Magnetism is a force which acts on *moving charges* in the presence of magnetic field **B**. Like gravitational and electric fields, magnetic fields are also sourced by the same type of entity they act upon i.e. moving charges (or current carrying wires). Magnetic field strength is measured in tesla (T). Earth's magnetic field is about 10^{-5} T, a small permanent bar magnet is about 0.01T to 0.2T. The strongest laboratory electromagnets are about 20T.

Magnets always appear in *dipoles*, *quadropoles* etc, but *never monopoles*. i.e. magnetic field lines (outwards from North, inwards to South) always form *closed loops*. The force between dipoles is proportional to the *gradient* of the magnetic field. The magnetic field in the far-field of a dipole decays as $1/r^3$ so the magnetic force between two dipoles decays as $1/r^4$.

The force **F** on a charge q in the presence of electric field **E** and magnetic field **B** is given by $\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$ where $\mathbf{v}$ is the velocity of the charge. i.e. the purely magnetic force has magnitude qvB, assuming $\mathbf{v}$ is perpendicular to **B**. If the moving charge is a current I travelling through a wire of length l at angle θ to a magnetic field of strength B, the force F (which acts perpendicular to both current and **B** field directions, as per *Fleming's Left Hand Rule*) is given by: $F = BIl\sin\theta$. Particle charges are typically multiples of the charge on the electron $e = 1.602\times 10^{-19}\,\mathrm{C}$.

Ampère's Theorem: $\oint_{loop} \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$ can be used to determine magnetic field strength, assuming no time varying electric fields are present. $d\mathbf{l}$ is a vector element of the circumference of a loop which encloses current I. μ_0 is the *permeability of free space*. $\mu_0 = 4\pi\times 10^{-7}\,\mathrm{NA}^{-2}$. Applying Ampere's Theorem to a current carrying cylinder of radius a: $B = \mu_0 Ir/2\pi a^2$ for $r < a$ and $B = \mu_0 I/2\pi r$ for $r > a$. The perpendicular distance from the wire is r.

The magnetic field inside a (long i.e. 'infinite') solenoid is $B = \mu\mu_0 nI$ where n is the number of coils per unit length. μ is the *relative permeability* and results from the presence of a ferromagnetic core such as iron ($\mu \approx 5000$), nickel ($\mu \approx 100$), or iron oxide (ferrite) ($\mu \approx 640$) which can be magnetized by an applied field, which it will then intensify. The net magnetic field $\mathbf{B}$ is the sum of an applied field $\mu_0\mathbf{H}$ and a magnetization field $\mu_0\mathbf{M}$. If a material is 'magnetically isotropic:' $\mathbf{B} = \mu\mu_0\mathbf{H}$.

Magnetic materials will lose their magnetism above their *Curie Temperature.* For Iron this is 1043K. Nickel is 627K and Neodymium is around 600K. Ferrites (Iron oxide + oxides of Mn, Mg, Cu, Ni) have Curie temperatures of between 573K to 858K.

The *Biot-Savart law* is a useful general-purpose expression for calculating magnetic fields from current line elements $Id\mathbf{l}$ if a geometry doesn't naturally lend itself to Ampere's Theorem being the simplest method of finding $\mathbf{B}$.

$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\int_{line} \frac{d\mathbf{l}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$ where $\mathbf{r}$ is the coordinate at which $\mathbf{B}$ is evaluated and $\mathbf{r}'$ is the location of the current element $Id\mathbf{l}$.

Classic applications of forces due to magnetic fields are: (i) *Hall probe*; (ii) *Mass spectrometer*; (iii) *Velocity selector*; (iv) *Cyclotron*; (v) *Electromagnets*; (vi) *Electric motors*. In electrical circuit applications, solenoids are known as *inductors* (see Electromagnetism problem sheet) and are one of the most basic components alongside resistors and capacitors.

Question 1

(i) Carefully sketch the magnetic field lines from (a) a permanent bar magnet; (b) an air cored solenoid; (c) the same solenoid in (b) but with an iron core.

(ii) A 10cm section of wire is placed perpendicular to a uniform magnetic field of strength B tesla. When 1.23A flows through the wire, a force of magnitude 2.0×10^{-4} N acts on the wire. (a) Draw a diagram to represent the situation and indicated the direction of the force. (b) Calculate B. (c) How does the force differ if the wire is tilted at an angle of 30° to the magnetic field?

(iii) 5.0A flows through a thick copper wire. (a) Sketch the magnetic field lines that manifest around the wire, and also sketch a graph of magnetic field strength B vs radius r, perpendicular from the wire. (b) Calculate the magnetic field strength (in T) at 1.0cm and 2.0cm from the wire.

(iv) An electromagnet is made from wrapping N turns of wire round an iron nail of length 15cm. If the current through the wire is 2.0A, what must N be for the magnetic field strength at the edge of the nail to be $B = 0.509\text{T}$? The relative permeability for (ferrite) iron is: $\mu = 800$.

(v) Two bar magnets repel each other with a force of 0.2N at a distance of 5.0cm. What is the expected force of repulsion at a separation of 1.0cm?

(vi) A steel permanent magnet is constructed with a long groove that is wide enough to run a wire through. The magnet, of mass 0.11kg, is placed on a mass-balance. A copper wire is suspended such that is passes through the groove, but without touching the metal. The groove is 6.0cm long. A 3.0A current is passes through the wire and the mass balance reading increases by 10%. Calculate the magnetic field in the groove. Determine the mass-balance value if 4.0A of current passes through the wire in the opposite direction. Take $g = 9.81\text{N/kg}$.

(vii) In an amplifier, a toroidal coil of 100 turns is wrapped around a ring of iron, with the same μ as in (iv). The radius of the core is 2.0cm and $I = 2.3\text{mA}$. What is the magnetic field strength within the torus? Toroidal coils tend to be more popular than cylindrical geometries in electrical engineering applications. Why do you think this should be?

(viii) Using a magnetic field is a useful way to distinguish alpha and beta radiation. Alpha particles are Helium nuclei (charge +2*e*) and beta particles are electrons (of charge -*e*) in this case. If the particles, moving at speed $v = |\mathbf{v}|$, enter a uniform magnetic field of strength B (acting perpendicular to velocity $\mathbf{v}$) they will be bent into a circular path of radius r (a) Show that $r = \dfrac{mv}{qB}$ where q is the particle charge. (b) Explain the differences in the trajectories of alpha and beta particles. $m_\alpha = 6.6\times10^{-27}\text{kg}$, $m_\beta = 9.1\times10^{-31}\text{kg}$.

(ix) The magnetic field strength on axis to a coil of N turns and radius R is given by: $B(z) = \tfrac{1}{2}\mu_0 NI \dfrac{R^2}{\left(R^2+z^2\right)^{\frac{3}{2}}}$ where z is the perpendicular distance (i.e. 'on axis') from the centre of the coil, which carries current I. A pair of *Helmholtz Coils* of radius $R = 0.15\text{m}$ are separated by distance $2z = 0.15\text{m}$. If $N = 130$ determine the ratio B/I for the field at the centre point between the coils, and sketch B vs I on a graph with appropriate scales.

(x) A *Fine Beam Tube* uses a voltage V to accelerate electrons emitted (effectively at rest on average) from a hot wire. The electrons, which ionize low density hydrogen gas in a spherical tube and result in a purple-blue beam, are bent into a circle of radius r by application of the Helmholtz coils in (ix).

(a) Show that: $e/m_e = 2V/B^2r^2$. (b) Calculate r (in cm) if $I = 1.000\text{A}$ and $V = 170\text{V}$.

Question 2 A *Hall Probe* consists of a cuboid of semiconductor material containing n electron charges per cubic metre. A current I amps passes through the block, which has dimensions $l\times w\times h$. The current direction is parallel to the l dimension. (a) Show that the velocity of electrons moving through the block is $v = I/enwh$. (b) The block is exposed to a magnetic field of strength B, perpendicular to the current direction. This causes a force evB to act on the electrons, which move to the outer edges of the block. This process of charge separation sets up an electric field $E = V_H/w$ where V_H is the Hall voltage between the sides of the block. Show that $B = enhV_H/I$. (c) A Hall probe has thickness $h = 0.10\text{mm}$ has 0.200A passing through it. A Hall Voltage of 1.78mV is developed when the probe is placed near a bar magnet magnetic field of field strength $1000\,\mu\text{T}$. Calculate n. (d) What is the Hall voltage for a 2.0T electromagnet?

Question 3 The *Cassini-Huygens* spacecraft sent to Saturn contained a mass-spectrometer to help analyze the chemical composition of frozen hydrocarbons such as methane (CH_4) present in the moons *Titan* and *Enceladus*. Methane molecules were ionized and accelerated by a voltage V before entering a chamber with uniform magnetic field B perpendicular to the ion velocity. Ion detectors were placed a distance d below the ion entry point. If a methane ion has mass 16.043*u*, where $u = 1.661\times10^{-27}\text{kg}$ and has a charge of + *e*, calculate the magnetic field strength if $d = 30.0\text{cm}$ and $V = 6{,}000\text{V}$. What is the magnetic field strength required to detect a 2+ charged methane molecule?

Question 4 An alpha particle has an energy of 10MeV and a charge of +2*e*. It enters via a thin slit into a vacuum chamber with uniform electric and magnetic fields acting at right angles, and both initially perpendicular to the velocity of the alpha particle. The magnetic field strength is 0.123T and the distance between charged plates which produce the electric field is d = 2.5cm. Calculate the voltage between the plates if the alpha particle passes straight through the chamber undeflected.

Question 5 A *Cyclotron* is a space-efficient mechanism for accelerating charges to high speeds. It was one of the main experimental tools in the pioneering phase of Particle Physics in the early to mid twentieth century. Particles of charge q and mass m enter a Cyclotron and are exposed to a perpendicular magnetic field of strength B. The magnetic field causes the particles to undergo circular motion of radius r. The cyclotron consists of two D-shaped vacuum chambers separated by a gap of distance l. Across this gap is an AC voltage source $V(t) = V_0 \cos(2\pi ft)$.

(a) Show that particles rotate about the Cyclotron at frequency $f_c = qB/2\pi m$, and explain how the AC voltage source can be used to accelerate the particles.

(b) Assuming $V_0 = 100\text{kV}$, calculate the number N of Cyclotron 'orbits' for a proton of mass $m = 1.637 \times 10^{-27}\,\text{kg}$, starting from rest, to attain a speed of 0.05*c*, where the speed of light $c = 2.998 \times 10^8\,\text{ms}^{-1}$.

(c) If $B = 0.2\text{T}$, calculate the *Cyclotron frequency* f_c, and the final radius r_N of the proton orbits.

(d) *Extension*: write a computer program to plot the trajectory of the proton. Take $l = 0.05\text{m}$.

Question 6 The south pole of an electromagnet, aligned West-East, is brought within distance r from a *tangent magnetometer*. The latter essentially a large, freely rotating compass. The needle of the magnetometer deviates from magnetic North (assumed to be a South pole!) and tilts towards the electromagnet by angle θ. The electromagnet has twenty coils around an impure iron nail $(\mu = 1000)$ of length 15cm. 2.0A of current pass through the coils. Assume the magnetic field is uniform up to 0.5cm from the end of the nail, and diminishes as $1/r^3$ thereafter. At r cm the magnetometer tilts by $\theta = 30^\circ$. If the Earth's magnetic field strength is $B_\oplus = 50\mu\text{T}$, calculate r.

Question 7 (a) Use *Ampère's Theorem* to determine the magnetic field strength (in T) at distance r from the centre of a uniform conductor of diameter 2.0cm carrying 1,000A of current. Carefully sketch the magnetic field strength vs r up to 8.0cm from the centre of the conductor. (b) Use *Ampère's Theorem* to show that the magnetic field strength in the centre of a toroidal inductor is $B = \dfrac{\mu\mu_0 NI}{2\pi r}$ where r is the radius, N is the number of coil turns, I is the current and μ is the relative permeability of a ferromagnetic core. (c) Use *Ampère's Theorem* to show that the magnetic field strength in the centre of an 'infinite' air-cored solenoid is $B = \mu_0 nI$ where n is the number of turns per unit length.

Question 8 A sphere of mass 0.321kg contains a powerful rare-earth magnet embedded in its base. When two identical sphere + magnet (s) are brought together, at a distance of 0.5cm they repel each other with a force of 15.0N.

(a) Assume the magnetic field of one sphere is $B = k/z^3$ where z is the sphere separation. Use a formula for magnetic repulsion force $F = -k\,dB/dz$ to determine k for the sphere.

(b) The sphere is now placed above a horizontal coil of $N = 2400$ turns of radius 2.0cm, wrapped around a ferrite core of $\mu = 1000$. The magnetic field strength on axis to a coil of N turns and radius R is given by:

$$B(z) = \tfrac{1}{2}\mu\mu_0 NI \frac{R^2}{\left(R^2 + z^2\right)^{\frac{3}{2}}}$$ where z is the perpendicular distance (i.e. 'on axis') from the centre of the coil,

which carries current I. The coil is wrapped around a ferromagnetic core of relative permeability μ.

What current is required to cause the sphere to levitate 1.0cm above the coil? What about 2.0cm? (Assume the sphere remains stable i.e. the embedded magnet is buried in such a way that the sphere cannot easily flip and be attracted to the coil). Take $g = 9.81\text{Nkg}^{-1}$.

Hint: Assume the magnetic levitation force on the sphere still has the form $F = -k\,dB/dz$, but the magnetic field is now that of the coil.