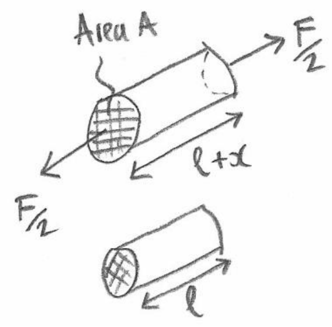
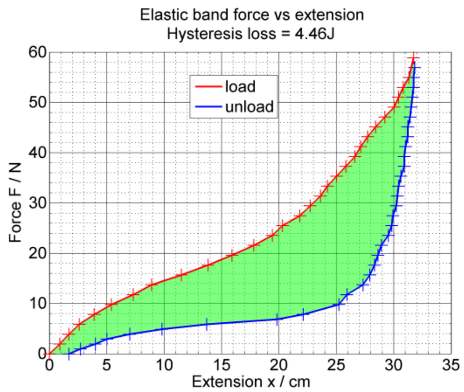
Stress σ = force / area (N/m^2). $\sigma = F/A$. *Strain* ε = extension / original length. $\varepsilon = x/l$. 'load' is the same as 'force. *Young's Modulus* Y = stress / strain. $Y = \sigma/\varepsilon$



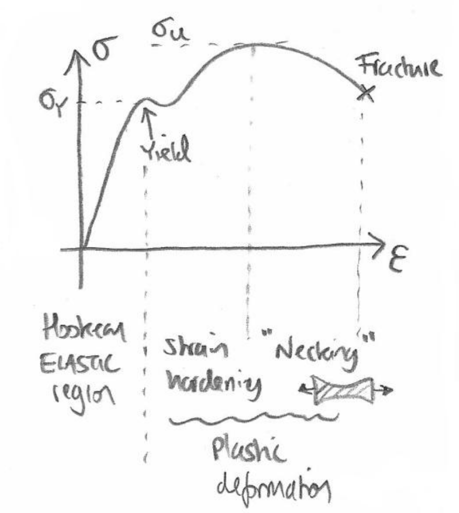
If a material obeys *Hooke's Law,* then applied force is proportional to extension. $F = kx$

Strain energy is $E = \int F(x)dx$ i.e. the area under a force vs extension graph. Materials like elastic bands will tend to release less energy during unloading than the work done during loading. This *hysteresis* loss will be in the form of heat.

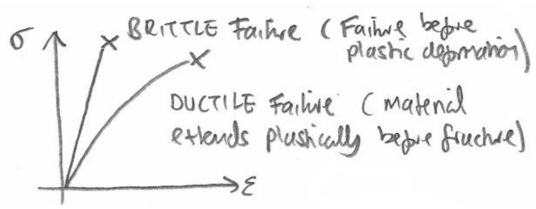


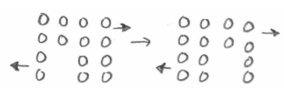
For a *Hookean* material, the *strain energy* is $E = \frac{1}{2}kx^2$ and the *strain energy density* (elastic strain energy per unit volume) is $U = \frac{1}{2}Y\varepsilon^2$. If a material is Hookean (or 'perfectly elastic'), Y is constant and the material will return to its original shape when unloaded.

Most materials follow a characteristic *stress-strain curve*. σ_Y (the 'yield' stress) is the lowest stress for permanent deformation. σ_u (the 'ultimate' stress) is the stress which leads to *ductile* or *brittle* fracture. In the *plastic deformation* region, a material is permanently deformed. A *ductile* material (e.g. copper) can be *drawn into wires* under modest load. A *brittle* material (e.g. ceramic) will *fracture prior to plastic deformation*, i.e. will not *yield*.



Fracture results from the propagation of *cracks*. These originate from *dislocations* in the atomic structure of a material, i.e. a *slip* of one plane of atoms over another.





Stiff materials have a high Young's modulus, and often fail in a brittle manner.

Strong materials (e.g. steel) have a high yield (and ultimate) stress. Most material stresses are in GPa ($1\text{GPa} = 10^9\text{N/m}^2$).

Tough materials are somewhat resilient to brittle fracture. e.g. many metals are tough, but ceramics are not.

Hard materials (like diamond) are very resistant to scratches and other surface deformations.

Question 1

(i) A small Hookean spring of original length 12.3cm is strained by 45% resulting from a load of 6.7N. Calculate (a) the extension x , (b) the spring constant k and (c) the elastic strain energy E /J.

(ii) A 10.000m steel rod in a bridge support structure has a Young's modulus of 200GPa. Calculate the tension (in N) to stretch the rod by 2mm, given the rod has a radius of 42mm and the *Poisson ratio* (the ratio of transverse to axial strain) is deemed to be effectively zero. What would happen is the Poisson ratio was not zero?

(iii) An Ernie Ball *Regular Slinky* guitar string (usually tuned to bottom E at about 82.4Hz) has a diameter of about 46 thousandths of an inch. 1 inch = 25.4mm. The string is clamped to a bench and is loaded by a heavy weight of mass m , dangled vertically via a pulley. If 3.14m of wire extends by 2.7mm, calculate the mass m , assuming $Y = 200\text{GPa}$. Assume $g = 9.81\text{N/kg}$.

(iv) Sketch representative stress vs strain curves (on the same graph) for (a) a slate roof tile; (b) a titanium bicycle frame; (c) a carbon fiber bicycle frame; (d) an aluminum drinks can (e) a plastic spoon (f) a latex swimming hat.

Plastics have Young's moduli between 0.5 and 3.0 GPa, and carbon fiber (suitably weaved and fabricated in a plastic such as epoxy resin) has a Young's modulus of about 230GPa. Titanium is about 120GPa.

(v) A solid steel rod of diameter 3.5mm is used in the design of a sports car. It is to be replaced by a carbon fiber rod of the same length. Calculate the diameter of the carbon fiber rod, and the fractional weight saving. Assume the steel rod has and the carbon fiber replacement must withstand similar forces and strain. Assume: $Y_{steel} = 200\text{GPa}, Y_{cf} = 230\text{GPa},\ \rho_{steel} = 8.00\text{g/cm}^3,\ \rho_{cf} = 2.00\text{g/cm}^3$. If the ultimate stress in the carbon fiber is 3GPa, what is the maximum load?

(vi) In the *Magic Roundabout,* Zebedee was a red-faced, mustached character mounted on a spring. If Zebedee was compressed with maximum downward force of 10N, and then released, he would rise 10cm in the air above his *unloaded* height. What is the spring constant k of his spring? Zebedee has a mass of 0.173kg, and take $g = 9.81\text{N/kg}$.

(vii) If a force of about 1.2N is applied to a nail (diameter 0.5mm), it will just about puncture human skin. Calculate the nails per square centimeter of a 'bed of nails' such that a 80kg person, with cross sectional area of 1.8m $\times$ 0.3m, could lie on without being pierced. What is the puncture stress of human skin?

Question 2 In the fairy tale by Hans Christian Andersen *The Princess and the Pea*, the Princess is bizarrely sensitive to the placement of a pea placed under twenty mattresses. Assume the surface area of a sleeping princess is about 1.6m $\times$ 0.3m, and mass about 55kg. The mattresses are all the finest quality pocket sprung type, and each spring has a diameter of 5.5cm, and are 15cm tall when uncompressed. When the Princess's handmaid checked the springs, she found that she could compress one of them by 5cm with a force of 15N. If one can ignore the effect of the stacked weight of springs (compared to the effect of the Princess), determine the amount the total compression of the bed. Compare this to the diameter of a pea (about 4.0mm).

Question 3 An elastic band is stretched and tension T (N) and extension x are recorded during loading and unloading phases. The units of x correspond to 10cm intervals. T vs x is given by the following equations:

$$T(x) = \begin{cases} 4(x-2)^3 + 32 & \text{load} \\ x^3 & \text{unload} \end{cases}$$

Sketch $T(x)$ and then calculate the energy(in J) lost due to *hysteresis* as the band is loaded, then unloaded.

Question 4 The Tudor warship *Mary Rose* sank in the Solent, just off the Isle of Wight, in 1545. She was raised from the seabed in 1982, and is now preserved in a museum in Portsmouth's historic dockyard. The wooden hull of The Mary Rose was raised using a steel cradle, which was lifted using a huge crane via four straps at each corner of the cradle. Assume the straps have a fixed rectangular cross-sectional area of 20cm $\times$ 0.2cm and are 12m long, with a Young's modulus of 196GPa. If the total density of the cradle + Mary Rose was 50% greater than the density of water, and the *change* in strap extension once the cradle was lifted clear of the water was 10.0mm, calculate the mass of the cradle + Mary Rose. Compare this to the estimated mass of the fully laden warship of about 600 tonnes.

Question 5 A pair of 0.666m wires are joined vertically. One is copper ($Y_c = 117\text{GPa}$) and one is aluminum ($Y_a = 69\text{GPa}$). A mass of 123kg is hung from the wire ensemble, and the combined length stretches by 10.0cm. If the copper wire has a diameter d_c of 0.400mm, calculate the diameter d_a of the aluminum wire.

Question 6 The exoplanet *55 Cancri e* is thought to consist of a huge layers of diamond, a silicon (and then molten iron) interior. Use the information below to calculate the time for a sound 'S' wave to travel between a source on the surface of the diamond layer, bounce of the *bottom* of the silicon layer, and arrive at a receiver on the diamond surface. The source and receiver along-the surface separation is 12,000km, the diamond depth is 3000km and the silicon depth is 4000km. The radius of the (assume spherical) planet is 10,000km. The speed of elastic shear waves is $c = \sqrt{G/\rho}$ where G is the shear modulus, and ρ is the density. For diamond: $G = 478\text{GPa},\ \rho = 3510\text{kgm}^{-3}$, and for silicon: $G = 65\text{GPa},\ \rho = 2380\text{kgm}^{-3}$. Assume the densities of the diamond and silicon remain constant with depth. Sketch the ray paths first, and don't forget Snell's law! You'll probably need a numerical method to solve this....