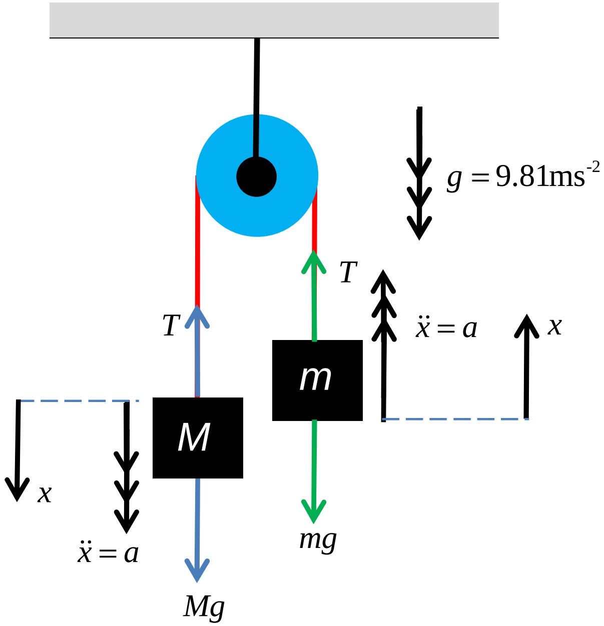
A **pulley system** uses wheels mounted upon axes to *change direction*, and indeed *magnify*, the mechanical effect of a cable under tension. In all examples we shall assume the cables are ***light and inextensible***, and the pulley wheels are ***frictionless***. Note these assumptions become less valid as more pulleys are added to increase the *mechanical advantage* of a system. i.e. the more cables pulling against a load, the longer (and hence heavier) the cable and more friction.



Since the cables are inextensible, then mass *m* must move up at the same rate as mass *M*. Also, there cannot be any difference in cable tension. Otherwise a *light* cable would accelerate apart and therefore not be inextensible!

Applying *Newton's Second Law* to each mass:

$$Ma = Mg - T$$

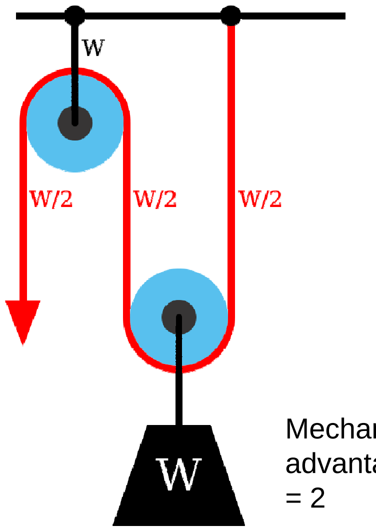
$$ma = T - mg$$

$$\therefore Ma + ma = Mg - mg$$

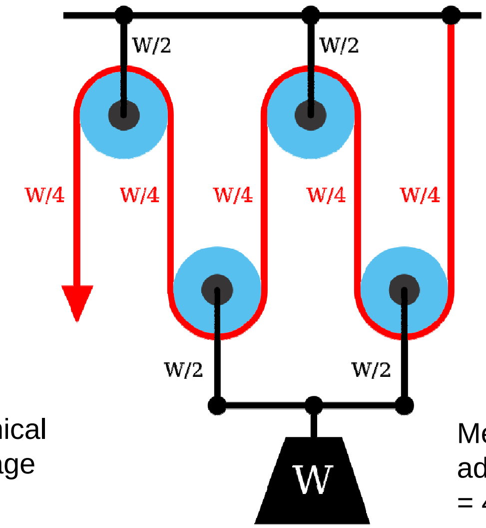
$$\therefore a = \frac{M - m}{M + m} g$$

If the masses are *equal* there is no acceleration. However the system is still useful as the motion of the lower mass causes an equal and opposite motion of the other mass. If mass *M* descends with a constant velocity, then mass *m* will rise with a constant velocity, as long as *M* = *m*.

The equilibrium situation is more useful with multiple pulleys as illustrated below. In this cases the cable tension required to balance the load is reduced by a factor equal to the number of pulleys in the system. The *mechanical advantage* is this factor.

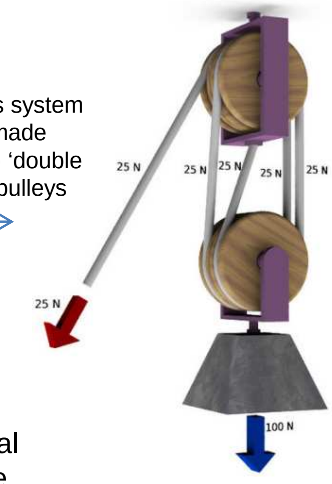


Mechanical advantage = 2

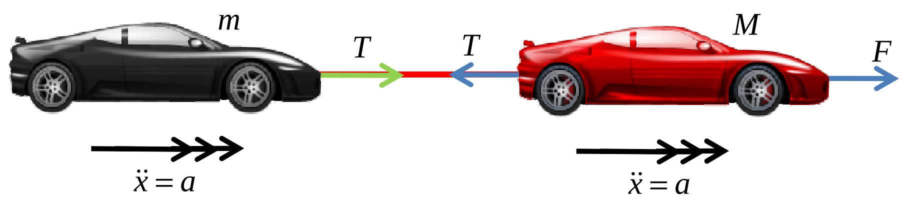


Mechanical advantage = 4

Note this system can be made from two 'double groove' pulleys



A similar analysis can be applied to any set of objects connected via a light, inextensible cable



The trick is to separate the system into parts either side of the cable, and apply Newton II separately to each part.

$$Ma = F - T$$

$$ma = T$$

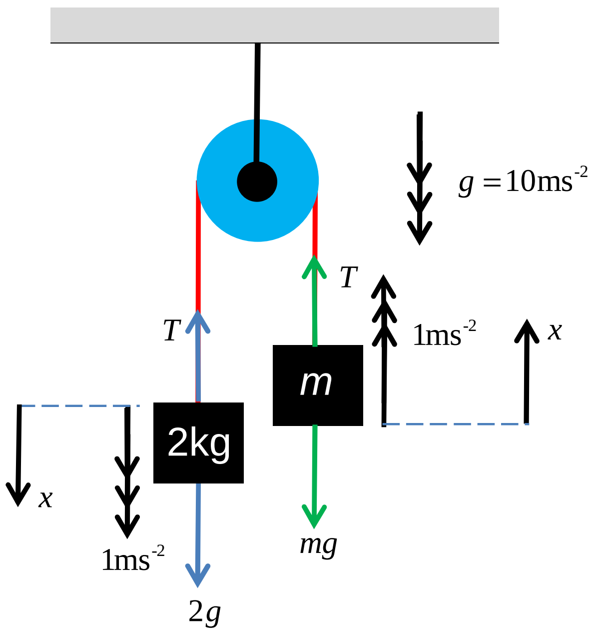
$$Ma + ma = F$$

$$\therefore a = \frac{F}{M + m}$$

$$\therefore T = \frac{mF}{M + m}$$

So if we know the masses and the driving force of the front car, we can compute the overall acceleration and the cable tension.

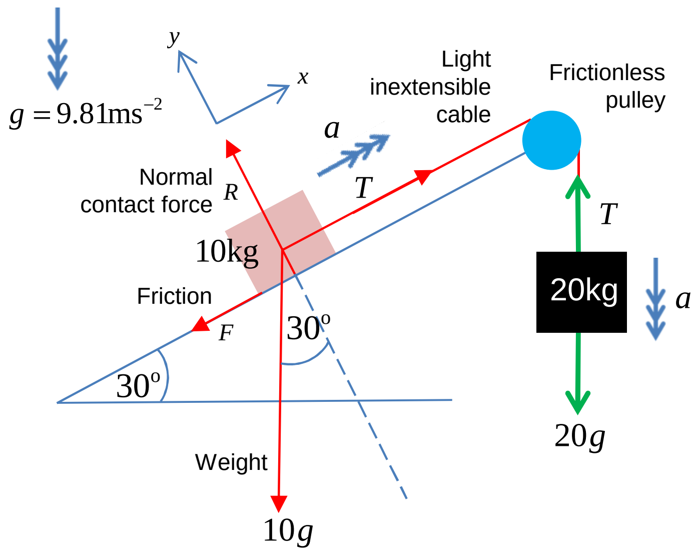
Example 1: The acceleration of mass *m* (kg) is 1ms^{-2}. Find *m*.



Applying *Newton's Second Law* to each mass:

$$2 = 2g - T$$
$$m = T - mg$$
$$\therefore 2 + m = 2g - mg$$
$$m(1+g) = 2(g-1)$$
$$m = \frac{2(g-1)}{1+g}$$
$$m = \frac{18}{11}$$
$$\boxed{m = 1\tfrac{7}{11}}$$

Example 2: Calculate the time taken for the 10kg mass to be dragged 4 metres uphill. Assume the coefficient of friction between the 10kg mass and the hill is $\mu = 0.2$.



Newton II for the 10kg mass:

$$x:\ 10a = T - 10g\sin 30^\circ - \mu R$$
$$y:\ 0 = R - 10g\cos 30^\circ$$
$$\therefore R = 5g\sqrt{3}$$
$$\therefore 10a = T - 5g - 5\mu g\sqrt{3}$$

Newton II for the 20kg mass:

$$20a = 20g - T$$
$$\therefore T = 20g - 20a$$

Hence, by substituting for *T* :

$$10a = 20g - 20a - 5g - 5\mu g\sqrt{3}$$
$$30a = 15g - 5\mu g\sqrt{3}$$
$$a = \frac{5g\left(3 - \mu\sqrt{3}\right)}{30}$$
$$a = \frac{5\times 9.81\times\left(3 - 0.2\sqrt{3}\right)}{30} \approx \boxed{4.34\text{ms}^{-2}}$$

If the 10kg mass starts from rest, it will travel 4 metres in *t* seconds via the equation of constant acceleration motion:

$$4 = \tfrac{1}{2}at^2$$
$$\therefore t = \sqrt{\frac{8}{a}} \approx \boxed{1.36\text{s}}$$