Newton's Laws of Motion, Friction and Inclined Planes

1. With no external force applied, a body will move with a constant velocity $\mathbf{v}$
2. mass x acceleration = sum of external forces*
3. If body A is in contact with body B and exerts a force upon it, the force upon A due to B is equal in magnitude and opposite in direction.

$$m\frac{d\mathbf{v}}{dt}=\sum_i \mathbf{f}_i$$

A body is said to be ***in equilibrium*** if the vector sum of external forces is zero

The force due to gravity upon a body of mass *m* is its **weight** and has magnitude *mg*, where *g* is the local gravitational field strength.

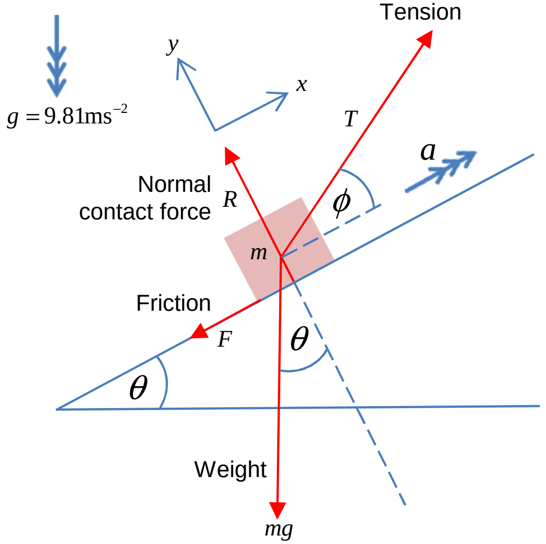
$$\mathbf{x}(t=0)=\mathbf{x}_0$$

$$\mathbf{v}=\frac{d\mathbf{x}}{dt} \qquad \mathbf{a}=\frac{d\mathbf{v}}{dt}=\frac{d^2\mathbf{x}}{dt^2}$$

Velocity

Acceleration

Displacement Velocity and Acceleration are **vector** quantities



Consider a block being pulled uphill via force *T*. The vector sum of forces results in an acceleration *a* directly up the hill. Surface contact is maintained at all times.

Rough inclined plane with **coefficient of friction** μ between block of mass *m* and plane.

Applying Newton Second Law in *x* and *y* directions

$$x: \quad ma=T\cos\phi-mg\sin\theta-F$$

$$y: \quad 0=R+T\sin\phi-mg\cos\theta$$

Sliding friction model (friction always *resists* motion)

$$F\le\mu R \quad \text{In equilibrium}$$

$$F=\mu R \quad \text{Sliding}$$

$$x: \quad ma=T\cos\phi-mg\sin\theta-\mu R$$

$$y: \quad mg\cos\theta-T\sin\phi=R$$

$$ma=T\cos\phi-mg\sin\theta-\mu\left(mg\cos\theta-T\sin\phi\right)$$

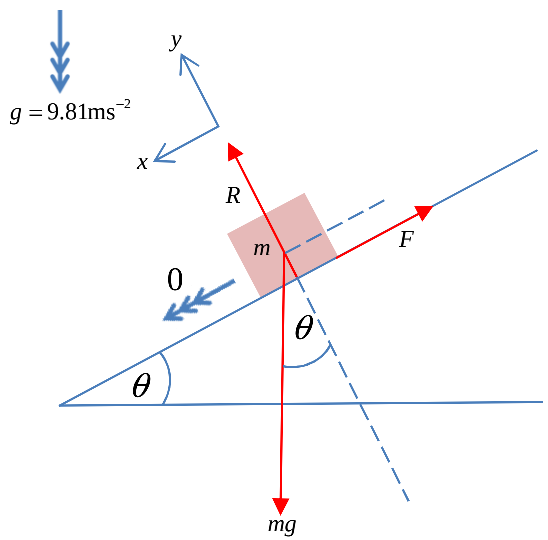
$$a=\frac{T}{m}\left(\cos\phi+\mu\sin\phi\right)-g\left(\sin\theta+\mu\cos\theta\right)$$

Example 1: Block is on the point of sliding, *T* = 0. Note *F* will point in the opposite direction as the block will slide downhill if θ is increased.

$$x: \quad 0=mg\sin\theta-\mu R \qquad \therefore \mu=\frac{mg\sin\theta}{R}$$

$$y: \quad 0=R-mg\cos\theta \qquad \therefore R=mg\cos\theta$$

$$\therefore \mu=\frac{mg\sin\theta}{mg\cos\theta} \qquad \therefore \quad \mu=\tan\theta$$

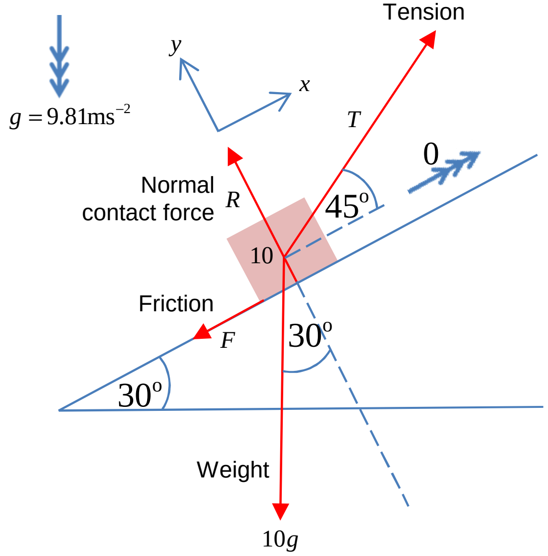


*Actually Newton #2 states the rate of change of *momentum* = sum of the external forces. $\frac{d(m\mathbf{v})}{dt}=\sum_i \mathbf{f}_i$ If mass remains constant the LHS is mass x acceleration.

Example 2: A block of 10kg is in equilibrium 'at the point of sliding' *uphill* (this is called limiting friction). If the plane is inclined at 30° and the tension is at 45° to the plane, what is T given a coefficient of friction of $\mu = 1/5$?

$$x: \quad 0 = \frac{T}{\sqrt{2}} - 10 \times \tfrac{1}{2} g - \tfrac{1}{5} R \qquad \therefore \; T = \left(5g + \tfrac{1}{5}R\right)\sqrt{2}$$

$$y: \quad 0 = R + \frac{T}{\sqrt{2}} - 10g\frac{\sqrt{3}}{2} \qquad \therefore R = 5g\sqrt{3} - \frac{T}{\sqrt{2}}$$



$$\therefore T = \left(5g + \tfrac{1}{5}\left(5g\sqrt{3} - \frac{T}{\sqrt{2}}\right)\right)\sqrt{2}$$

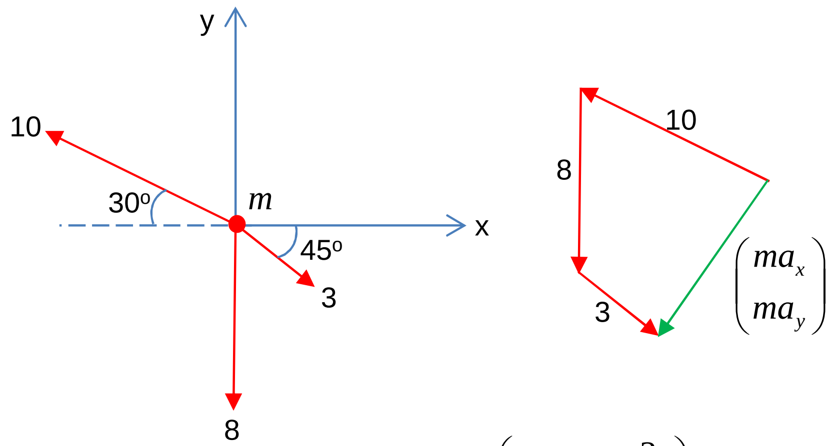
$$T = 5\sqrt{2}g + g\sqrt{6} - \tfrac{1}{5}T$$

$$\tfrac{6}{5}T = g\left(5\sqrt{2} + \sqrt{6}\right)$$

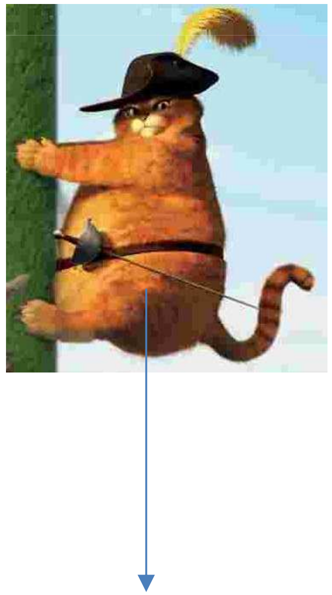
$$T = \frac{5\left(5\sqrt{2} + \sqrt{6}\right)}{6}g$$

$$T \approx 7.93g$$

Resolving forces



$$\begin{pmatrix} ma_x \\ ma_y \end{pmatrix} = \begin{pmatrix} -10\cos 30^\circ + 3\cos 45^\circ \\ 10\sin 30^\circ - 3\sin 45^\circ - 8 \end{pmatrix} = \begin{pmatrix} -5\sqrt{3} + \dfrac{3}{\sqrt{2}} \\ -3 - \dfrac{3}{\sqrt{2}} \end{pmatrix}$$



The weight of the puss