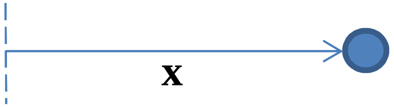
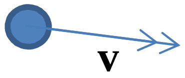
Kinematics, from the Greek κίνημα, (*kinema)* literally means the study of motion. This is done via analysis of the mathematical relationships between *time*, *displacement*, *velocity* and *acceleration*. For a kinematic analysis, the internal motion of a body (e.g. deformation, rotation etc) is ignored; only the bulk motion of the centre of mass is considered. i.e. a *particle* model.

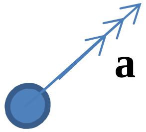
Displacement, velocity, acceleration are *vector* quantities. i.e. they have both magnitude and direction



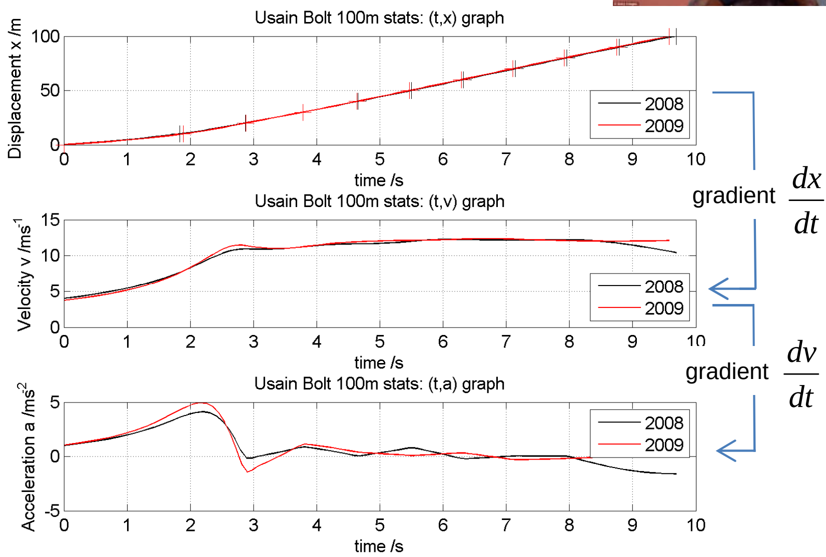
Displacement is the vector between a fixed origin and the point of interest. If an object is moving, the displacement will vary with time t



Velocity is the *rate of change of displacement*. If velocity is in the same direction as displacement, it is the gradient of a *(t,x)* graph.



Acceleration is the *rate of change of velocity*. If acceleration is in the same direction as velocity, it is the gradient of a *(t,v)* graph.



In calculus notation: $\boxed{\mathbf{v} = \frac{d\mathbf{x}}{dt} \qquad \mathbf{a} = \frac{d\mathbf{v}}{dt} \qquad \mathbf{a} = \frac{d^2\mathbf{x}}{dt^2}}$

In many situations displacement, velocity and acceleration will all be *parallel*. We can therefore use *scalar* quantities. Direction (i.e. forwards or backwards) is indicated by a positive or negative sign.

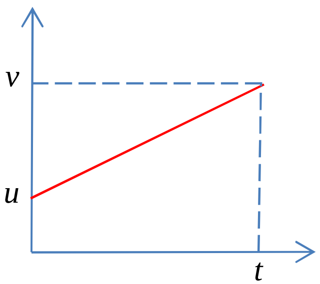
$$\boxed{v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2x}{dt^2} \qquad a = v\frac{dv}{dx}}$$

Displacement is therefore the *anti-derivative* of velocity and velocity is the anti-derivative of acceleration. In other words, the area under a *(t,v)* graph is the displacement and the area under a *(t,a)* graph is velocity.

$$\boxed{x = \int v\,dt \qquad v = \int a\,dt}$$

i.e. $v = 1 - 2t + 3t^2 \Rightarrow \boxed{x = x_0 + t - t^2 + t^3}$

Constant acceleration motion. It is almost *always* a good idea to start with a *(t,v)* graph. Let velocity increase at the same rate a from u to v in t seconds.



The acceleration is the gradient: $a = \frac{v-u}{t} \quad \therefore \boxed{v = u + at}$

The area under the graph is the displacement.
Since this a trapezium shape:

$$\boxed{x = \tfrac{1}{2}(u+v)t}$$

We can work out other useful relationships for constant acceleration motion

$$x = \tfrac{1}{2}(u + u + at)t$$

$$\boxed{x = ut + \tfrac{1}{2}at^2}$$

$$x = ut + \tfrac{1}{2}at^2$$

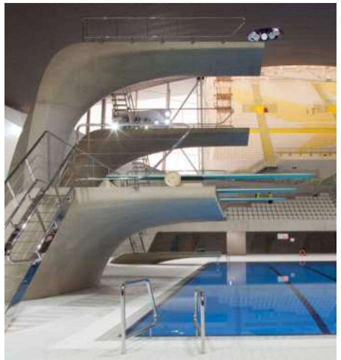
$$2ax = 2uat + a^2t^2$$

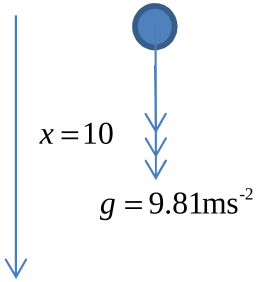
$$v^2 = (u+at)^2 = u^2 + 2uat + a^2t^2$$

$$\therefore \boxed{v^2 = u^2 + 2ax}$$

These equations are ONLY VALID when acceleration *a* is CONSTANT

Example 1: Falling due to gravity
If we ignore the effect of air resistance, what speed does a diver strike the water after a 10m jump? How long is the diver in the air for?





Since acceleration is constant $a = g$ and the diver starts from rest, i.e. $u = 0$

$$v^2 = u^2 + 2ax$$
$$\therefore \quad v = \sqrt{2gx} \approx 4.43\sqrt{x}$$
$$\therefore \quad \boxed{v \approx 14\text{ms}^{-1}}$$
$$v = u + at$$
$$\therefore t = \frac{\sqrt{2gx}}{g} = \sqrt{\frac{2x}{g}} \approx 0.45\sqrt{x}$$
$$\therefore \boxed{t \approx 1.43\text{s}}$$

Useful speed conversions:

1 ms^{-1}
= 2.34 miles per hour

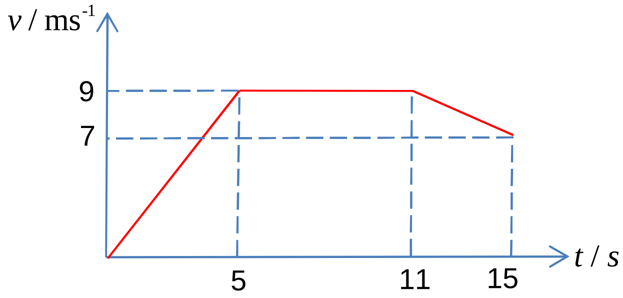
1 ms^{-1}
= 3.6 km per hour

$$t\,/\,\text{min} = 60 \times \frac{x\,/\,\text{miles}}{v\,/\,\text{mph}}$$

Speed in mph	Time in minutes per 10 miles
10	60
20	30
30	20
40	15
50	12
60	10
70	8.57

$$t\,/\,\text{min} = \frac{60}{\frac{1}{10}v\,/\,\text{mph}}$$

Example 2: What was the average speed of the following athlete?



Acceleration is constant in each 'phase' but not over the whole motion. We much treat each trapezium of the *(t,v)* graph separately.

Displacement is

$$x = \tfrac{1}{2}(5)(9) + (9)(6) + \tfrac{1}{2}(9+7)(4)$$
$$\boxed{x = 108.5\text{m}}$$

And therefore average speed is

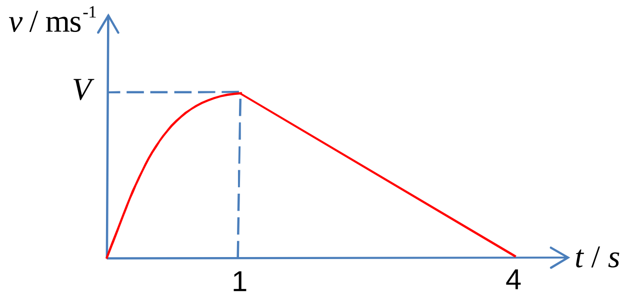
$$\bar{v} = \frac{108.5\text{m}}{15\text{s}} = \boxed{7.23\text{ms}^{-1}}$$

Example 3: A cyclist starts an uphill climb at 10ms^{-1}. The hill causes him to lose speed at 0.5ms^{-2}. How far could he cycle before he stops? How long will it take him to cycle 96 metres?

$$v = 10 - \tfrac{1}{2}t$$
$$x = 10t - \tfrac{1}{4}t^2$$
$$v = 0 \Rightarrow t_{\max} = \boxed{20}$$
$$\therefore x_{\max} = \boxed{100}$$

$$96 = 10t - \tfrac{1}{4}t^2$$
$$16 \times 6 \times 4 = 40t - t^2$$
$$t^2 - 40t + 16 \times 24 = 0$$
$$(t-16)(t-24) = 0$$
$$\text{Since } t \le 20, \quad \boxed{t = 16}$$

Example 4: Between 0 and 1 seconds a cat accelerates to *V* metres per second. The acceleration of the cat diminishes linearly during this time, such that at exactly 1 second, it is moving at constant speed. After 1 second the cat decelerates at a constant rate until it stops, at 4 seconds. If the cat travels 13 metres in total, find *V*



Between t=0 and t=1 seconds, acceleration a is

$a = a_0 - At$ Cat is moving at constant speed at t=1

$0 = a_0 - A \quad \therefore \boxed{a_0 = A}$

Hence in first three seconds

$\boxed{a = A - At} \;\therefore\; \boxed{v = At - \tfrac{1}{2}At^2} \;\therefore\; \boxed{x = \tfrac{1}{2}At^2 - \tfrac{1}{6}At^3}$

Which means $\boxed{V = \tfrac{1}{2}A}$

The cat travels $\tfrac{3}{2}V$ metres in the last 3 seconds, hence

$$13 = \tfrac{1}{2}A - \tfrac{1}{6}A + \tfrac{3}{2}V$$
$$13 = \tfrac{1}{2}A - \tfrac{1}{6}A + \tfrac{3}{4}A$$
$$13 = \tfrac{13}{12}A \qquad \therefore \boxed{A = 12}$$
$$\therefore \boxed{V = 6}$$