Yet more matrix transforms!

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1. Multiply out the following matrices. Elements may be left in terms of unknowns *a* and *b*

(i) $2\begin{pmatrix}-1 & 3\\ 5a & 2\end{pmatrix}\begin{pmatrix}7 & 2a\\ 0 & -1\end{pmatrix}$

$$= 2\begin{pmatrix}-7 & -2a-3\\ 35a & 10a^2-2\end{pmatrix}$$

$$= \begin{pmatrix}-14 & -4a-6\\ 70a & 20a^2-4\end{pmatrix}$$

(ii) $\begin{pmatrix}1 & -2 & 5\\ 0 & -3 & -9\\ 7 & 8 & -1\end{pmatrix}\begin{pmatrix}-1 & 12 & -2\\ -6 & -1 & 0\\ 1 & 5 & 7\end{pmatrix}$

$$= \begin{pmatrix}16 & 39 & 33\\ 9 & -42 & -63\\ -56 & 71 & -21\end{pmatrix}$$

(iii) $\begin{pmatrix}-4a & 7\\ 5 & 3b\end{pmatrix}^{-1}$

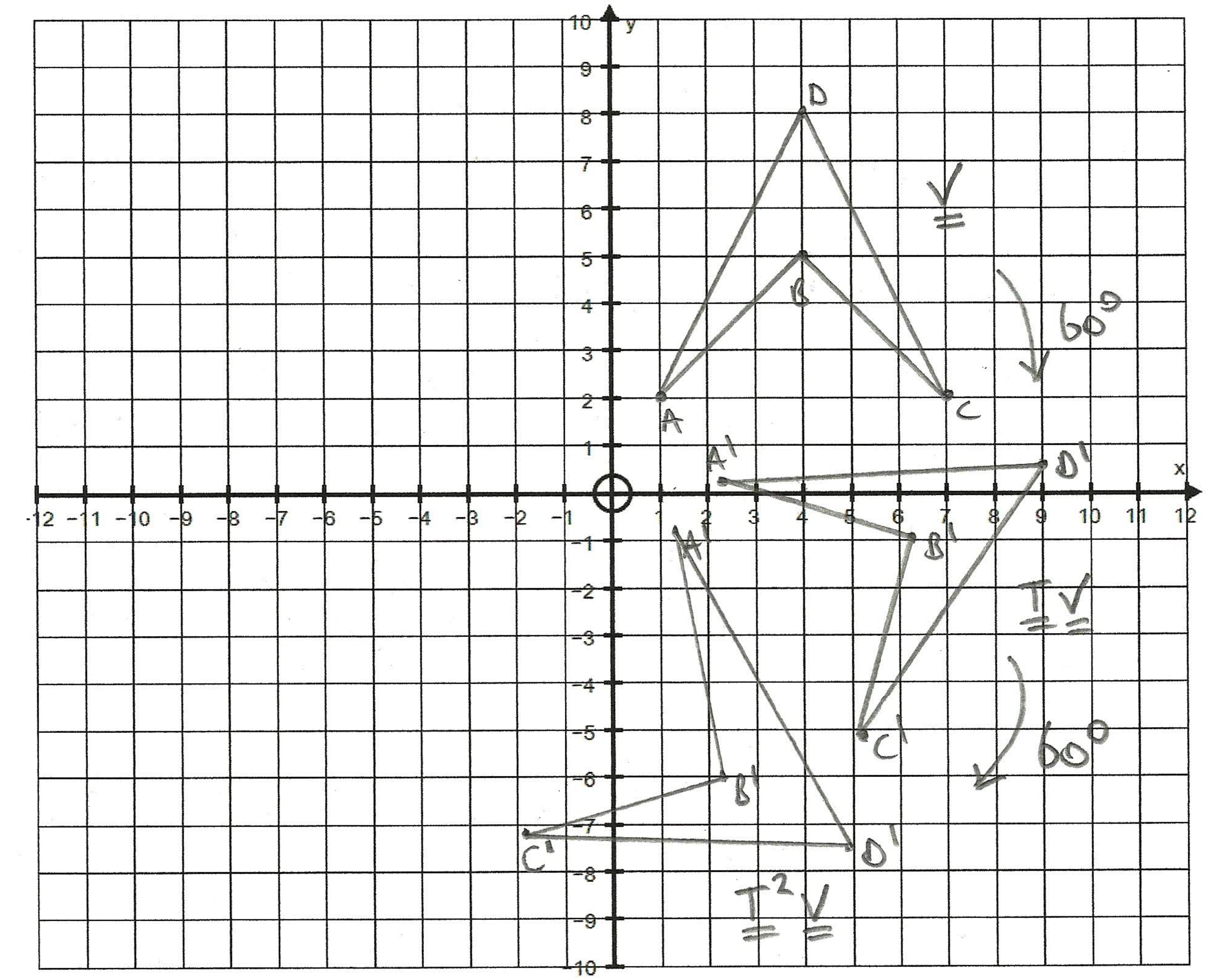
$$= \frac{1}{-12ab-35}\begin{pmatrix}3b & -7\\ -5 & -4a\end{pmatrix}$$

$$= \begin{pmatrix}\frac{-3b}{12ab+35} & \frac{7}{12ab+35}\\ \frac{5}{12ab+35} & \frac{4a}{12ab+35}\end{pmatrix}$$

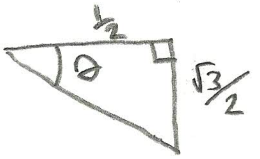
2. Vertices A,B,C,D of a shape in (x,y) coordinates are collected together in a 2 x 4 matrix

$$\mathbf{V} = \begin{pmatrix} 1 & 4 & 7 & 4 \\ 2 & 5 & 2 & 8 \end{pmatrix}$$

(a) Plot the shape on the axes below and label all vertices [2 marks]



(b) A matrix transformation has form



$\theta = \tan^{-1}(\sqrt{3})$

$= \boxed{60°}$

$$\mathbf{T} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \simeq \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix}$$

Evaluate to 2.d.p the elements of **T** and therefore work out **TV** . Plot this on the graph above. Describe in words the transformation.

$$\underline{\underline{T}}\,\underline{\underline{V}} \simeq \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix} \begin{pmatrix} 1 & 4 & 7 & 4 \\ 2 & 5 & 2 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 2.24 & 6.35 & 5.24 & 8.96 \\ 0.13 & -0.98 & -5.09 & 0.52 \end{pmatrix}$$

60° clockwise rotation about (0,0)

(ii) Calculate $\mathbf{T}^2\mathbf{V}$ and plot it on the graph above.

$$\underline{\underline{T}}^2 \simeq \begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix}\begin{pmatrix} 0.5 & 0.87 \\ -0.87 & 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} -0.51 & 0.87 \\ -0.87 & -0.51 \end{pmatrix}$$

This is a 120° rotation clockwise about (0,0)

$$\therefore \underline{\underline{T}}^2\underline{\underline{V}} = \begin{pmatrix} -0.51 & 0.87 \\ -0.87 & -0.51 \end{pmatrix}\begin{pmatrix} 1 & 4 & 7 & 4 \\ 2 & 5 & 2 & 8 \end{pmatrix} = \begin{pmatrix} 1.23 & 2.32 & -1.81 & 4.93 \\ -1.88 & -6.01 & -7.10 & -7.54 \end{pmatrix}$$

(iii) Work out what value of n results in $\mathbf{T}^n\mathbf{V} = \mathbf{V}$

i.e. the effect of n applications of transformation $\mathbf{T}$ on $\mathbf{V}$ brings them back to where they started. Write down $\mathbf{T}^n$ in this case and name this special transformation!

$6 \times 60° = 360° \quad \therefore \quad n = 6.$

$$\underline{\underline{T}}^6 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ the identity transformation.

(v) Without evaluating the matrices, describe the transformations

(i) $\mathbf{T}^4$

$4 \times 60° = 240°$ rotation clockwise about (0,0)

(ii) $\mathbf{T}^{-3}$

$-3 \times 60° = -180°$ rotation about (0,0)

[This could be clockwise or anticlockwise of course!]

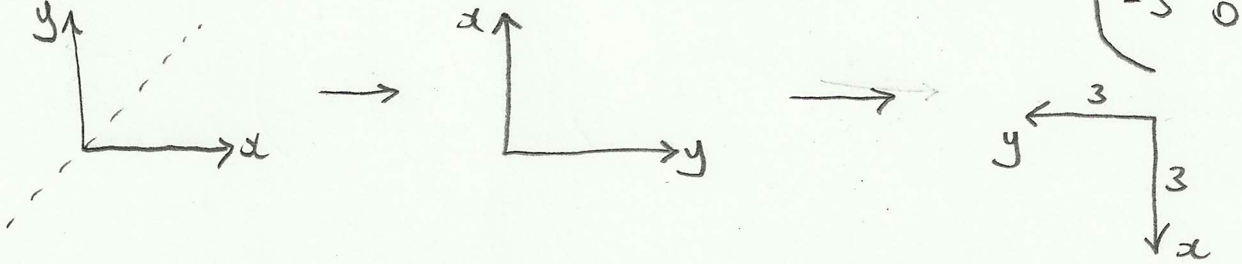
(iii) $\mathbf{T}^{1.234}$

$1.234 \times 60° = 74.04°$ rotation clockwise about (0,0)

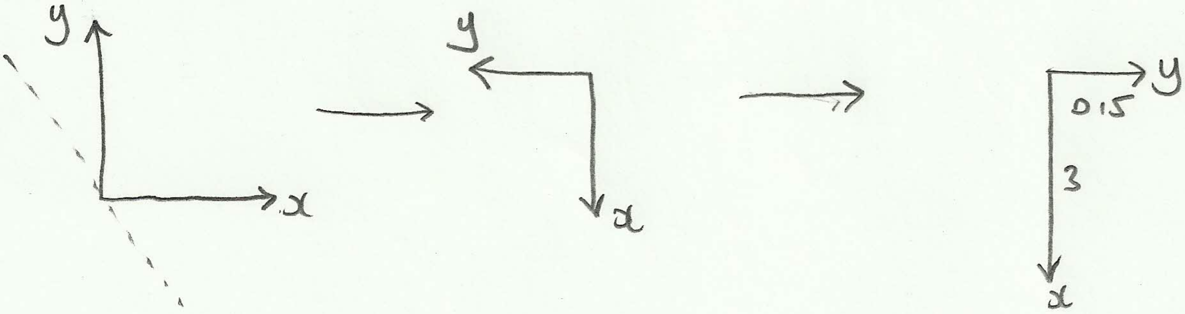
3) Using 'red and blue basis vector diagrams' derive matrices which represent the following transformations

(i) Reflection in the y = x , then enlargement scale factor -3 about the origin

$$\begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix}$$



(ii) Reflection in the y = -x, then a stretch of scale factor 3 parallel to the y axis and -0.5 parallel to the x axis.



$$\begin{pmatrix} 0 & 0.5 \\ -3 & 0 \end{pmatrix}$$

4) Consider the matrices below

$$\mathbf{T} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} -1 & -4 & -7 & -4 \\ 2 & 5 & 2 & 8 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} -4 & -4 & -4 & -4 \\ 5 & 5 & 5 & 5 \end{pmatrix}$$

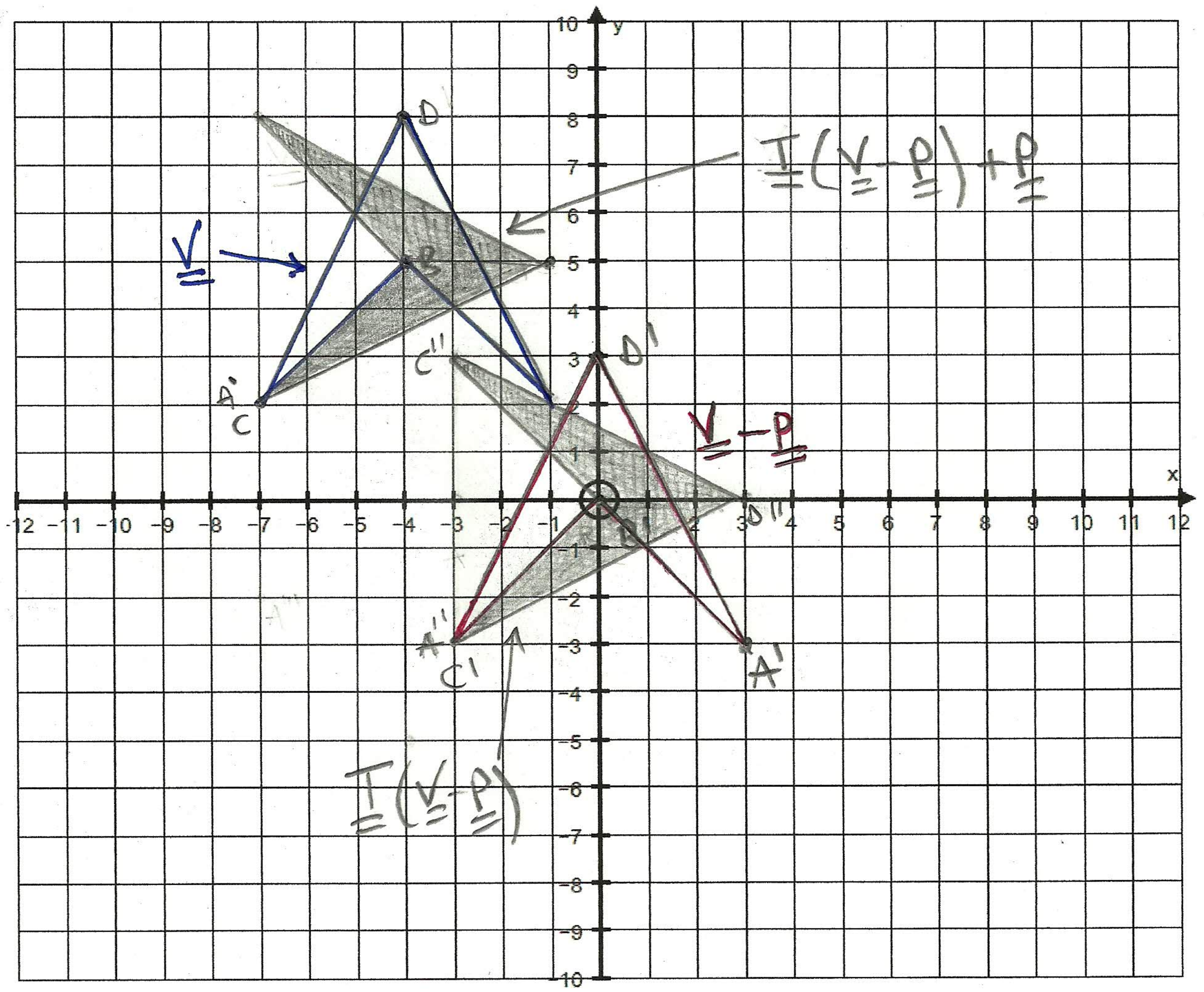
Work out the following in stages and plot each result on the graph below:

1. $\mathbf{V}$

2. $\mathbf{V} - \mathbf{P}$ $\begin{pmatrix} 3 & 0 & -3 & 0 \\ -3 & 0 & -3 & 3 \end{pmatrix}$

3. $\mathbf{T}(\mathbf{V} - \mathbf{P})$ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 & -3 & 0 \\ -3 & 0 & -3 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 0 & -3 & 3 \\ -3 & 0 & 3 & 0 \end{pmatrix}$

4. $\mathbf{T}(\mathbf{V} - \mathbf{P}) + \mathbf{P}$ $= \begin{pmatrix} -7 & -4 & -7 & -1 \\ 2 & 5 & 8 & 5 \end{pmatrix}$



Describe the transformation $\mathbf{T}(\mathbf{V}-\mathbf{P})+\mathbf{P}$

90° clockwise rotation about point (-4,5)

What does a transformation of the form $\mathbf{T}(\mathbf{V}-\mathbf{P})+\mathbf{P}$ allow us to do in general?

A matrix transformation (represented by $\underline{\underline{T}}$) of $\underline{\underline{V}}$ about any point (x_*, y_*)

$$\underline{\underline{P}} = \underbrace{\begin{pmatrix} x_* & x_* & \cdots & x_* \\ y_* & y_* & \cdots & y_* \end{pmatrix}}_{\#\text{ columns } = \#\text{ coordinates of } \underline{\underline{V}}}$$