

Write your answers on this paper. Credit will be given for **Neatness**, **Organization** of your mathematical argument and clearly shown **Workings** out!

Many questions will be based upon the following information:

Vertices A,B,C,D of a shape in (x,y) coordinates are collected together in a 2 x 4 matrix

$$V = \begin{pmatrix} 1 & 5 & 8 & 2 \\ 1 & 1 & 4 & 6 \end{pmatrix}$$

Transformation matrices are defined by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} -1.5 & 0 \\ 0 & 1.5 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix}$$

The *inverse* of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$ad-bc$ is the determinant of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Questions

1. Evaluate the following:

(a) AV

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 8 & 2 \\ 1 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 & 8 & 2 \\ -1 & -1 & -4 & -6 \end{pmatrix}$$

(b) BV

$$= \begin{pmatrix} -1.5 & 0 \\ 0 & 1.5 \end{pmatrix} \begin{pmatrix} 1 & 5 & 8 & 2 \\ 1 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -1.5 & -7.5 & -12 & -3 \\ 1.5 & 1.5 & 6 & 9 \end{pmatrix}$$

(c) CV

$$= \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix} \begin{pmatrix} 1 & 5 & 8 & 2 \\ 1 & 1 & 4 & 6 \end{pmatrix}$$

1. Evaluate the following:

(a) C^{-1}

$$= \begin{pmatrix} 1.37 & 3.37 & 7.48 & 6.22 \\ -0.37 & -3.85 & -4.96 & 1.26 \end{pmatrix}$$

$$= \frac{1}{0.25 + 0.7569} \begin{pmatrix} 0.5 & -0.87 \\ 0.87 & 0.5 \end{pmatrix} \approx \begin{pmatrix} 0.5 & -0.87 \\ 0.87 & 0.5 \end{pmatrix}$$

This should be

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$\text{so } \det(C) = 1$$

$$\left[\frac{1}{4} + \frac{3}{4} = 1 \right]$$

$$(b) \quad C^2 = \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix} \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} -0.5 & 0.87 \\ -0.87 & -0.5 \end{pmatrix}$$

$$(c) \quad C^{-1}V = \begin{pmatrix} 0.5 & -0.87 \\ 0.87 & 0.5 \end{pmatrix} \begin{pmatrix} 1 & 5 & 8 & 2 \\ 1 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -0.37 & 1.63 & 0.52 & -4.22 \\ 1.37 & 4.85 & 8.96 & 4.74 \end{pmatrix}$$

$$(d) \quad C^2V = \begin{pmatrix} -0.5 & 0.87 \\ -0.87 & -0.5 \end{pmatrix} \begin{pmatrix} 1 & 5 & 8 & 2 \\ 1 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 0.37 & -1.63 & -0.52 & 4.22 \\ -1.37 & -4.85 & -8.96 & -4.74 \end{pmatrix}$$

(d) What is the *determinant* of

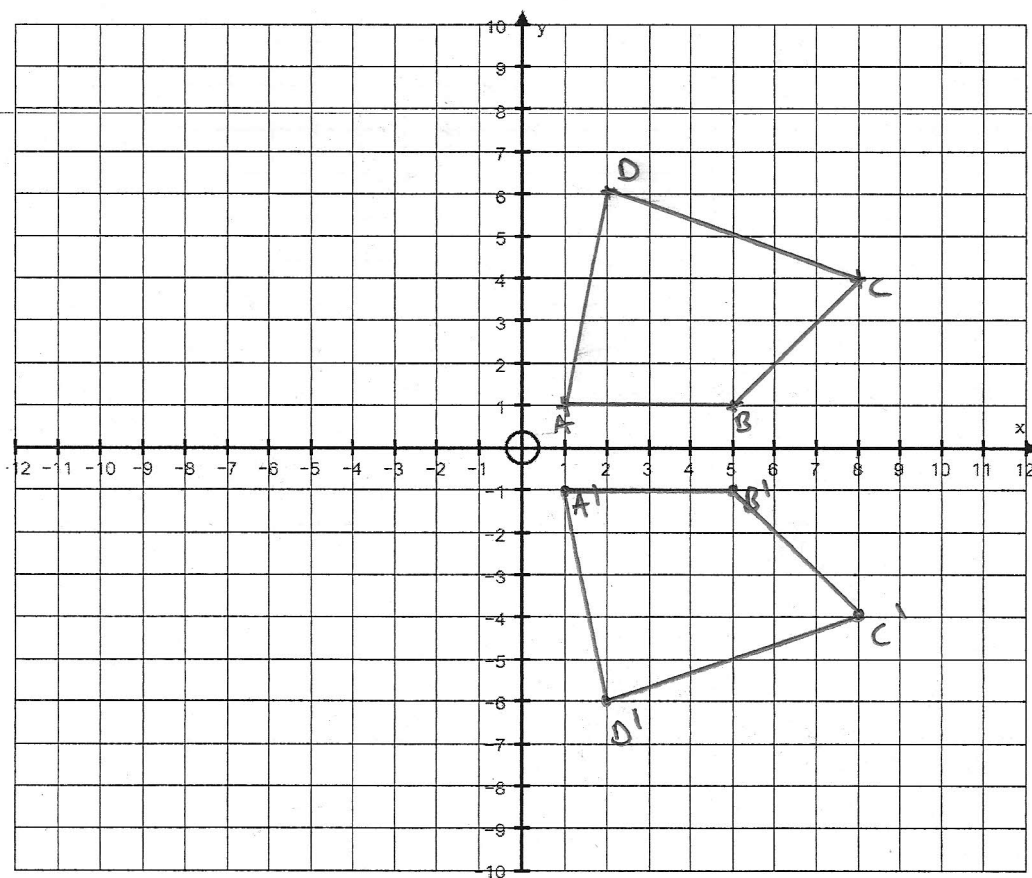
(i) $A \quad -1$

(ii) $C \quad \approx 1$

(iii) $C^{-1} \quad \approx 1$

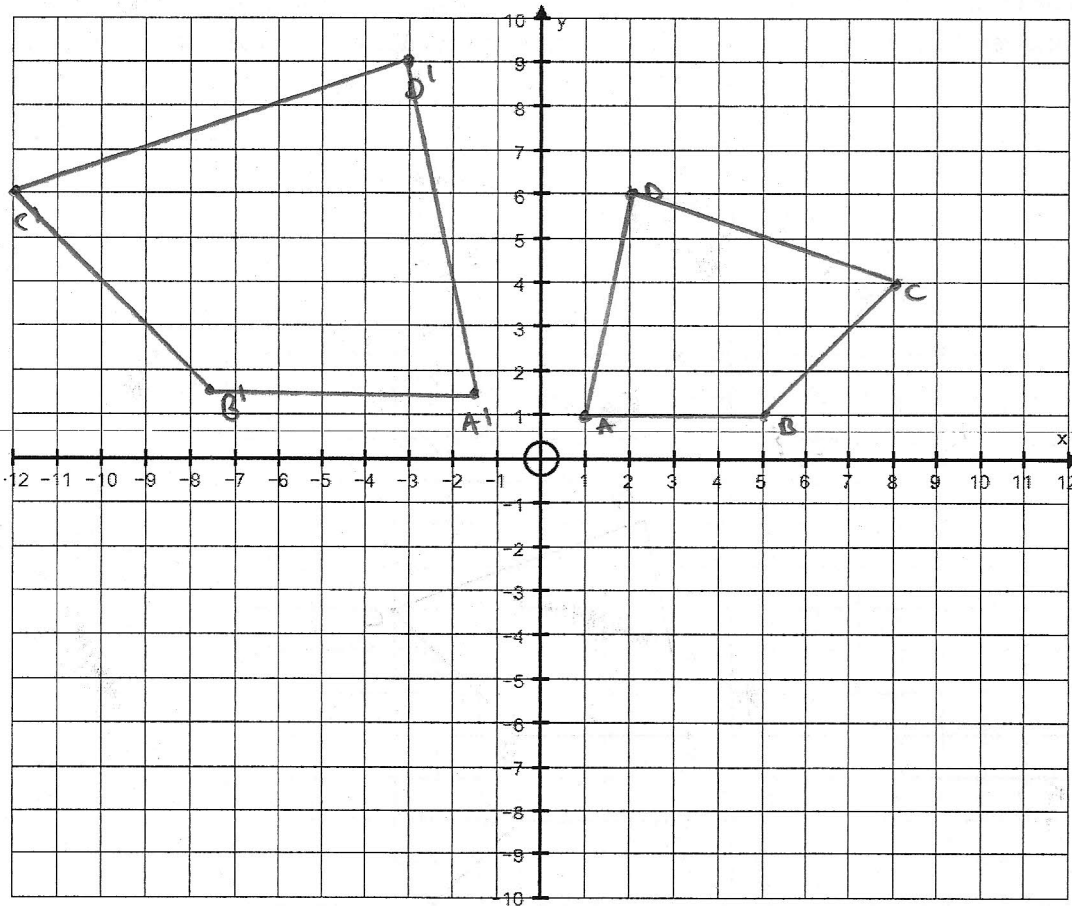
2. On the graphs below plot the shape ABCD and then where this shape transforms to under the matrix transformations given. In all cases describe carefully in words the transformation and label new vertices A', B', C', D'

AV



Reflection in x axis

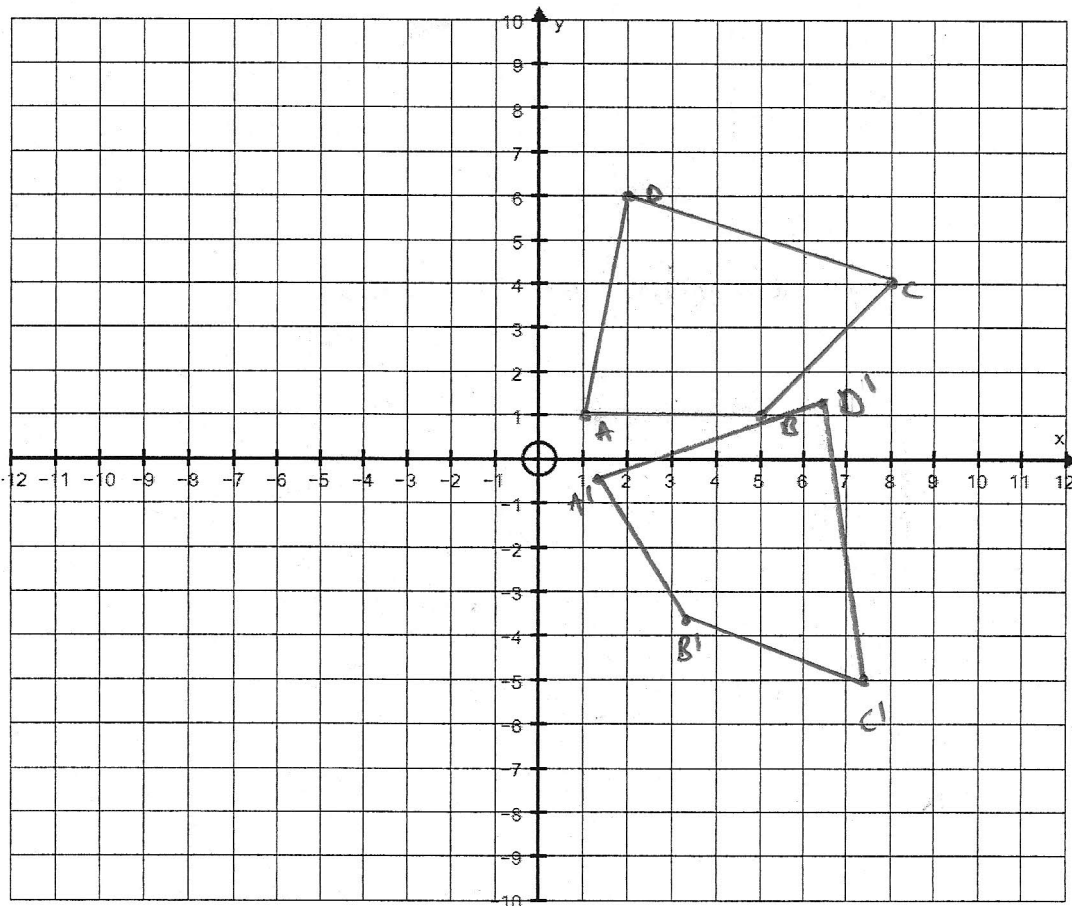
BV



Stretch
 $-\frac{3}{2} \parallel x$
 $\frac{3}{2} \parallel y$

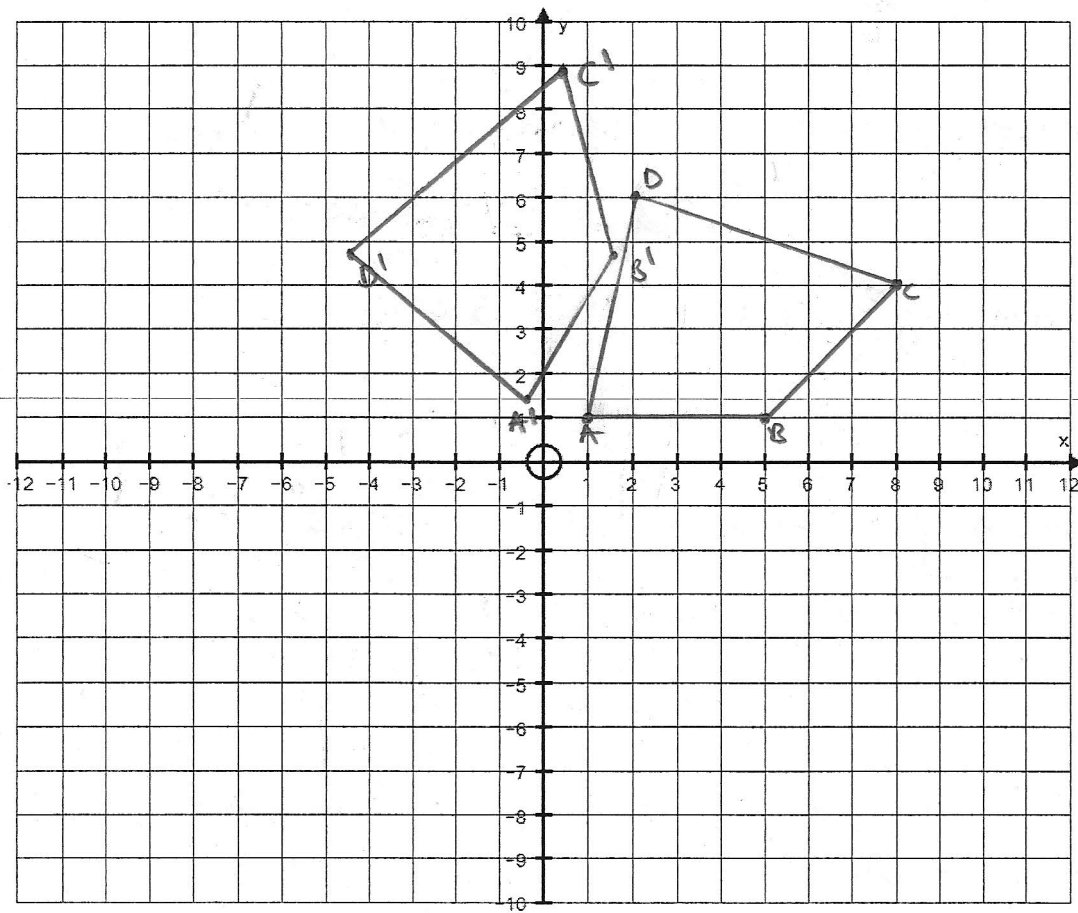
or enlargement
 SF $\frac{3}{2}$ from
 (0,0), then
 reflection in
 y axis.

CV



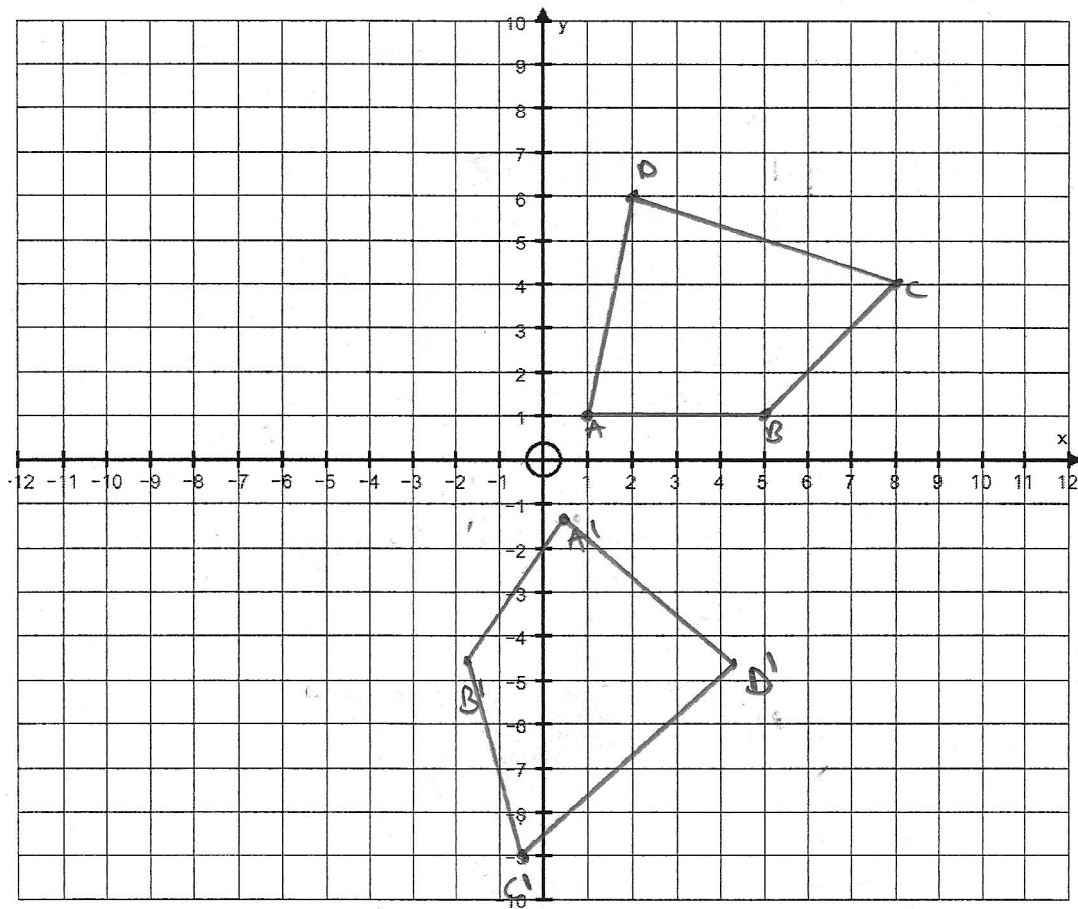
Rotation
 clockwise by
 60°
 about (0,0)

C⁻¹V



Rotation 60°
anti clockwise
about (0,0)

C²V



Rotation 120°
clockwise
about (0,0)

3. What transformation do you think the following matrices represent? Hint: don't try to evaluate them - try and spot the pattern using the work you have already done!

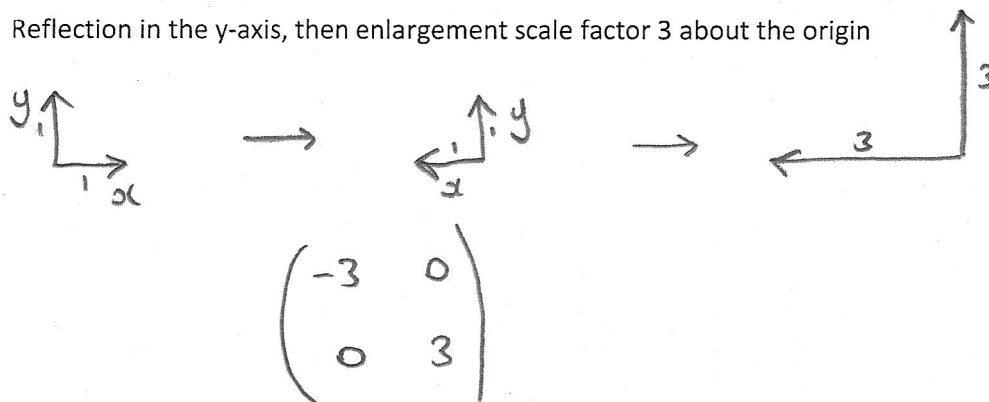
(a) C^4 Rotation of $4 \times 60^\circ = 240^\circ$ clockwise about (0,0)

(a) C^{-5} Rotation of $5 \times 60^\circ = 300^\circ$ anti clockwise about (0,0)

(a) C^{17} Rotation of $17 \times 60^\circ = 1020^\circ$ (2 & 3° rotations) clockwise about (0,0)

4. Using 'red and blue basis vector diagrams' derive matrices which represent the following transformations

- (i) Reflection in the y-axis, then enlargement scale factor 3 about the origin



- (ii) Reflection in $y=x$ then rotation clockwise by 90 degrees, then stretch in y axis by 0.5. (All stages about the origin).

