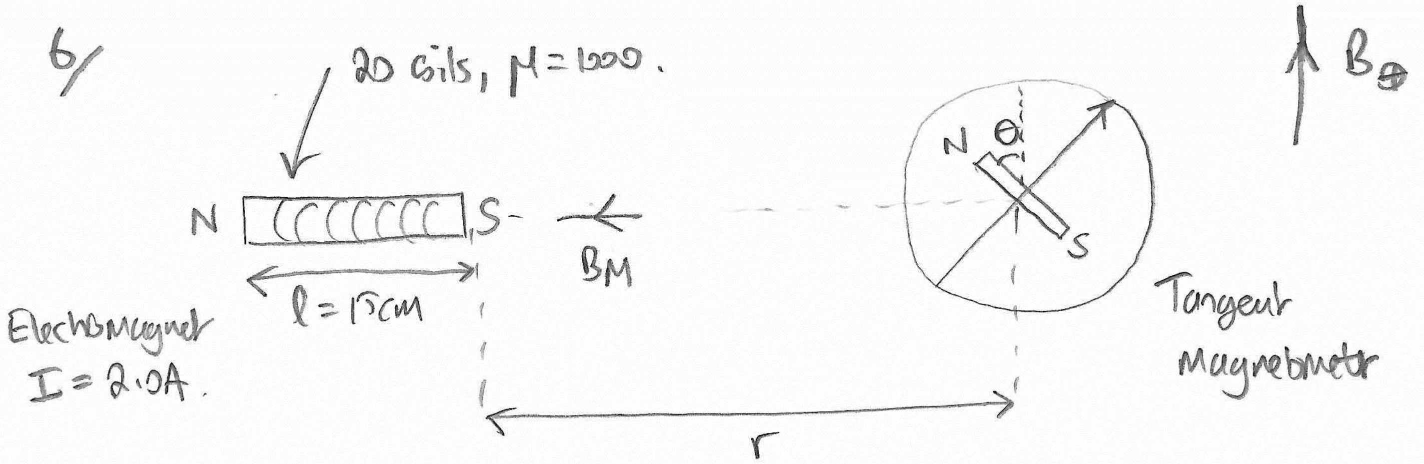
$$r_{12} = \frac{mv}{Bq} = \frac{1.637 \times 10^{-27} \times 0.0511 \times 2.998 \times 10^8}{0.2 \times 1.602 \times 10^{-19}}$$

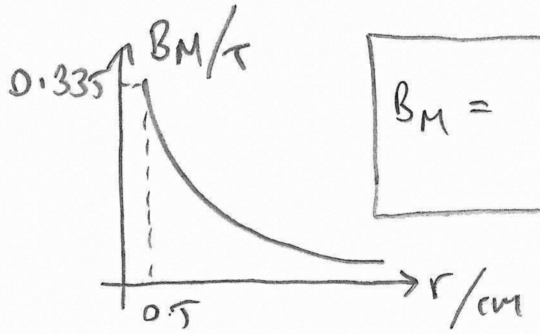
$$= \boxed{0.783 \, m}$$

(So 'desktop sized' indeed).

d) See cyclotron orbit MATLAB Model.

6/



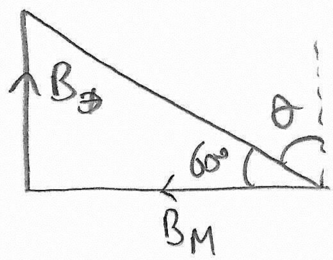

$$\boxed{B_M = B_*\left(\frac{r}{0.5\,\text{cm}}\right)^{-3}}$$

$$B_* = \mu\mu_0 \frac{20}{15\times10^{-2}} \times 2.0$$

$$B_* = \frac{1000 \times 4\pi\times10^{-7} \times 20 \times 2.0}{15\times10^{-2}}$$

$$\boxed{B_* = 0.335\ \text{T}}$$

Balance of magnetic fields on magnetometer




So if $\theta = 30^\circ$

$$\tan 60^\circ = \frac{B_⊕}{B_M}$$

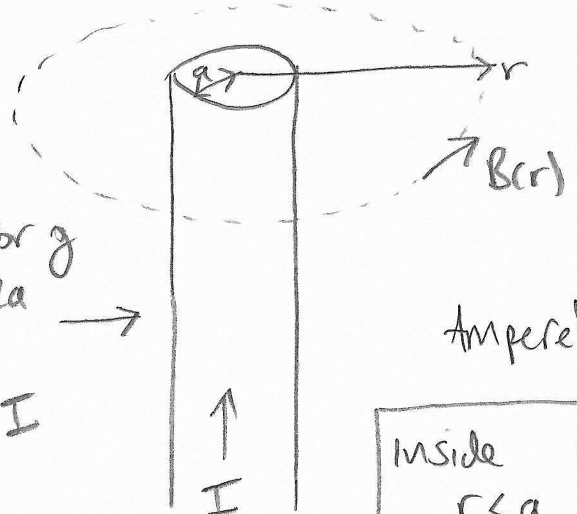
$$\therefore \tan 60^\circ = \frac{B_⊕}{B_*\left(\frac{r}{0.5\,\text{cm}}\right)^{-3}}$$

$$\frac{B_*}{B_⊕}\tan 60^\circ = \left(\frac{r}{0.5\,\text{cm}}\right)^{3}$$

$$\therefore \quad r = 0.5\,\text{cm} \times \sqrt[3]{\frac{B_* \tan 60^\circ}{B_⊕}}$$

$$r = 0.5\,\text{cm} \times \sqrt[3]{\frac{0.335 \tan 60^\circ}{50\times10^{-6}}} = \boxed{11.3\,\text{cm}}$$

7/ a)





conductor of diameter 2a carrying current I

By symmetry assume $\underline{B}$ is tangential.

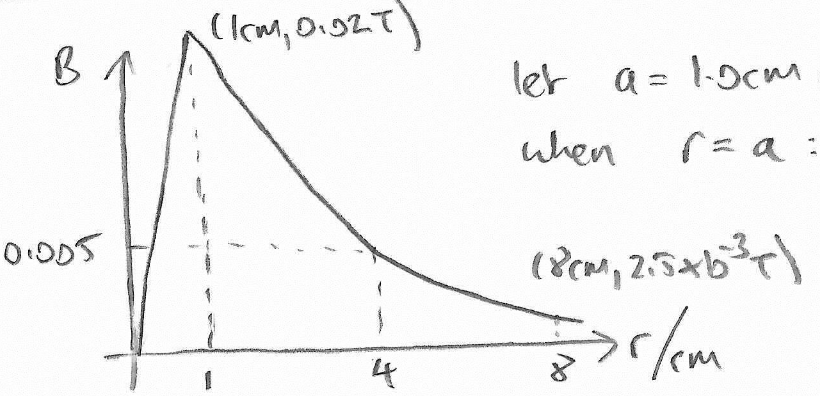
Ampere's Theorem $\boxed{\oint \underline{B}.d\underline{l} = \mu_0 I}$

$\boxed{\text{Inside conductor: } r \leq a}$  $I \to \frac{\pi r^2}{\pi a^2} I$

$\therefore \quad B \times 2\pi r = \frac{r^2}{a^2} I$

$\therefore \quad \boxed{B = \frac{I}{2\pi a^2} r}$

$\boxed{\text{outside conductor: } r > a}$  $B \times 2\pi r = \mu_0 I \quad \therefore \quad \boxed{B = \frac{\mu_0 I}{2\pi r}}$




let $a = 1.0\text{cm}$, $I = 1000\text{A}$

when $r = a$: $B = \frac{\mu_0 I}{2\pi a}$

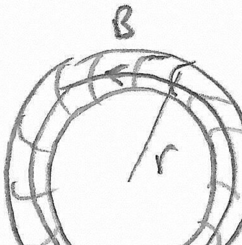
$$B = \frac{4\pi \times 10^{-7} \times 1000}{2\pi \times 1.0 \times 10^{-2}}$$

$\boxed{B = 0.02\,\text{T}}$

$\boxed{\text{so for } r > 1.0\text{cm} \quad (B/\text{T}) = 0.02/(r/\text{cm})}$

$\boxed{\text{for } r < 1.0\text{cm} \quad B/\text{T} = 0.02\left(r/\text{cm}\right)}$

b)




N coils

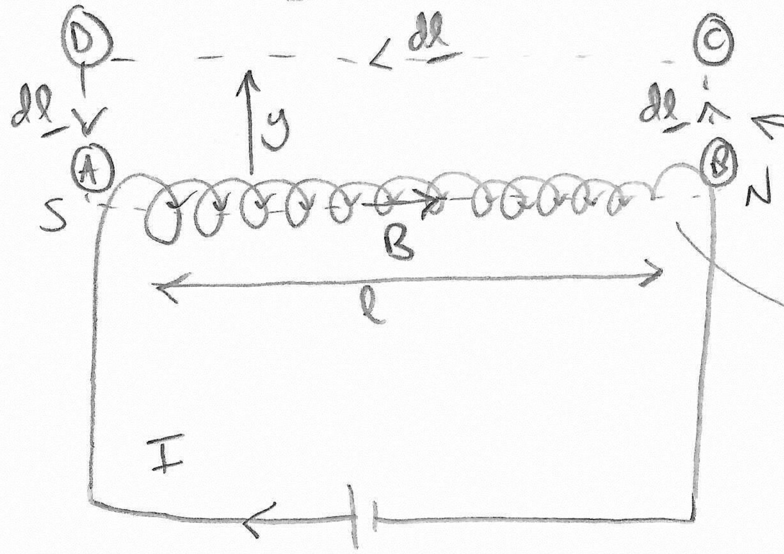
Ampere: $\boxed{\oint \underline{B}.d\underline{l} = \mu\mu_0 NI}$ if N loops in torus of radius r, around a ferromagnetic core of relative permeability $\mu$.

$\therefore \quad 2\pi r \times B = \mu\mu_0 N I$

$$\Rightarrow \boxed{B = \frac{\mu\mu_0 N I}{2\pi r}}$$

Magnetic field in centre of torus.

c)




"Amperian circuit ABCD"

air cored solenoid of n turns in length $l$.

$l$ is long enough (and n large enough) for $B \simeq$ uniform inside coils.

Ampere: $\oint \underline{B} \cdot \underline{dl} = nl\mu_0 I$

$$\int_A^B \underline{B} \cdot \underline{dl} + \int_B^C \underline{B} \cdot \underline{dl} + \int_C^D \underline{B} \cdot \underline{dl}$$

$$+ \int_D^A \underline{B} \cdot \underline{dl} = nl\mu_0 I$$

Now by symmetry $\int_B^C \underline{B} \cdot \underline{dl} = -\int_D^A \underline{B} \cdot \underline{dl}$

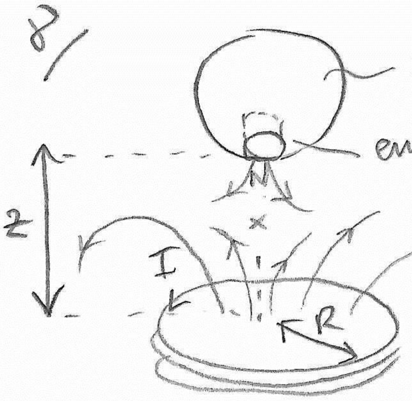
Also, we can take $y \to \infty$ without invalidating Ampere's ~~theorem~~. when $y \to \infty$, $|\underline{B}| \to 0$

$\therefore$ in this limit $\int_C^D \underline{B} \cdot \underline{dl} \to 0$

Hence: $\int_A^B \underline{B} \cdot \underline{dl} = nl\mu_0 I$. Assuming $\underline{B}$ to be uniform in the coils

$$\Rightarrow Bl = nl\mu_0 I \quad \therefore \boxed{B = \mu_0 n I}$$

8/




Sphere of mass 0.321 kg

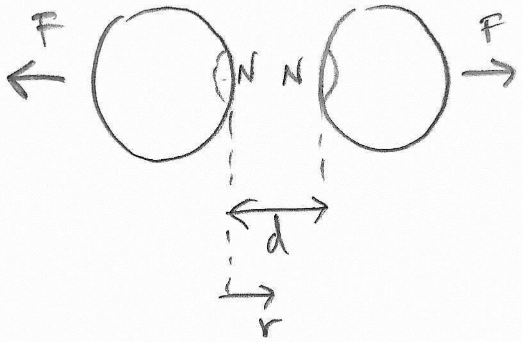
embedded magnet

$g = 9.81$ N/kg

$N = 1200$ turns, $R = 2.0$ cm & iron core $\mu = 1000$

$$\boxed{B_c(z) = \frac{\frac{1}{2}\mu_0 N I R^2 \mu}{(R^2+z^2)^{3/2}}}$$

Magnetic field on axis to coil, height $z$ above.




Magnet has repulsion force

$$F = \frac{k}{d^4}$$

$F = 15.0$ N when $d = 0.5$ cm

$$\therefore \quad k = 15 \times (0.5\times10^{-2})^4$$

$$\boxed{k = 9.375\times10^{-9} \text{ N m}^4}$$

Now $\quad F \propto \dfrac{dB}{dr}$ $\quad$ let $B$ due to one magnet vary as $B = \alpha / r^3$

$$\therefore \quad \frac{dB}{dr} = \frac{-3\alpha}{r^4}$$

Much like gravity and electromagnetism, the 'magnetic dipole moment' is the equivalent to mass or charge.

$\uparrow$

$\alpha$ $\quad$ So $\quad F = \dfrac{3\alpha^2}{r^4}$ $\quad\quad \left(F = -\alpha \dfrac{dB}{dr} \quad B = \dfrac{\alpha}{r^3}\right)$

So $\quad 3\alpha^2 = k$

$$\Rightarrow \quad \alpha = \sqrt{15\times(0.5\times10^{-2})^4/3} = \boxed{5.59\times10^{-5}\ \text{N}^{\frac{1}{2}}\text{m}^2}$$

So for sphere to be in equilibrium above the coil

$$mg = -\alpha \frac{dB}{dz}$$

$$\frac{dB}{dz} = \frac{1}{2}\mu_0 NI \left(-\frac{3}{2}\right) \frac{R^2}{(R^2+z^2)^{5/2}} (\cancel{2}z)$$

$$\therefore \quad mg = \frac{\frac{3}{2}\mu_0 NIR^2 z}{(R^2+z^2)^{5/2}} \alpha M$$

$$\therefore \quad \boxed{I = \frac{\frac{2mg}{3}(R^2+z^2)^{5/2}}{\mu_0 NR^2 z \alpha M}}$$

$\therefore$ if $z = 1.0$ cm

$$I = \frac{\frac{2 \times 0.321 \times 9.81}{3}\left((2\times10^{-2})^2 + (1.0\times10^{-2})^2\right)^{5/2}}{4\pi\times10^{-7} \times 2400 \times (2\times10^{-2})^2 \times 1.0\times10^{-2} \times 5.59\times10^{-5} \times 1000}$$

$$= \boxed{17.4\,A}$$

if $z = 2.0$ cm, $\boxed{I = 28.2A}$