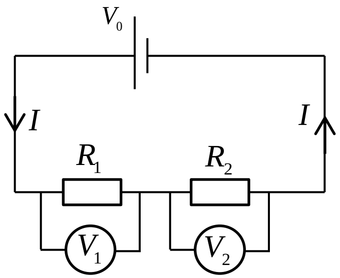
**Maximum power theorem**

Consider a series electrical circuit consisting of two resistors.

$$R_1 = xR$$
$$R_2 = R$$



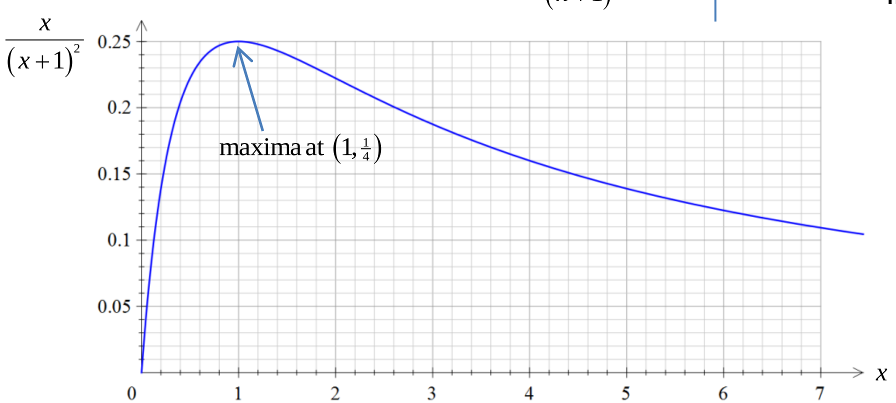

What value of $x$ results in the *maximum power* dissipated by resistor $R_1$ ?

$$P_1 = V_1 I$$
$$P_1 = I^2 xR$$
$$I = \frac{V_0}{xR + R}$$
$$\therefore P_1 = \frac{V_0^2}{R^2(x+1)^2} xR$$
$$\therefore \boxed{P_1 = \frac{V_0^2}{R}\frac{x}{(x+1)^2}}$$

Note since the resistances must be positive $x > 0$

Define: $P_1 = \frac{V_0^2}{R} y$

$$y = \frac{x}{(x+1)^2}$$




$$y = \frac{x}{(x+1)^2}$$
$$\frac{dy}{dx} = \frac{(x+1)^2(1) - x(2)(x+1)}{(x+1)^4}$$
$$\frac{dy}{dx} = \frac{(x+1)(x+1-2x)}{(x+1)^4}$$
$$\boxed{\frac{dy}{dx} = \frac{1-x}{(x+1)^3}}$$

$$\frac{dy}{dx} = 0 \Rightarrow \frac{1-x}{(x+1)^3} = 0$$
$$\therefore x = 1$$
$$y(1) = \frac{1}{(1+1)^2} = \tfrac{1}{4}$$
$$\therefore \boxed{\text{maxima at } (1, \tfrac{1}{4})}$$

Therefore **maximum power** dissipated in resistor $R_1$ is

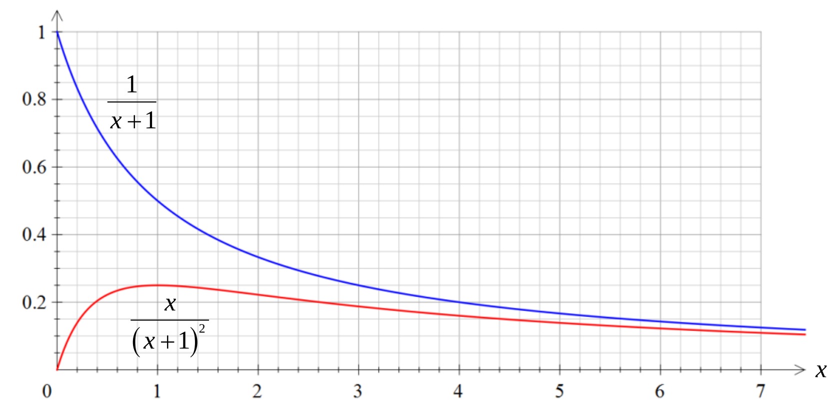
$$\boxed{P_1 = \frac{V_0^2}{4R}}$$

i.e. when **both resistors have the same value**

$$\boxed{R_1 = R_2 = R}$$

The total power dissipated by *both* resistors is

$$P = P_1 + P_2$$
$$P = I^2(R_1 + R_2)$$
$$I = \frac{V_0}{xR + R}, \quad R_1 = xR, \quad R_2 = R$$
$$\therefore P_1 = \frac{V_0^2}{R^2(x+1)^2} R(x+1)$$
$$\boxed{P = \frac{V_0^2}{R}\frac{1}{x+1}}$$




The graphs above describe the situation. The total power dissipated decays *hyperbolically* with $x$. The power in $R_1$ increases until $x = 1$. At this point both resistors dissipate the same power.

When $x > 1$, the fraction of total power dissipated in $R_1$ increases, but the total power dissipated by $R_1$ decreases, since the total power dissipated is also decreasing.