MP outers. More transformations using matrices.

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Write your answers on this paper. Credit will be given for **N**eatness, **O**rganization of your mathematical argument and clearly shown **W**orkings out!

Many questions will be based upon the following information:

Vertices A,B,C,D of a shape in (x,y) coordinates are collected together in a 2 x 4 matrix

$$\mathbf{V} = \begin{pmatrix} 1 & 5 & 5 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix}$$

Transformation matrices are defined by $\mathbf{A} = \begin{pmatrix} 0 & -1.5 \\ -1.5 & 0 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 1 & \sqrt{3} \\ 0 & 1 \end{pmatrix}$

A shear transformation about (0,0) parallel to the x axis of angle θ clockwise from the y axis is given by

$$\mathbf{S} = \begin{pmatrix} 1 & \tan\theta \\ 0 & 1 \end{pmatrix}$$

Note a 'shear factor ' is sometimes used. This = $\tan\theta$

Questions

1. Evaluate the following:

(a) $\mathbf{A\,V}$ $= \begin{pmatrix} 0 & -1.5 \\ -1.5 & 0 \end{pmatrix}\begin{pmatrix} 1 & 5 & 5 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix}$

$= \begin{pmatrix} -1.5 & -1.5 & -6 & -6 \\ -1.5 & -7.5 & -7.5 & -1.5 \end{pmatrix}$

(c) $\mathbf{B\,V}$ $= \begin{pmatrix} 1 & \sqrt{3} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 & 5 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 2.73 & 6.73 & 11.93 & 7.93 \\ 1 & 1 & 4 & 4 \end{pmatrix}$

1. Evaluate the following:

(a) $\mathbf{A}^{-1}$ $-\frac{1}{2.25}\begin{pmatrix} 0 & 1.5 \\ 1.5 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -2/3 \\ -2/3 & 0 \end{pmatrix}$

(b) $\mathbf{B}^{-1}$ $= \begin{pmatrix} 1 & -\sqrt{3} \\ 0 & 1 \end{pmatrix}$

(c) $\mathbf{A}^{-1}\,\mathbf{V}$ $= \begin{pmatrix} 0 & -2/3 \\ -2/3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 5 & 5 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix}$

$= \begin{pmatrix} -0.67 & -0.67 & -2.67 & -2.67 \\ -0.67 & -3.33 & -3.33 & -0.67 \end{pmatrix}$

(d) $\mathbf{B}^{-1}\,\mathbf{V}$ $= \begin{pmatrix} 1 & -\sqrt{3} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 5 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix}$

$= \begin{pmatrix} -0.73 & 3.27 & -1.93 & -5.93 \\ 1 & 1 & 4 & 4 \end{pmatrix}$

(d) What is the *determinant* of

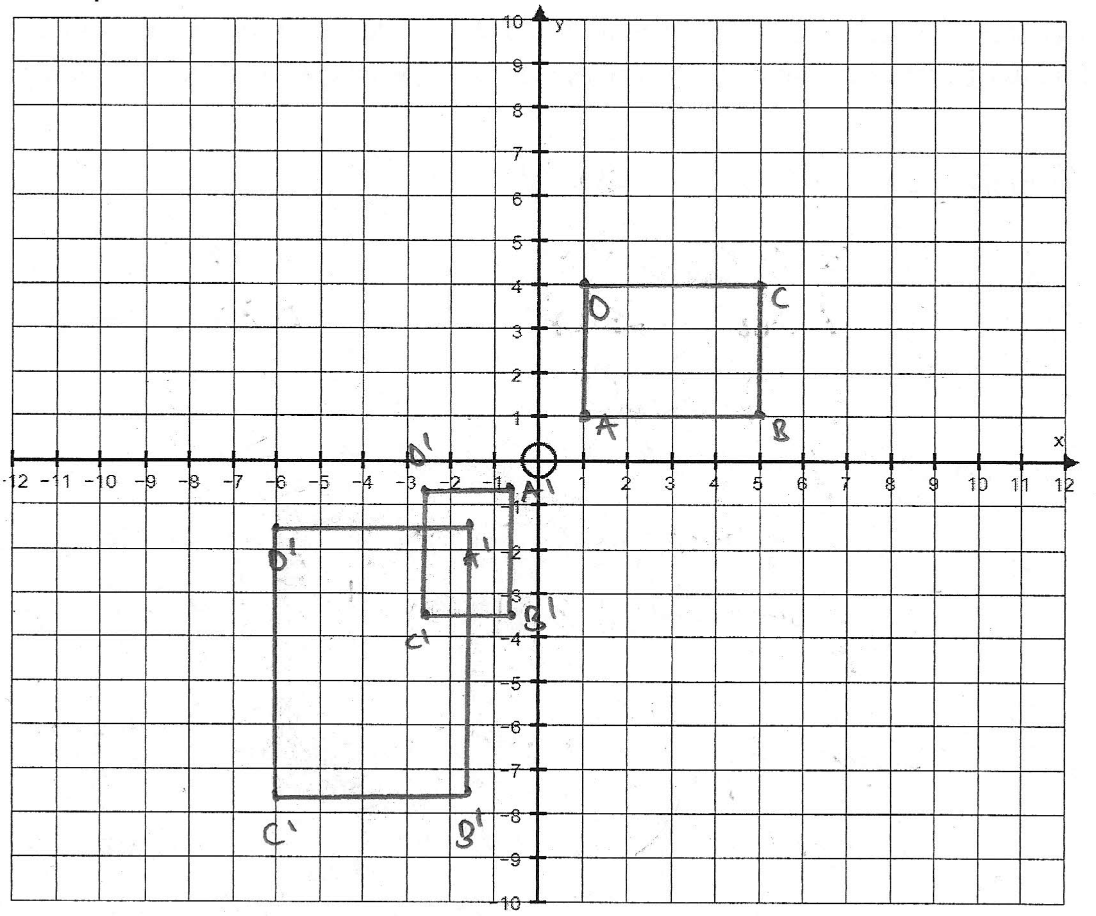
(i) $\mathbf{A}$ $-9/4$

(ii) $\mathbf{B}$ 1

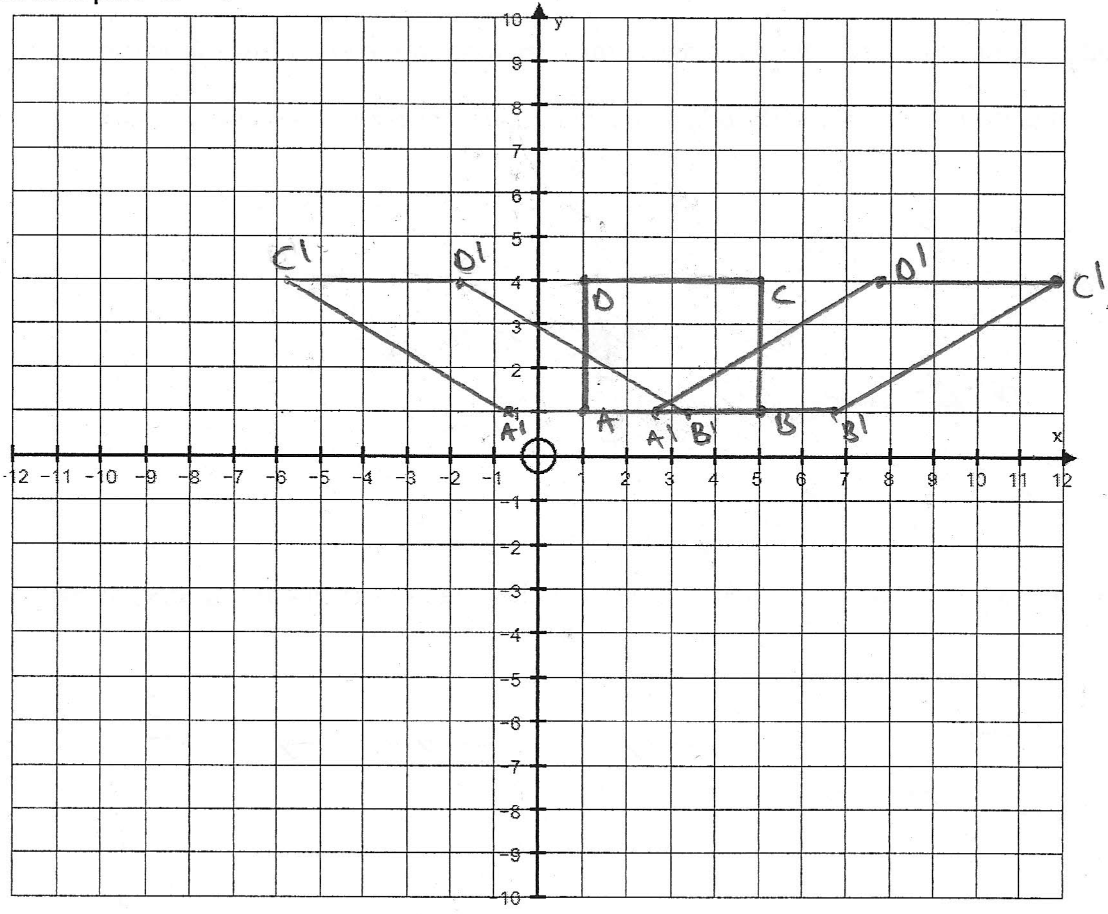
(iii) $\mathbf{B}^{-1}$ 1

2. On the graphs below plot the shape ABCD and then where this shape transforms to under the matrix transformations given. In all cases describe carefully in words the transformation and label new vertices A', B', C', D'

A V and then plot A^{-1} V



B V and then plot B^{-1} V



3. In order to rotate a shape about the origin by 42 degrees clockwise, we can use a transformation matrix defined by:

$$\mathbf{R} = \begin{pmatrix} \cos 42^\circ & \sin 42^\circ \\ -\sin 42^\circ & \cos 42^\circ \end{pmatrix}$$

Without multiplying out the matrices, write out in words what you think the following transformations are. ** For a virtus point, work out using a calculator (make sure it is in degrees mode!) what these matrices are. Show all working! **

(a) $\mathbf{R}^4$ Rotation by $4 \times 42^\circ$ clockwise $= 168^\circ$

** $\underline{\underline{R}}^4 = \begin{pmatrix} \cos 168^\circ & \sin 168^\circ \\ -\sin 168^\circ & \cos 168^\circ \end{pmatrix} \approx \begin{pmatrix} -0.98 & 0.21 \\ -0.21 & -0.98 \end{pmatrix}$

(a) $\mathbf{R}^{-3.5}$ Rotation by $3.5 \times 42^\circ$ anticlockwise $= 147^\circ$

** $\underline{\underline{R}}^{-3.5} = \begin{pmatrix} \cos(-147^\circ) & \sin(-147^\circ) \\ -\sin(-147^\circ) & \cos(-147^\circ) \end{pmatrix} \approx \begin{pmatrix} -0.84 & -0.54 \\ 0.54 & -0.84 \end{pmatrix}$

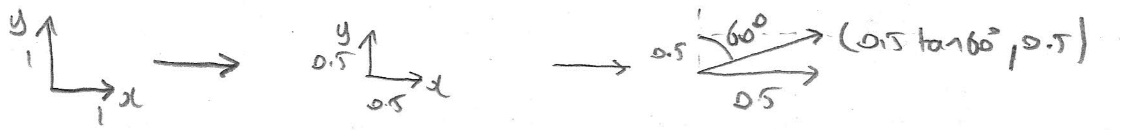
(a) $\mathbf{R}^{\pi}$

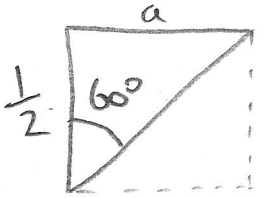
$\underline{\underline{R}}^{\pi}$ Rotation by $\pi \times 42^\circ$ clockwise $\approx 132^\circ$

** $\underline{\underline{R}}^{\pi} \approx \begin{pmatrix} \cos(42^\circ \times \pi) & \sin(42^\circ \times \pi) \\ -\sin(42^\circ \times \pi) & \cos(42^\circ \times \pi) \end{pmatrix} \approx \begin{pmatrix} -0.67 & 0.74 \\ -0.74 & -0.67 \end{pmatrix}$

4. Using 'red and blue basis vector diagrams' derive matrices which represent the following transformations

(i) Enlargement scale factor 0.5 about the origin, shear of angle 60 degrees parallel to the x axis.



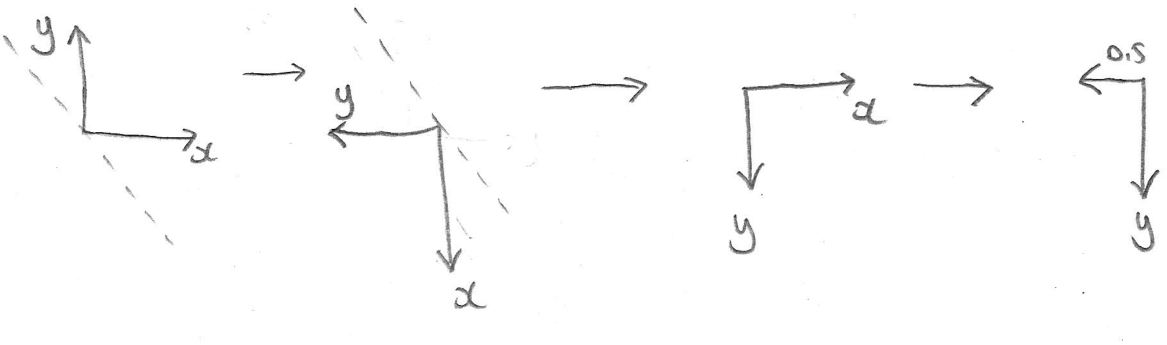


$\tan 60^\circ = \dfrac{a}{\frac{1}{2}}$

$\Rightarrow a = \frac{1}{2}\tan 60^\circ$

$\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\tan 60^\circ \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{1}{2} \end{pmatrix}$

(ii) Reflection in y=-x then rotation anticlockwise by 90 degrees, then stretch in x axis by -0.5. (All stages about the origin).



$\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -1 \end{pmatrix}$