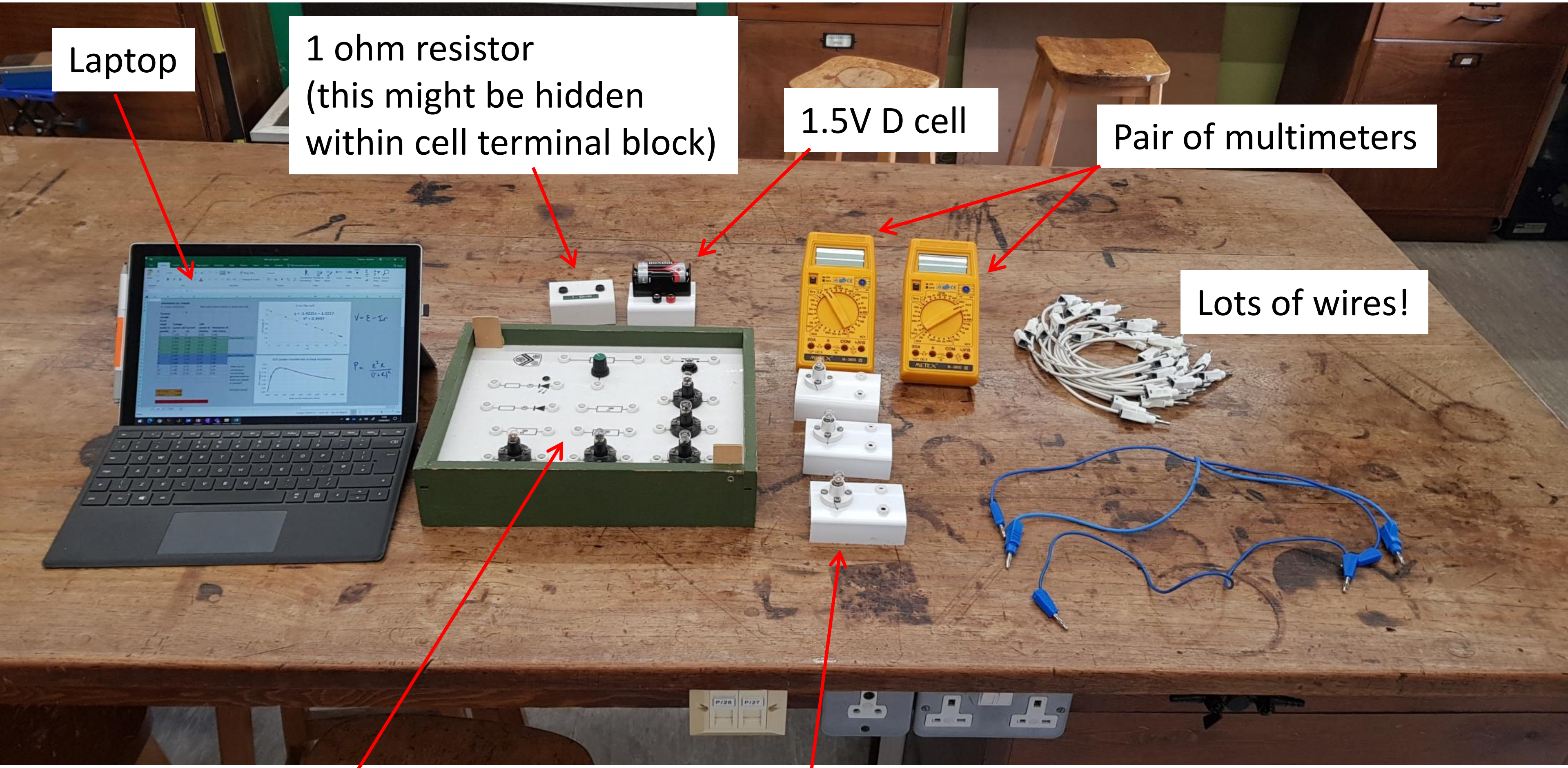
TEACHER NOTES

Youtube video

Max power from a cell

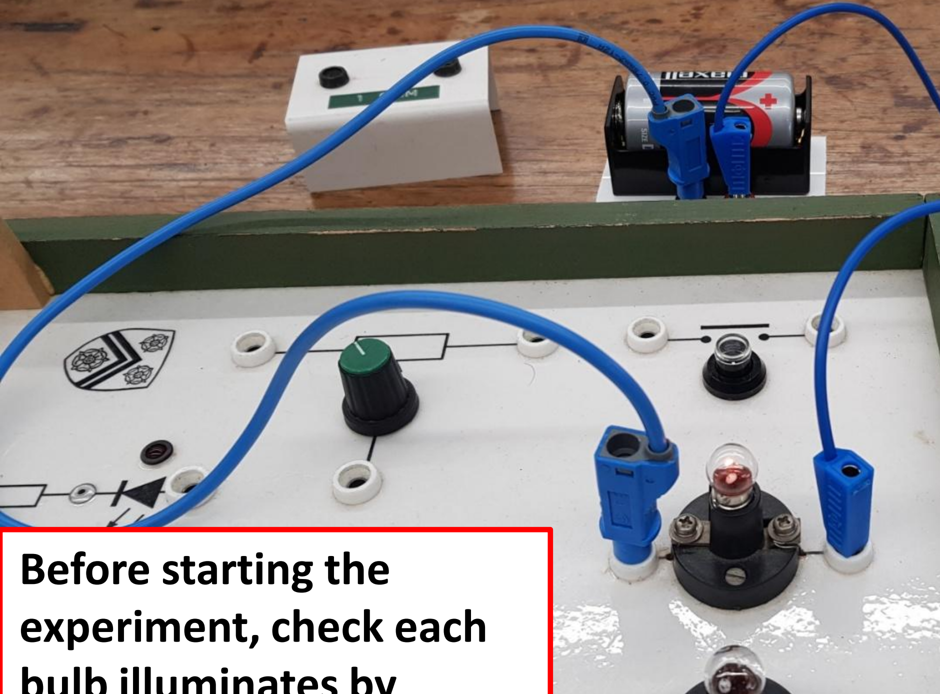
Dr Andrew French. September 2021.

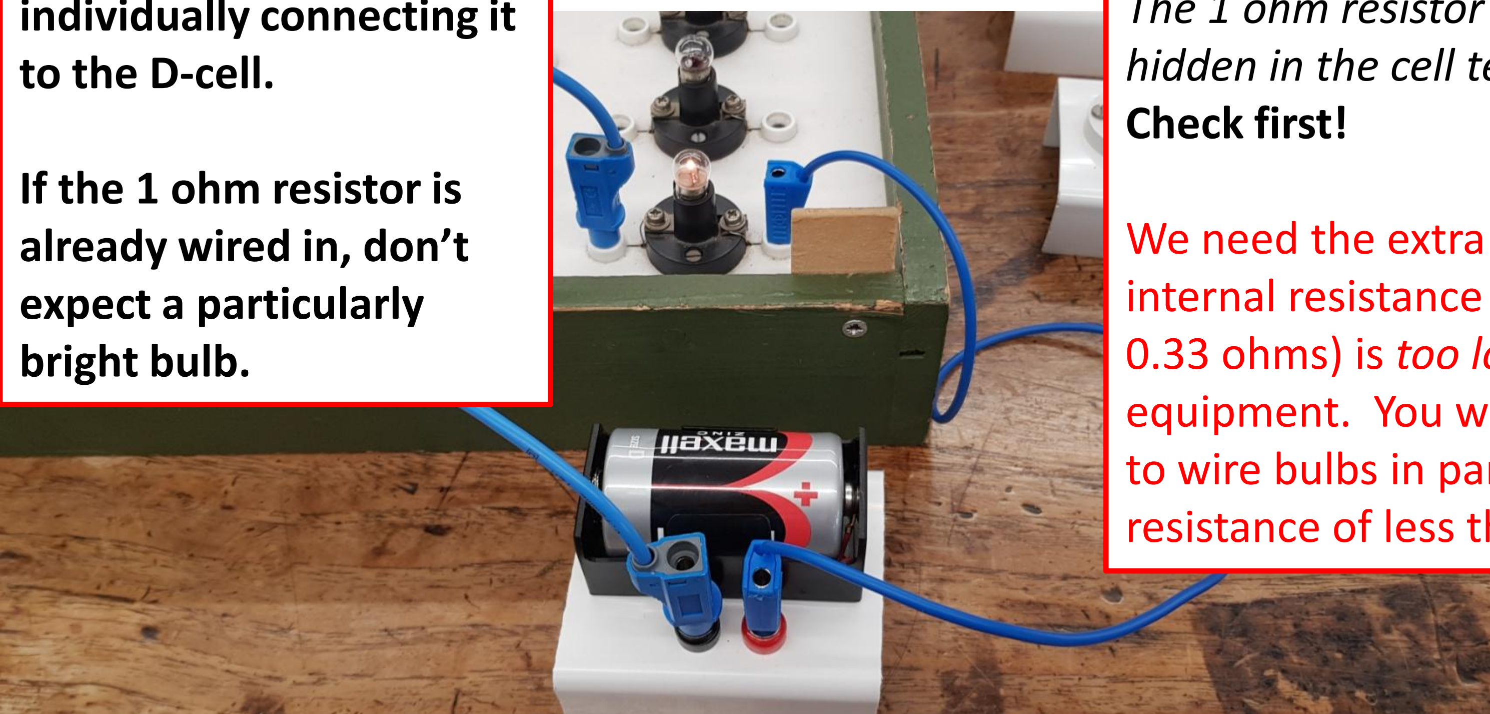
Equipment



Green Boards – with five bulbs.

Three 1.5V bulbs in terminal blocks





Before starting the experiment, check each bulb illuminates by individually connecting it to the D-cell.

If the 1 ohm resistor is already wired in, don't expect a particularly bright bulb.

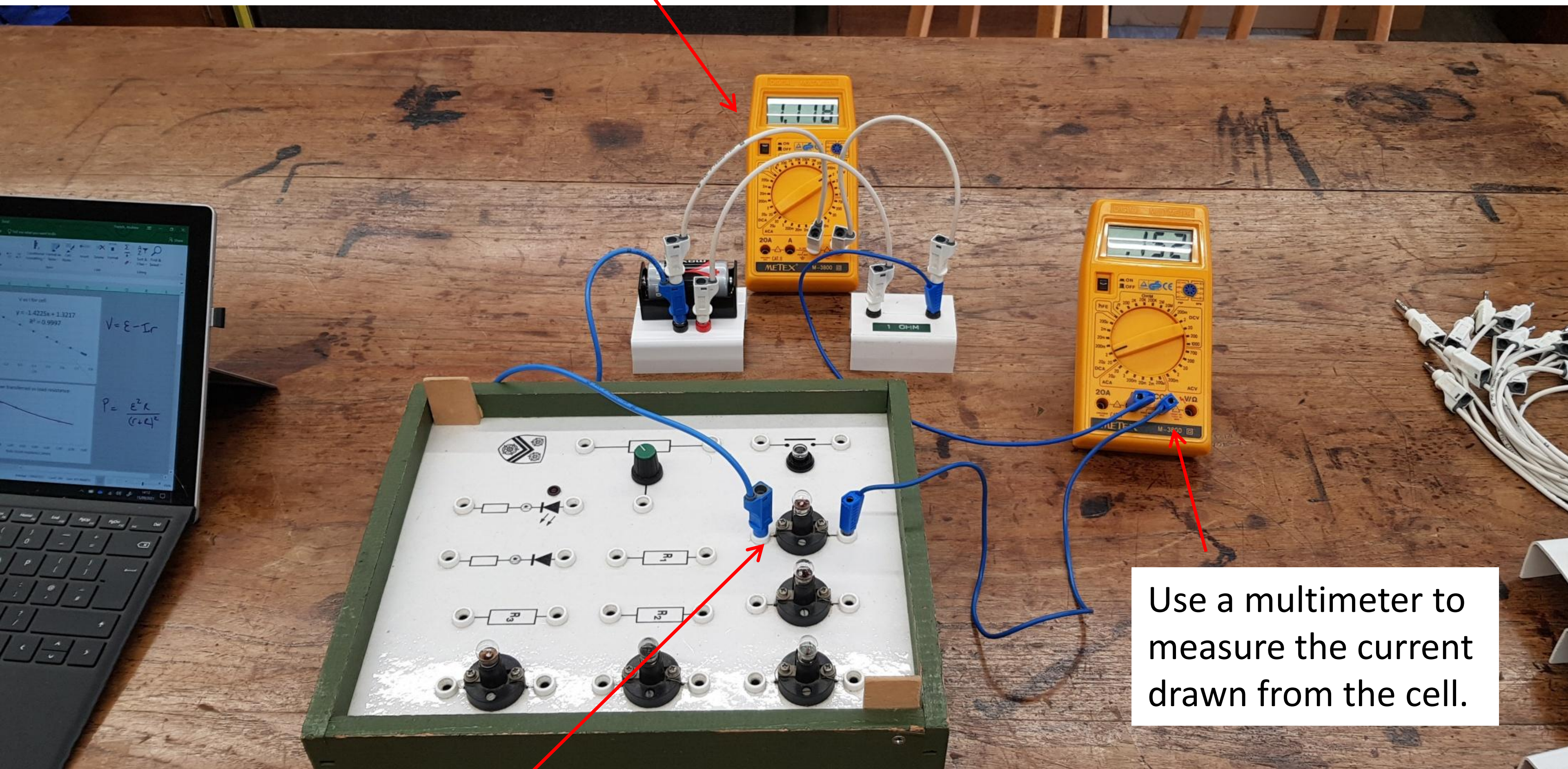
1.5V D cell + a 1 ohm resistor

The 1 ohm resistor will probably be hidden in the cell terminal block. **Check first!**

We need the extra resistor since the internal resistance of the D cell (about 0.33 ohms) is *too low* for this equipment. You will probably struggle to wire bulbs in parallel to attain a resistance of less than an ohm or so.

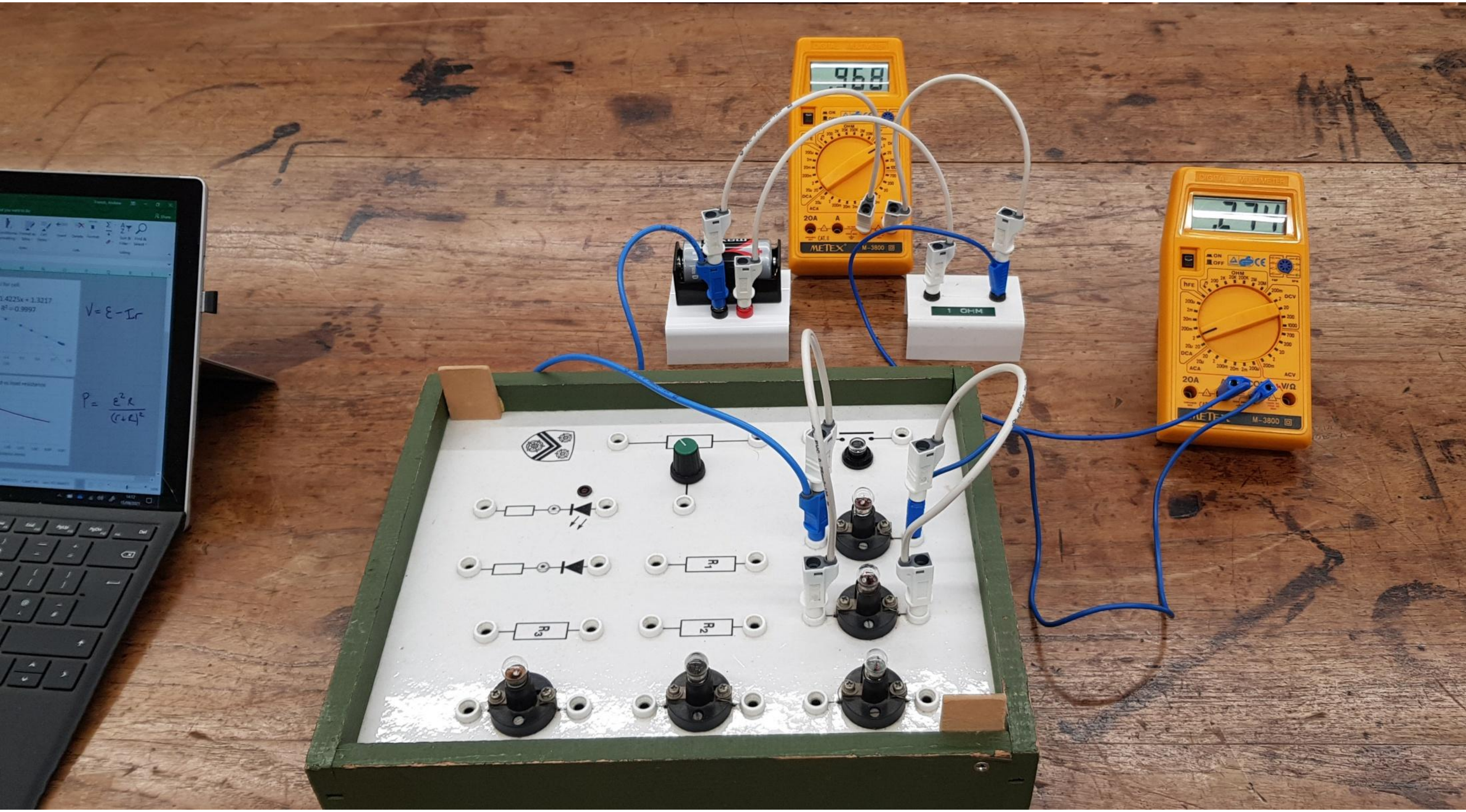
Use a multimeter to measure the potential difference across the D cell, *with the 1ohm resistor in series with the cell*

It might already be wired and hidden!

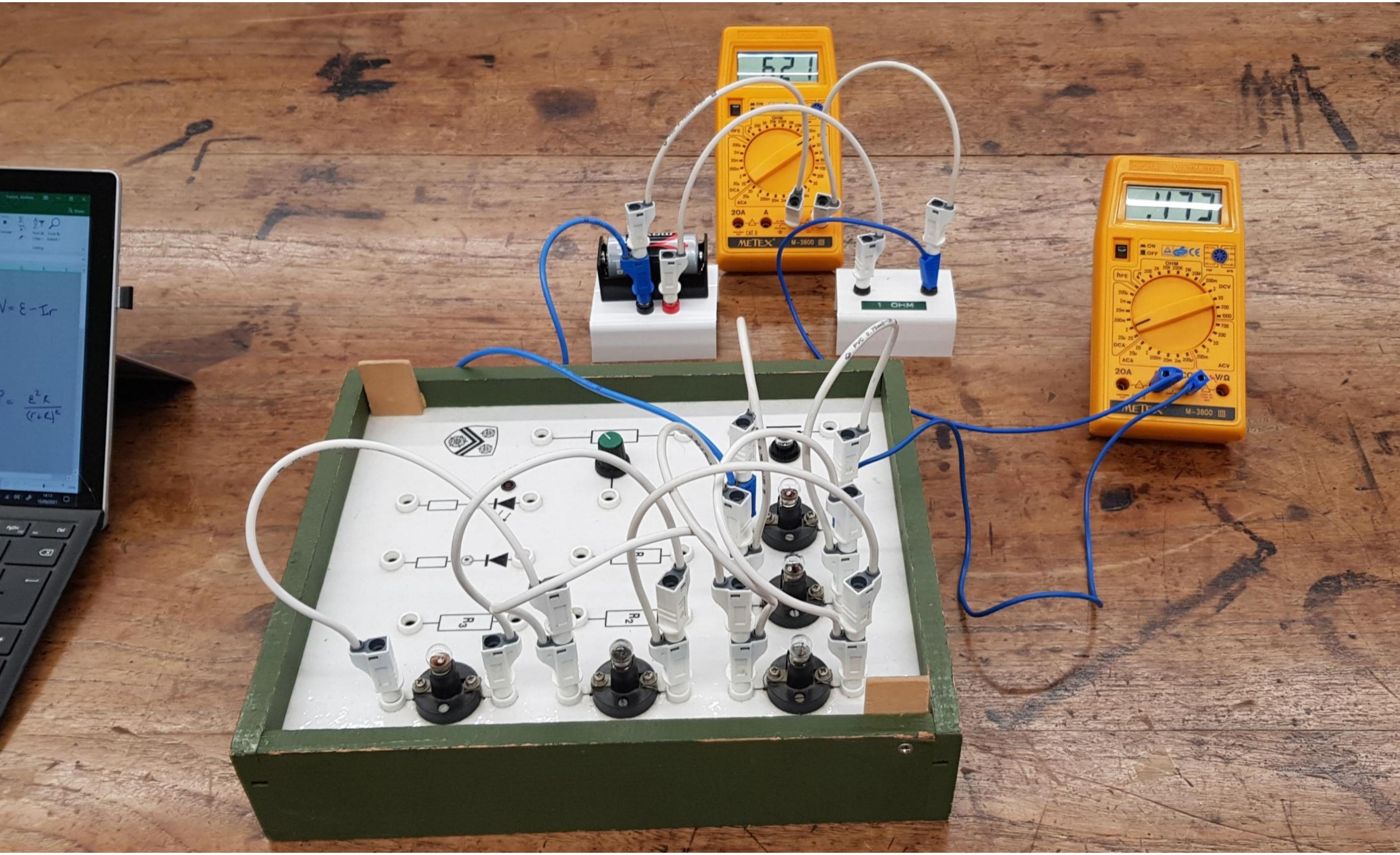


Use a multimeter to measure the current drawn from the cell.

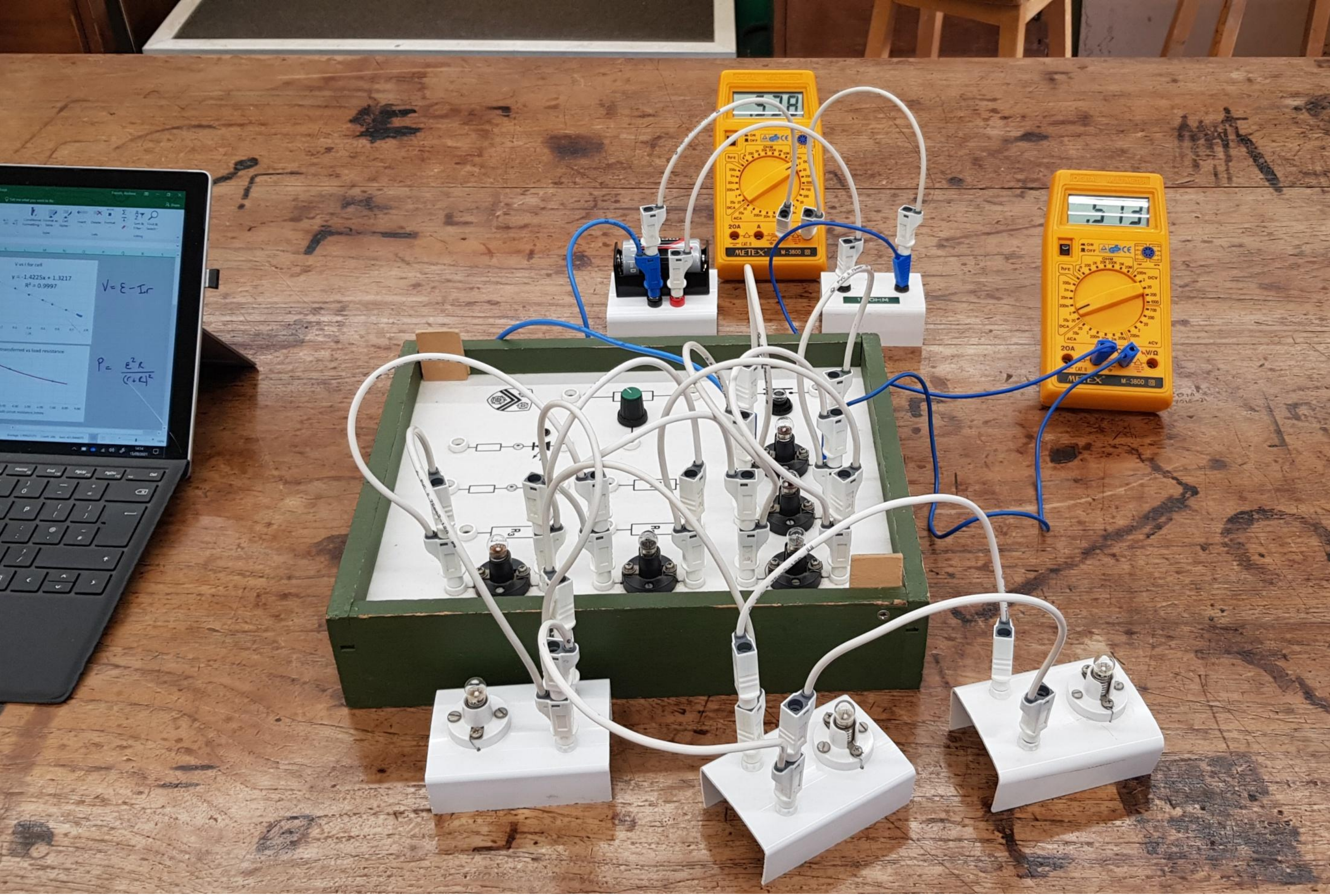
Start with just a single bulb



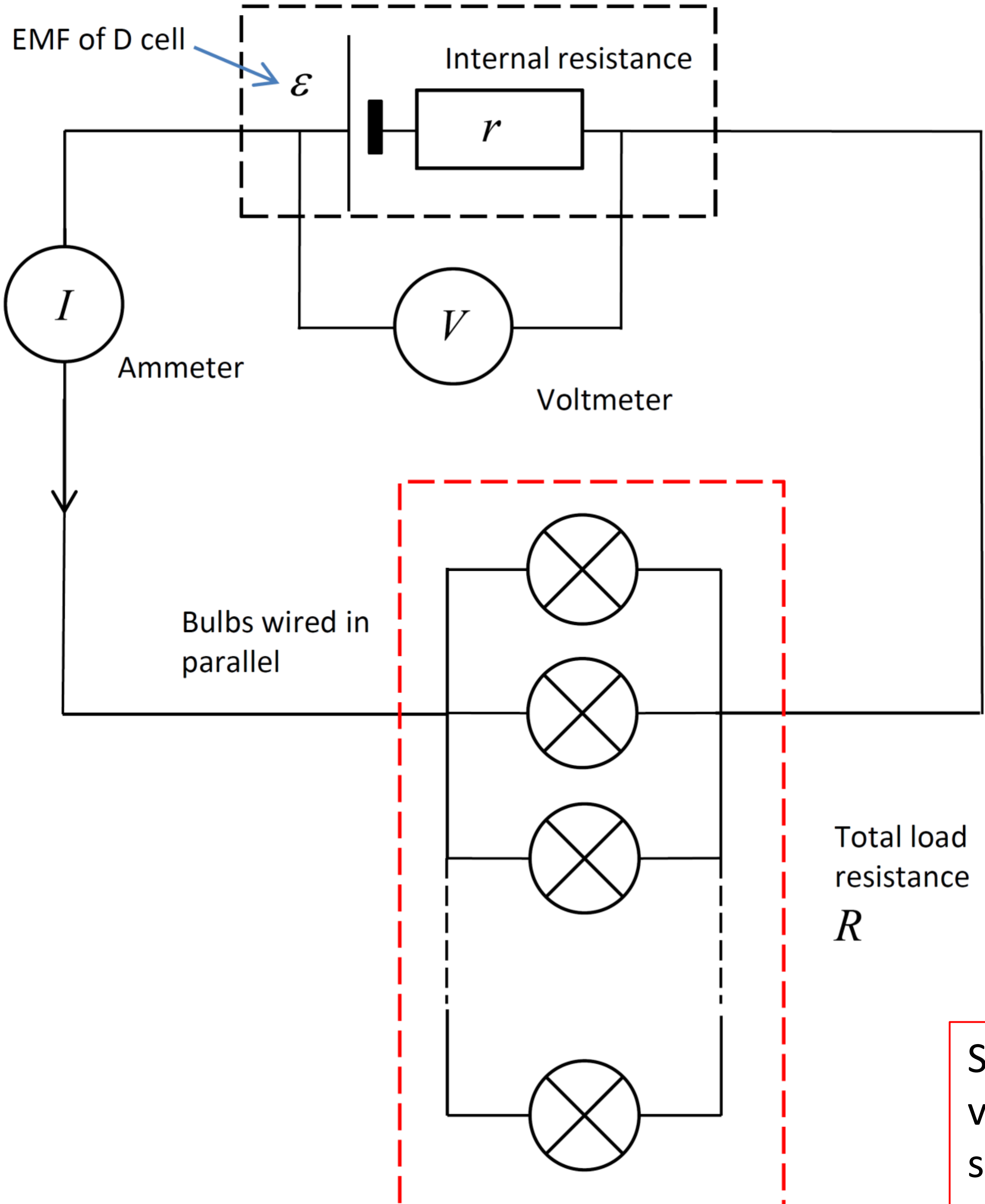
Now add another bulb in parallel, and record the new voltage and current measurements.



Eventually wire up all five bulbs in the Green Board in parallel and connect to the cell.



In an attempt to reduce the overall cell *load* further, add another three bulbs in parallel.



By **Kirchhoff's Second law** (essentially *conservation of energy*)

"EMF applied to any circuit loop is the sum of the currents multiplied by the resistances for all the electrical components in the loop."

$$\varepsilon = Ir + IR$$

$$V = IR$$

$$\therefore \boxed{V = \varepsilon - Ir}$$

So plot V vs I (as number of bulbs are varied, varying both V and I), and this should be a straight line of gradient $-r$ and vertical intercept ε.

MAXIMUM CELL POWER

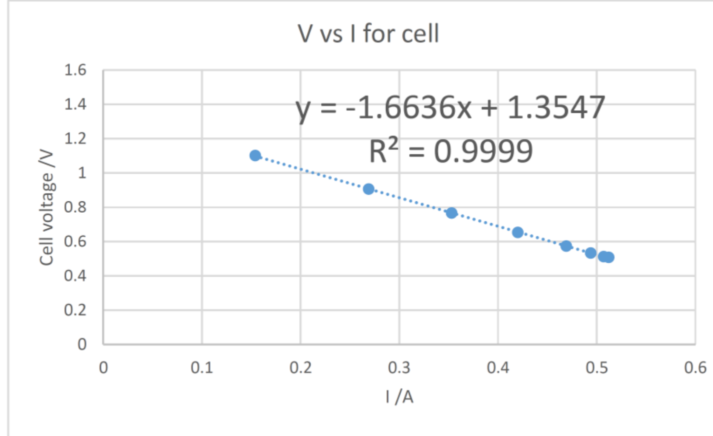
A. French 15/9/2021 1ohm resistor in series with cell

Number of cells (1.5V rated bulbs) in parallel	Voltage across cell /V	Current /A	Cell power IV (Watts)	Resistance of cells /ohms	
0	1.362		0.00	0.00	
1	1.101	0.154	0.17	7.15	Green boards
2	0.906	0.269	0.24	3.37	
3	0.766	0.353	0.27	2.17	
4	0.653	0.42	0.27	1.55	
5	0.573	0.469	0.27	1.22	
6	0.533	0.494	0.26	1.08	Extra 1.5V rated bulbs
7	0.511	0.507	0.26	1.01	
8	0.508	0.512	0.26	0.99	

EMF /volts	**1.355**
r /ohms	**1.664**

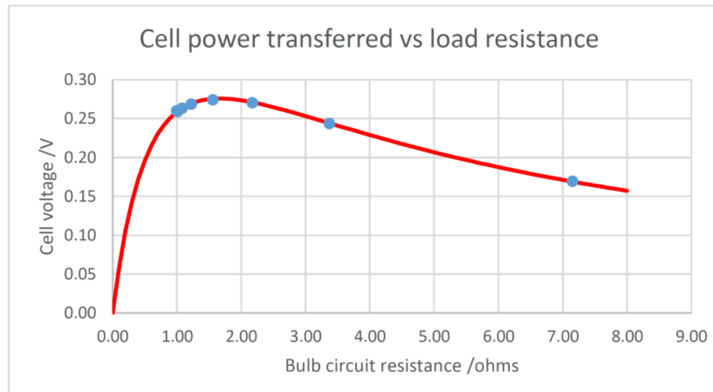
MODEL P vs R (cell power vs load resistance)

R /ohms	P /watts
0.000	0.000
0.100	0.059
0.200	0.106
0.300	0.143
0.400	0.172
0.500	0.196
0.600	0.215
0.700	0.230



$$V = \varepsilon - Ir$$

Find ε and r from a line of best fit.

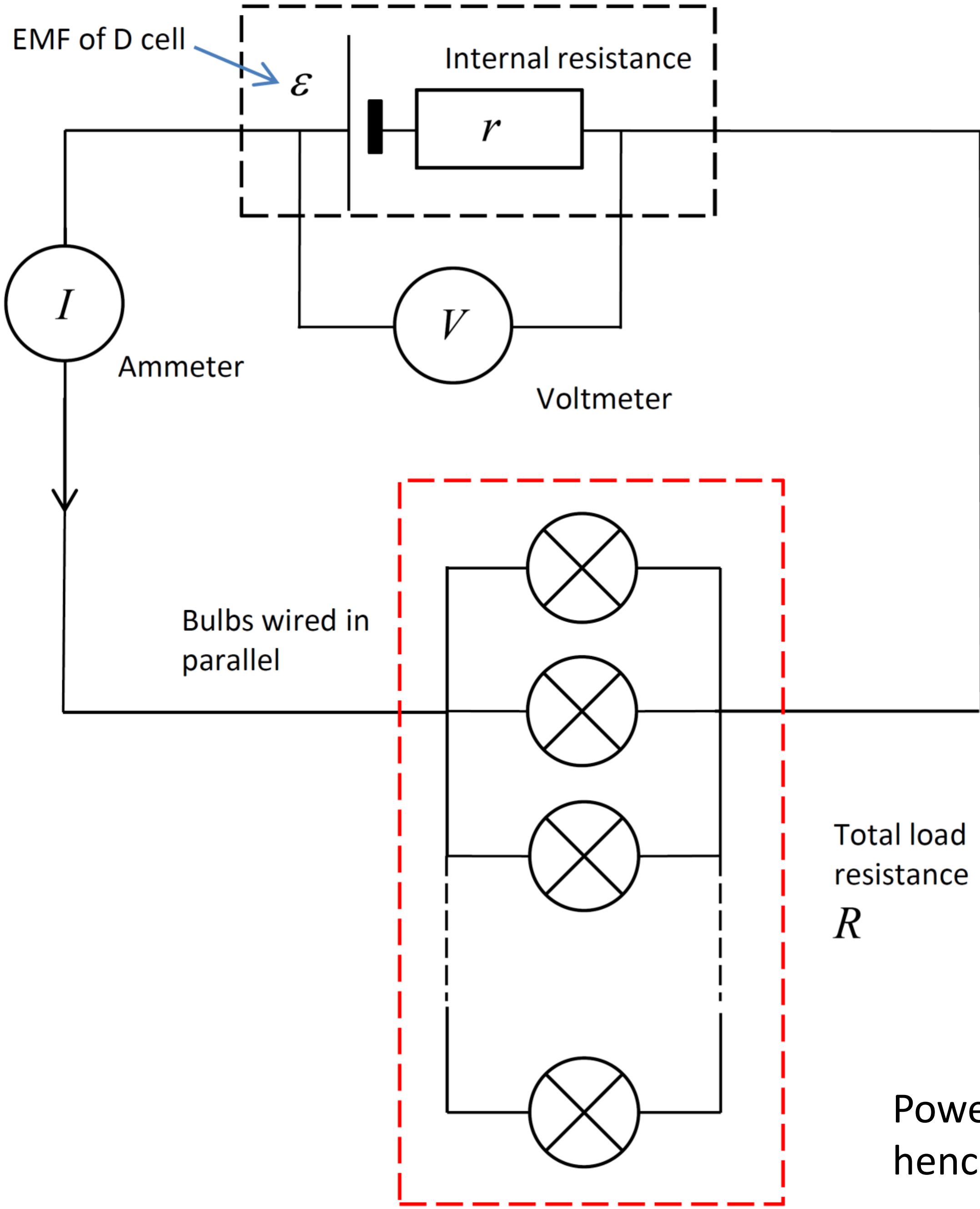


$$P = \frac{\varepsilon^2 R}{(r+R)^2}$$

- Record data (ideally in a spreadsheet format) and calculate:
(i) power P delivered from the cell and
(ii) the resistance R of the combination of bulbs wired in parallel.

- Then plot V vs I to determine the cell EMF ε and internal resistance , and cell power P vs load resistance R.

- From ε and r you can determine a **model curve** which can be used to *underlay* the P vs R measurements.



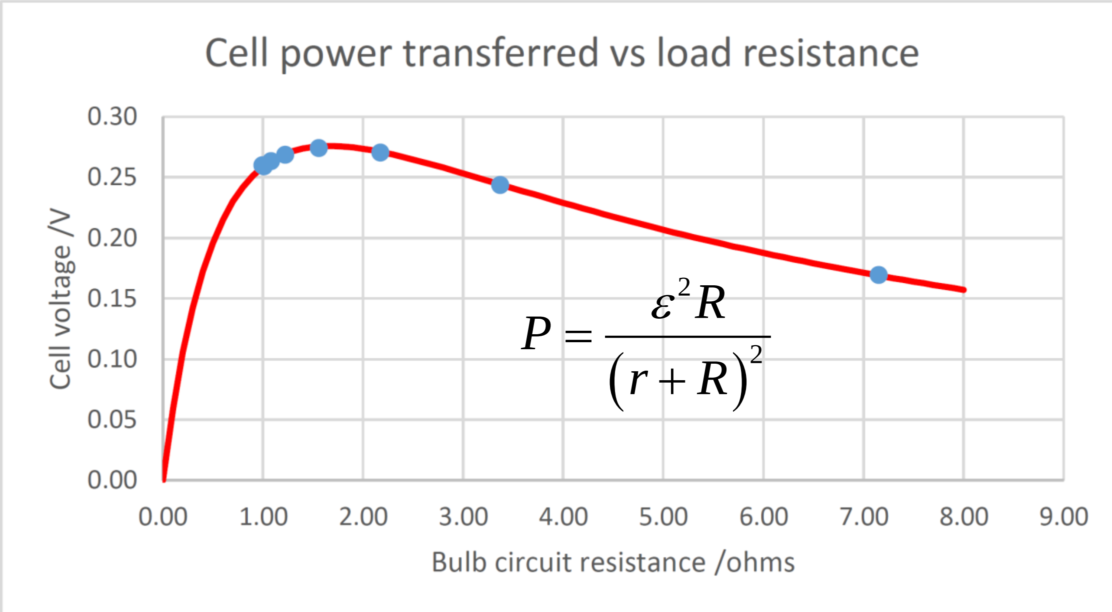
$$\varepsilon = Ir + IR$$

$$\therefore I = \frac{\varepsilon}{r + R}$$

$$P = I^2 R$$

$$\therefore \boxed{P = \frac{\varepsilon^2 R}{(r + R)^2}}$$

Power P transformed by load R (and hence the power delivered by the cell)



$\varepsilon = 1.335\text{V}$ ← cell EMF ('Electro Motive Force')

$r = 1.664\Omega$ ← cell (+ 1 ohm resistor) internal resistance

$$P=\frac{\varepsilon^2 R}{(r+R)^2}$$

$$\frac{dP}{dR}=\frac{\varepsilon^2}{(r+R)^4}\left((r+R)^2-2R(r+R)\right)$$

$$\frac{dP}{dR}=\frac{\varepsilon^2}{(r+R)^3}(r+R-2R)$$

$$\boxed{\frac{dP}{dR}=\frac{\varepsilon^2}{(r+R)^3}(r-R)}$$

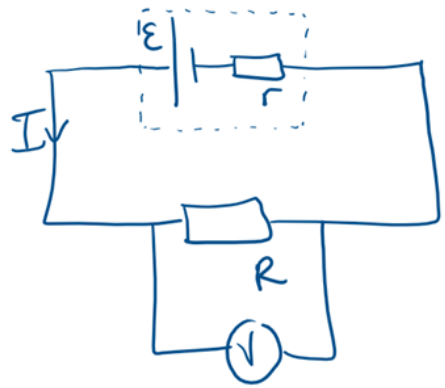
$$\therefore \frac{dP}{dR}=0 \text{ when } r=R$$

$$\boxed{P_{max}=\frac{\varepsilon^2}{4r}}$$

Max power from cell when **load resistance equals the internal resistance of the cell.**

This is called the
MAXIMUM POWER THEOREM.

Non calculus derivation



$\mathcal{E} = V + Ir \quad \therefore \quad V = \mathcal{E} - Ir$

$P = IV$ power dissipated in load R

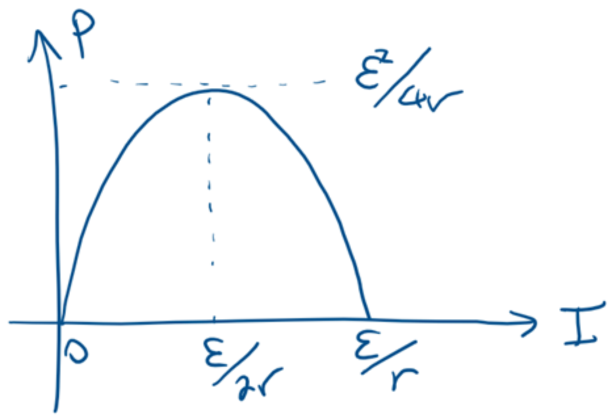
$\therefore P = I(\mathcal{E} - Ir)$

$P = I\mathcal{E} - I^2 r$

$= -r\left\{ I^2 - I\frac{\mathcal{E}}{r} \right\}$

$= -r\left\{ \left(I - \frac{\mathcal{E}}{2r}\right)^2 - \frac{\mathcal{E}^2}{4r^2} \right\}$

$$\therefore \boxed{P = -r\left(I - \frac{\mathcal{E}}{2r}\right)^2 + \frac{\mathcal{E}^2}{4r}}$$



So $P_{max} = \mathcal{E}^2/4r$ when $I = \mathcal{E}/2r$

$\therefore$ Since $P = I^2 R$ $\therefore$ when $P = P_{max}$

$$\mathcal{E}^2/4r = \frac{\mathcal{E}^2}{4r^2} R$$

$$1 = R/r \quad \therefore \boxed{R = r}$$

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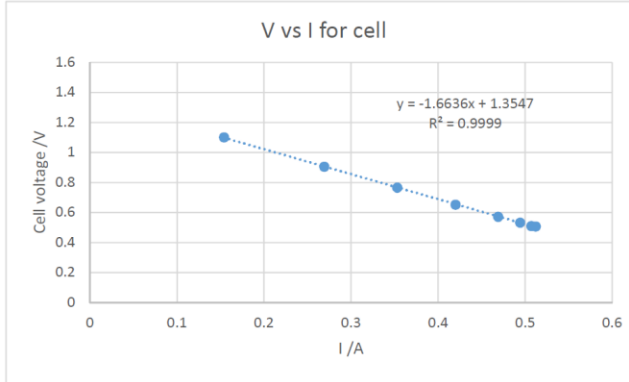
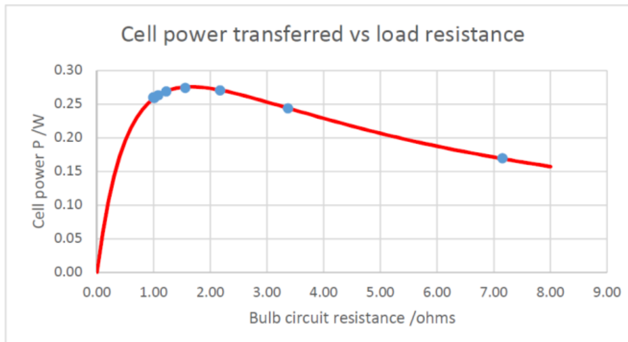
EMF /volts	1.355		Imax /A	0.814
r /ohms	1.664		Pmax /W	0.28

MODEL P vs R (cell power vs load resistance)

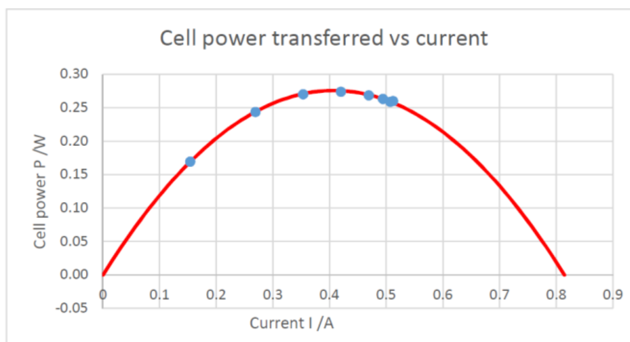
R /ohms	P /watts		I /A	P /watts	
0.000	0.000		0.000	0.000	0
0.100	0.059		0.008	0.011	0.01
0.200	0.106		0.016	0.022	0.02
0.300	0.143		0.024	0.032	0.03
0.400	0.172		0.033	0.042	0.04
0.500	0.196		0.041	0.052	0.05
0.600	0.215		0.049	0.062	0.06
0.700	0.230		0.057	0.072	0.07
0.800	0.242		0.065	0.081	0.08
0.900	0.251		0.073	0.090	0.09
1.000	0.259		0.081	0.099	0.1
1.100	0.264		0.090	0.108	0.11
1.200	0.269		0.098	0.117	0.12
1.300	0.272		0.106	0.125	0.13
1.400	0.274		0.114	0.133	0.14
1.500	0.275		0.122	0.141	0.15
1.600	0.276		0.130	0.148	0.16
1.700	0.276		0.138	0.156	0.17
1.800	0.275		0.147	0.163	0.18



$$V = \varepsilon - Ir$$



$$P = \frac{\varepsilon^2 R}{(r+R)^2}$$



$$P = -r\left(I - \frac{\varepsilon}{2r}\right)^2 + \frac{\varepsilon^2}{4r}$$