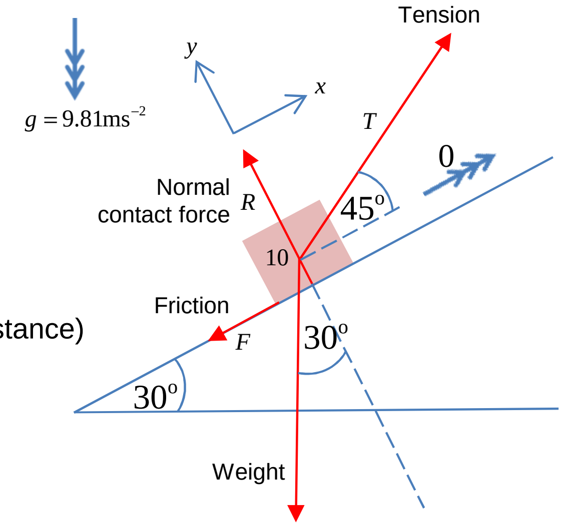
## A recipe for solving mechanics problems

Mechanics problems can be solved without fuss if the following recipe is followed. If steps are missed out, all manner of confusion can result. Note the pure maths part is not until step 9.

1. Represent the mechanics problem via a **clear diagram.** Make it large enough so any geometry can be easily interpreted
2. **Define all variables in the diagram**, which will become your mathematical universe
3. Idealize objects being considered. If you a considering a *particle* model, a single circular blob may be sufficient. However a bit of realism can give important physical context. e.g. draw a car, not a rectangle.
4. Clearly label all **forces**. Solid arrows using a red pen is good practice.
5. Define a **coordinate system** (e.g. Cartesian *x,y* or polar *r*, $\theta$ ) and choose a sensible orientation to reflect the **symmetry** of the physical system being modelled. e.g. If a block is sliding (or about to slide) down an inclined plane, choose 'x' to be downhill.
6. Mark any **external fields e.g. gravitation** and indicate their strength.
7. Indicate any *kinematic variables* in addition to displacement i.e. **velocity** (double arrow) and **acceleration** (triple arrow).
8. Invoke a **Law of Physics** and write down a system of equations in terms of variables defined in the diagram: e.g :
   1. *Conservation of energy* (scalar
   2. *Conservation of linear momentum* (vector)
   3. *Conservation of mass* (scalar)
   4. *Newton's Second Law* i.e. mass x acceleration = vector sum of forces (vector)
   5. *Conservation of angular momentum* (vector)
   6. Rate of change of angular momentum = net torque (vector)
9. Solve the equations to find desired unknowns. Note any additional relationships such as:

$$F = \mu R$$

**Example:** A block of 10kg is in equilibrium 'at the point of sliding' *uphill* (this is called limiting friction). If the plane is inclined at 30° and the tension is at 45° to the plane, what is *T* given a coefficient of friction of $\mu$ = 1/5 ?




1. Clear diagram
2. All variables defined in diagram
3. Block idealized as a shaded rectangle
4. All *significant* forces labelled (e.g. ignore air resistance)
5. Coordinate system defined (uphill is sensible 'x' direction)
6. Gravitational acceleration indicated
7. Acceleration (albeit = zero i.e. equilibrium) defined

8. Invoke Law of Physics and form a system of simultaneous equations

Newton's second law (mass x acceleration = vector sum of forces) with components resolved in x and y directions.

$$x: \quad 0 = \frac{T}{\sqrt{2}} - 10 \times \tfrac{1}{2} g - \tfrac{1}{5} R \qquad \therefore\ T = \left(5g + \tfrac{1}{5} R\right)\sqrt{2}$$

$$y: \quad 0 = R + \frac{T}{\sqrt{2}} - 10g\frac{\sqrt{3}}{2} \qquad \therefore R = 5g\sqrt{3} - \frac{T}{\sqrt{2}}$$

$$\therefore T = \left(5g + \tfrac{1}{5}\left(5g\sqrt{3} - \frac{T}{\sqrt{2}}\right)\right)\sqrt{2}$$

$$T = 5\sqrt{2}g + g\sqrt{6} - \tfrac{1}{5} T$$

$$\tfrac{6}{5} T = g\left(5\sqrt{2} + \sqrt{6}\right)$$

$$T = \frac{5\left(5\sqrt{2} + \sqrt{6}\right)}{6} g$$

$$T \approx 7.93g$$

9. Solve equations. In this case the goal is to find the tension *T* which causes the block to be at the point of sliding uphill.