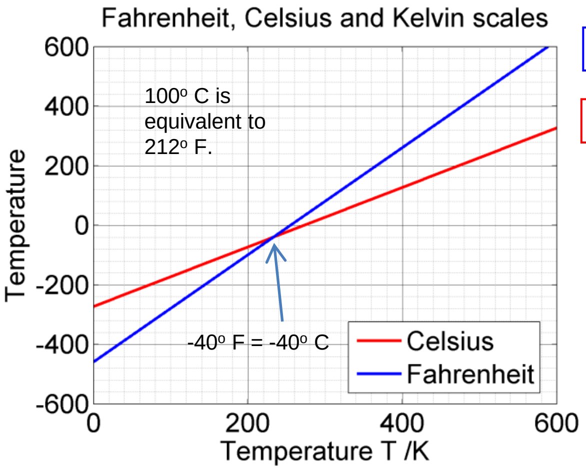
Measuring temperature



$$\boxed{T_F=\tfrac{9}{5}T_C+32}$$

$$\boxed{T_C=T-273}$$

The **Kelvin** temperature scale (or "absolute" scale) is proportional to the **mean kinetic energy** of **molecules**.

$$\boxed{U=\tfrac{1}{2}\alpha nRT}$$

Internal energy of n moles of gas

Number of degrees of freedom of molecular motion (e.g $\alpha = 3$ for x,y,z translation)

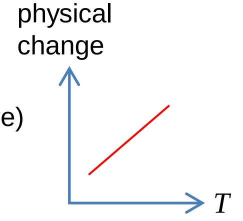
1 mol = 6.02 x 10^{23} molecules. i.e. **Avogadro's Number**. So average kinetic energy of a *molecule* is:

$$\boxed{u=\tfrac{1}{2}\alpha nk_BT}$$

$$k_B=R/6.02\times10^{23}=1.38\times10^{-23}\,\text{JK}^{-1}$$ Boltzmann's constant

Fahrenheit is a **temperature scale**, where 32° F is the freezing point of water and 212° F is the boiling point of water, defined at sea level at **standard atmospheric pressure** (101,325Pa). It was proposed in 1724 by Daniel Gabriel Fahrenheit.
0° F corresponded to the lowest temperature he could cool brine (salt water) and 100° F was the average human body temperature (37°C).

A more popular scale is the **Celsius** scale, with 0° C and 100° C representing the freezing and boiling points of water at standard atmospheric pressure.

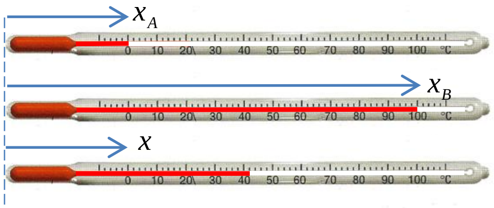


To measure a *third temperature*, given two known fixed points, (e.g. freezing point of ice and boiling point of water at standard atmospheric pressure) all we require is a sensor with a **physical change which varies *linearly* with temperature T over the range of interest.**

In-glass thermometer

This is a glass cylinder containing a very thin internal cylinder which contains a liquid. Mercury or alcohol, typically dyed red, are common choices. The idea is the **thermal expansion** of these liquids is approximately **linear** with temperature, and the melting and boiling points are *beyond the range of temperatures of interest.*

Liquid	Melting temp. / °C	Boiling temp. / °C
Ethyl alcohol	-114.4	78.3
Mercury	-38.9	356.6
Water	0	100

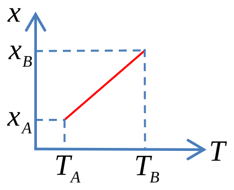


$T=T_A$ e.g. 0°C

$T=T_B$ e.g. 100°C

$$\boxed{T(x)=\frac{x-x_A}{x_B-x_A}(T_B-T_A)+T_A}$$

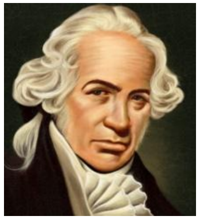
Linear relationship with temperature T and liquid column length x

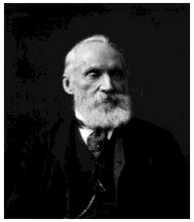


$$\boxed{\begin{array}{l} x_A=5.20\text{cm},\ \ x_B=12.70\text{cm},\ T_A=0.00^\circ\text{C},\ T_B=100^\circ\text{C} \\ x=8.35\text{cm}\ \ \therefore T=\dfrac{8.35-5.20}{12.70-5.20}\times\left(100^\circ\text{C}-0^\circ\text{C}\right)=42^\circ\text{C}\end{array}}$$

We can enhance the **sensitivity** of a thermometer by **(i) reducing the diameter of the liquid cylinder** and **(ii) increasing the volume of liquid**. This means for a change in temperature we *increase* the change in liquid column length x. This means smaller changes in temperature are more easily measurable, but the thermometer may become too long for practical use if we make these sensitivity enhancements in excess.

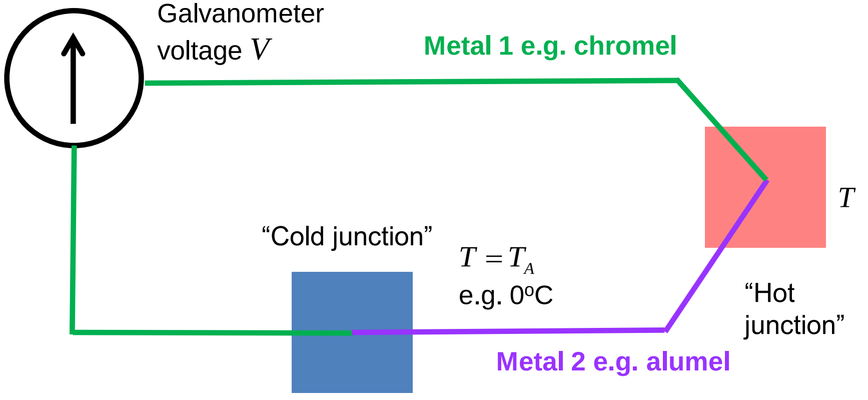

Anders Celsius
1701-1744


Daniel Fahrenheit
1686-1736


William Thompson
(Lord Kelvin)
1824-1907

Thermocouple

The *thermoelectric effect* is the creation of **voltage differences** from **temperature differences**. This is in evidence in a **thermocouple**, which can be constructed from **two different conductors**, which join at a *"hot junction"* at the point where temperature needs to be measured, and at a *"cold junction"* of known temperature.



Galvanometer voltage V

Metal 1 e.g. chromel

T

"Cold junction"

$T = T_A$
e.g. 0°C

"Hot junction"

Metal 2 e.g. alumel

At known reference temperature
e.g. iced water at 0°C

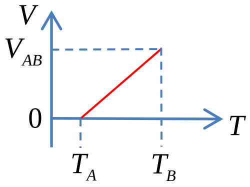
The idea is to determine the galvanometer voltage at a known hot junction temperature (e.g. 100°C) and assume **linearity** of voltage to temperature difference.

e.g. if V = 12.0mV for Hot junction at 100°C and Cold at 0°C; a V of 7.00mV would be: 100°C x 7.00/12.0 = 58.3°C

$$V = V_{AB},\ T = T_B$$

$$V = 0,\ T = T_A$$

$$T(V) = \frac{V}{V_{AB}}(T_B - T_A) + T_A$$



Linear relationship with "hot junction" temperature T and galvanometer voltage V

The reference temperature at the cold junction might be determined via a *thermistor* for a typical digital thermometer. i.e. there is no need to place one end of the wires in a vessel of known temperature. The sensor is already *calibrated*.

Chromel-alumel are the most common thermocouple conductor pairs (Type K), available in ranges -200°C to 1350°C. However, one of the constituents to these alloys is **Nickel**, which is magnetic. When *pure* Nickel exceeds it's **Curie temperature** (356°C) this will cause a *deviation* in the thermoelectric effect (i.e. Ni will cease being permanently magnetic above the Curie temperature). The Curie temperature for K-Type thermocouples is about 185°C. **Copper-Constantan** thermocouples (Type T) are effectively *non-magnetic* (Constantan has a Curie Temperature of -238°C) but work over a more restricted range -200°C to 350°C.

Useful properties of thermocouples

1. High melting point, so much larger range than a liquid-in-glass thermometer
2. Can be made small, so can measure local temperatures effects "at a point"
3. Small thermal mass, so will respond quickly to changes in temperature
4. Not liquid & glass, so perhaps more robust to harsh environmental conditions

Seebeck effect

"Temperature gradient is proportional to EMF induced"

This is one of three distinct phenomena which make up the ***thermoelectric effect.*** The others are the **Peltier effect** (heat generated at the junction of two distinct conductors when current flows between them) and the **Thomson (Kelvin) effect** (heat is produced when current flows in a conductor and there is a spatial temperature gradient).

Thomas Seebeck
1770-1831

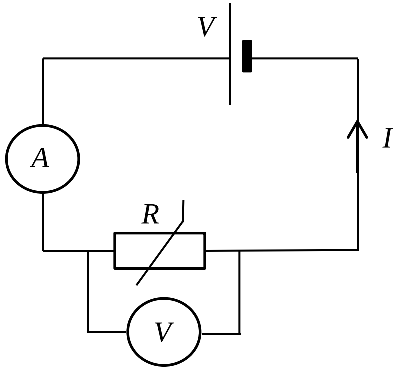
Jean Peltier
1785-1845

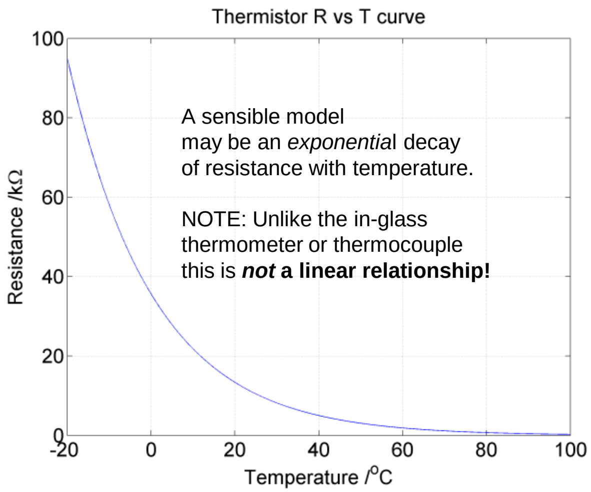
Chromel: **90% Ni, 10% Cr** Alumel: **95% Ni, 2% Al, 2% Mn, 1% Si**

https://en.wikipedia.org/wiki/Thermocouple

Thermistor

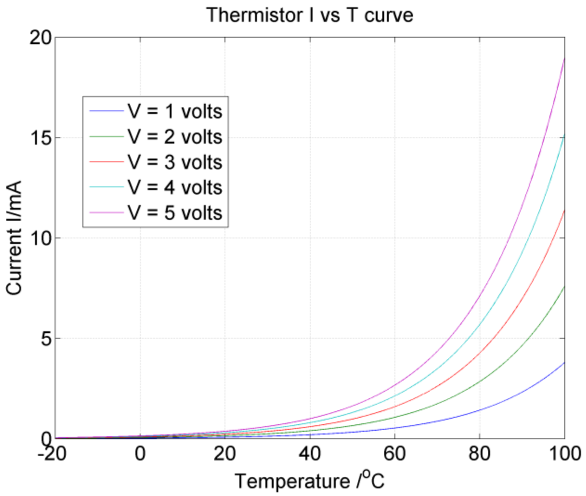
The electrical resistance of a *thermistor* will change with temperature, and the consequential change in current drawn through a circuit can be used to measure the temperature, assuming a *calibration process* has occurred to relate the current measured to temperature.





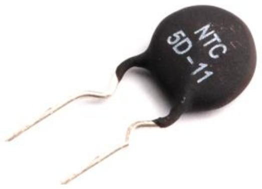
A sensible model may be an *exponential* decay of resistance with temperature.

NOTE: Unlike the in-glass thermometer or thermocouple this is ***not*** **a linear relationship!**

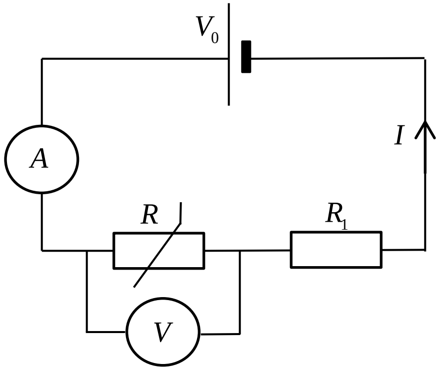


So apply a fixed voltage to the circuit, measure current using a sensitive ammeter and then use the graph to determine the temperature measurement.

Note a typical thermistor is has a *highly non-linear* resistance vs temperature relationship.



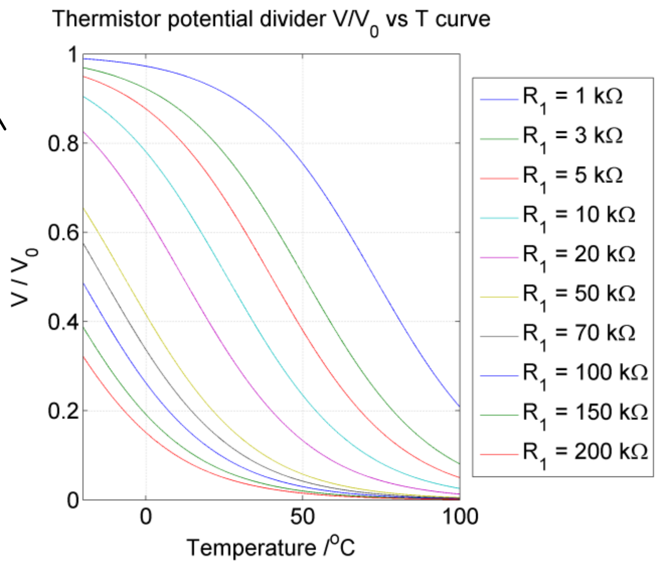
An alternative electrical measurement scheme is to use *voltage* rather than current. This may be preferable if the resistance of the sensor is very low, which means there may be a risk of drawing dangerously large currents through the sensor circuit. To vary voltage as resistance of the sensor changes, we use a **potential divider**.



$$V_0 = I\left(R + R_1\right)$$ Ohm's Law

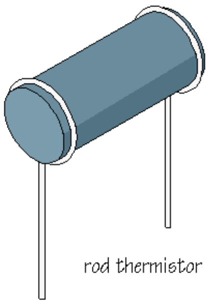
$$V = IR$$

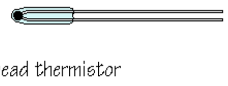
$$\boxed{\therefore \frac{V}{V_0} = \frac{R}{R + R_1}}$$

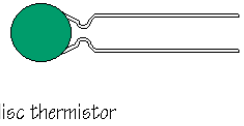


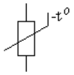
The sensor can be said to ***sensitive*** if the electrical measurement (voltage in this case) variation is *large* given a variation in the parameter being measured. i.e. in our case the (T,V) graph varies significantly (i.e. a large % change) over the temperature range -20°C to 100°C.

To optimize sensitivity choose a fixed resistor to be **close to the average expected resistance of the thermistor** in the temperature range of interest.


rod thermistor


bead thermistor


disc thermistor



thermistor circuit symbol