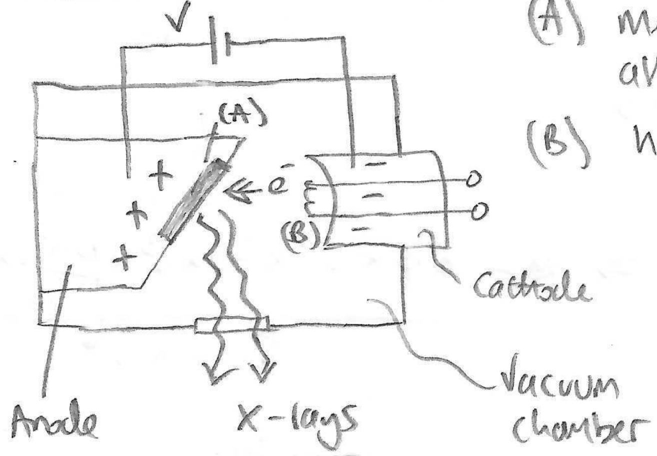
MEDICAL IMAGING

1/

(i)



(A) metal target (eg tungsten) attached to anode

(B) hot filament, which releases electrons

X-ray schematic

a) if 100% of electron K.E converts into X-ray photons, these photons will have the smallest wavelength λ_{min}

So $$\underbrace{\frac{hc}{\lambda_{min}}}_{\text{photon energy}} = \underbrace{eV}_{\text{electron KE gained from electric field between anode and cathode}}$$

$$\Rightarrow \boxed{\lambda_{min} = \frac{hc}{eV}}$$

$$\therefore \quad \lambda_{min} = \frac{6.626\times10^{-34} \times 2.998\times10^{8}}{1.602\times10^{-19} \times 70.0\times10^{3}} \quad (m)$$

$$= \boxed{1.77\times10^{-11}\ m} \quad = \quad 17.7\ pm$$

(Note: $\dfrac{\lambda_{min}}{pm} = \dfrac{1240}{V/kV}$ so $\lambda_{min} = \dfrac{1240}{70}$ pm $= 17.7$ pm)

b) X-ray energy $= \boxed{IVkt}$

$I = 8.0\times10^{-3}$ A, $V = 70\times10^{3}$ Volts $k = \frac{0.8}{100}$

t is duration of X-ray scan.

Absorbed energy is DM where D is 5.00×10^{-6} J/kg and M is mass in kg.

$$\therefore \quad \underbrace{DM}_{\text{Energy absorbed}} = \underbrace{IVkt}_{\text{Total X-ray energy}} \quad \Rightarrow \quad M = \frac{IVkt}{D}$$

well, one assumes it passes through the body to get to the photographic plates!

$$\Rightarrow M = \frac{8.0\times10^{-3}\times70\times10^{3}\times\frac{0.8}{100}\times0.320}{5.00\times10^{-6}}$$

$$= \frac{1.43\text{ J}}{5.00\times10^{-6}\text{ J/kg}}$$

$$= 2.87\times10^{5}\text{ kg (!)}$$

So there is clearly something about Sieverts that I don't understand! Perhaps it is the energy absorbed by the body (ie via ionization), and not what ends up passing through and interacting with photographic plates?

↳ question modified to calculate $\boxed{1.43\text{ J}} = IVkt$

ie the total energy of the X-rays produced.

(ii) Photon energy $E = \frac{hc}{\lambda}$

$$\therefore \quad E/\text{keV} = \frac{hc}{1000e\left(\frac{\lambda}{\text{nm}}\right)\times10^{-9}}$$

$$\left(\frac{E}{\text{keV}}\right) = \frac{6.626\times10^{-34}\times2.998\times10^{8}}{1000\times1.602\times10^{-19}\times10^{-9}}\ \frac{1}{(\lambda/\text{nm})}$$

$$\boxed{\left(\frac{E}{\text{keV}}\right) = 1.240\Big/\left(\lambda/\text{nm}\right)}$$

Tungsten K lines:

$$E_\beta = \frac{1.240}{0.0184} \text{ (keV)} = \boxed{67.4 \text{ keV}}$$

$$E_\alpha = \frac{1.240}{0.0209} \text{ (keV)} = \boxed{59.3 \text{ keV}}$$

Silver K lines

$$\boxed{\lambda/\text{nm} = \frac{1.240}{E/\text{keV}}}$$

$$\lambda_\beta = \frac{1.240}{24.9} = \boxed{0.0498 \text{ nm}}$$

$$\lambda_\alpha = \frac{1.240}{22.2} = \boxed{0.0558 \text{ nm}}$$

we can use the same formula if 100% of e^- KE $\rightarrow$ x-rays

λ_{min} for the 70.0 kV silver tube is:

$$\lambda_{min} = \frac{1.240}{70} \text{ (nm)}$$

$$= \boxed{0.0177 \text{ nm}}$$

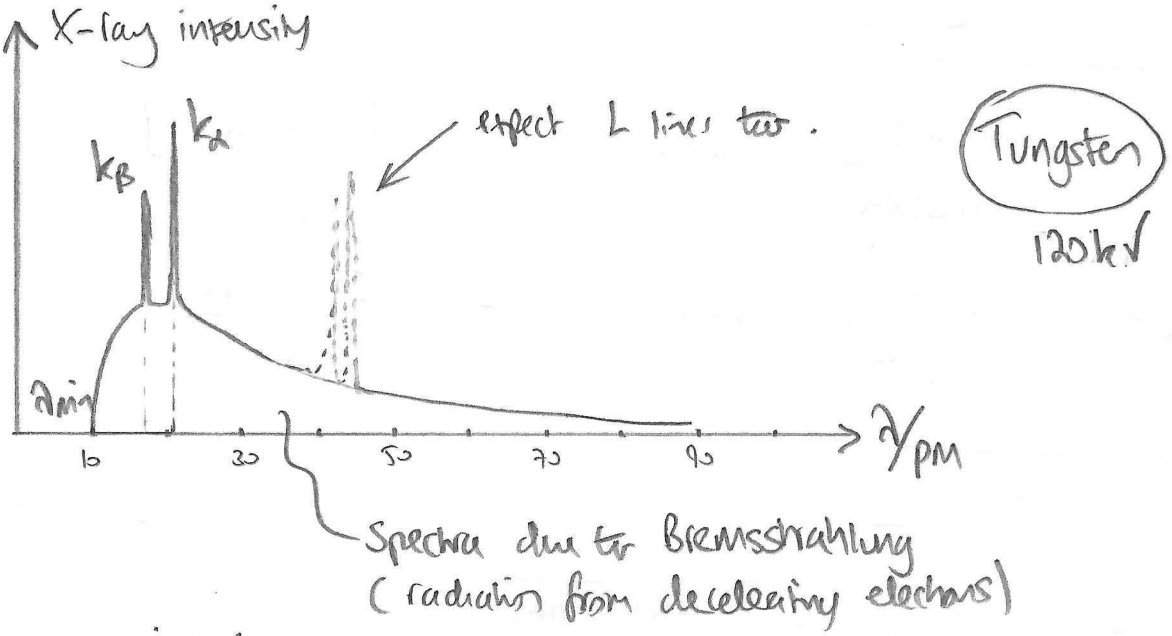
λ_{min} for the 120 kV tungsten tube is:

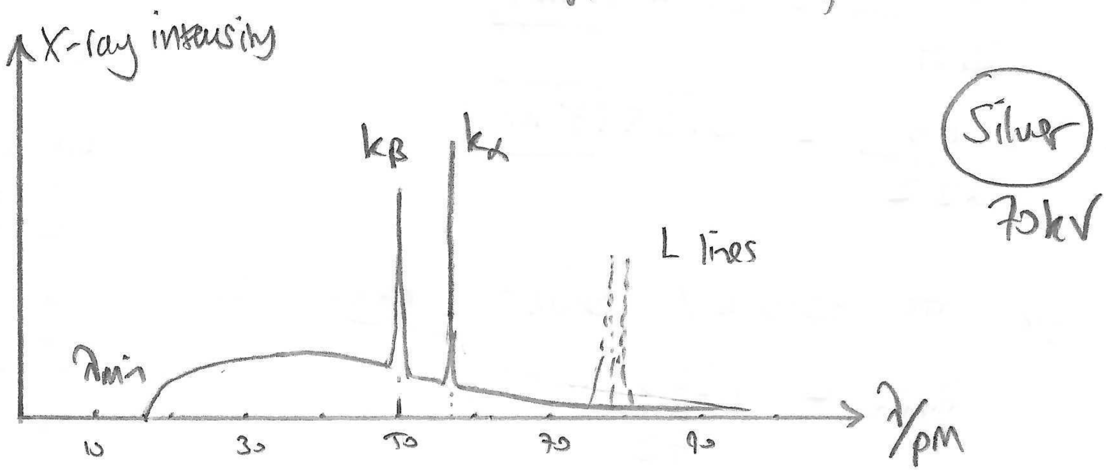
$$\lambda_{min} = \frac{1.240}{120} \text{ (nm)} = \boxed{0.0103 \text{ nm}}$$

In summary:

	Tungsten 120kV	Silver 70kV
λ_α	20.9	55.8
λ_β	18.4	49.8
λ_{min}	10.3	17.7

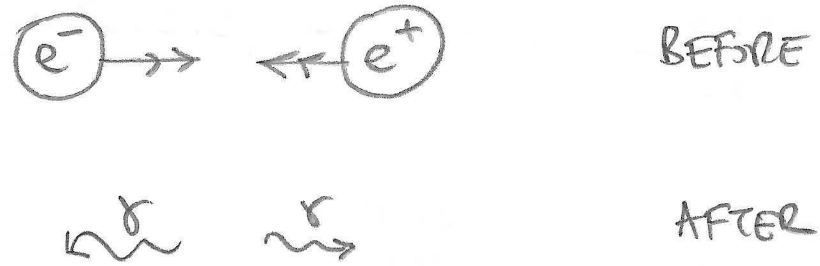
(wavelengths in pm $= 10^{-12}$ m $= 1$ nm$/1000$)





Expect lower area under spectra for silver, since lower accelerating potential ie less total X-ray energy.

(iii)



Momentum must be conserved as well as energy.
So the gamma photons must have the same net momentum as the electron and positron, which means they must travel in opposite directions.

[If the electron approaches the positron at relativistic speeds ie $KE \simeq mc^2$, does this mean the γ rays must have $\underline{\text{different energies}}$, to conserve momentum?]

photon momentum $\boxed{p = h/\lambda}$ (de-Broglie's relation)

Relativistic momentum * is $\boxed{\gamma m u}$

$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$

* for massive particles.

If e^- and e^+ annihilate at speeds $\ll c$

$\Rightarrow$ $E = mc^2$ is the photon energy.

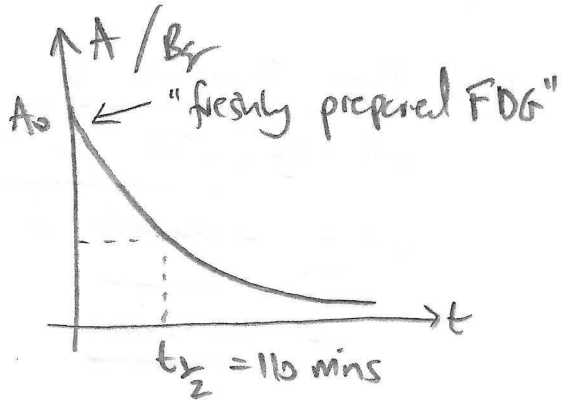
$$\therefore \quad E/\text{MeV} = \frac{9.109 \times 10^{-31} \times (2.998 \times 10^8)^2}{10^6 \times 1.602 \times 10^{-19}}$$

$$= \boxed{0.505}$$

(iv) Positron decay of Fluorine-18

$${}^{18}_{9}F \to {}^{18}_{8}O + {}^{0}_{1}e^+ + \nu_e + \gamma$$

in fluorodeoxyglucose (FDG)



Activity $\boxed{A = A_0 / 2^{t/t_{1/2}}}$

let A_{30} be activity before the scan

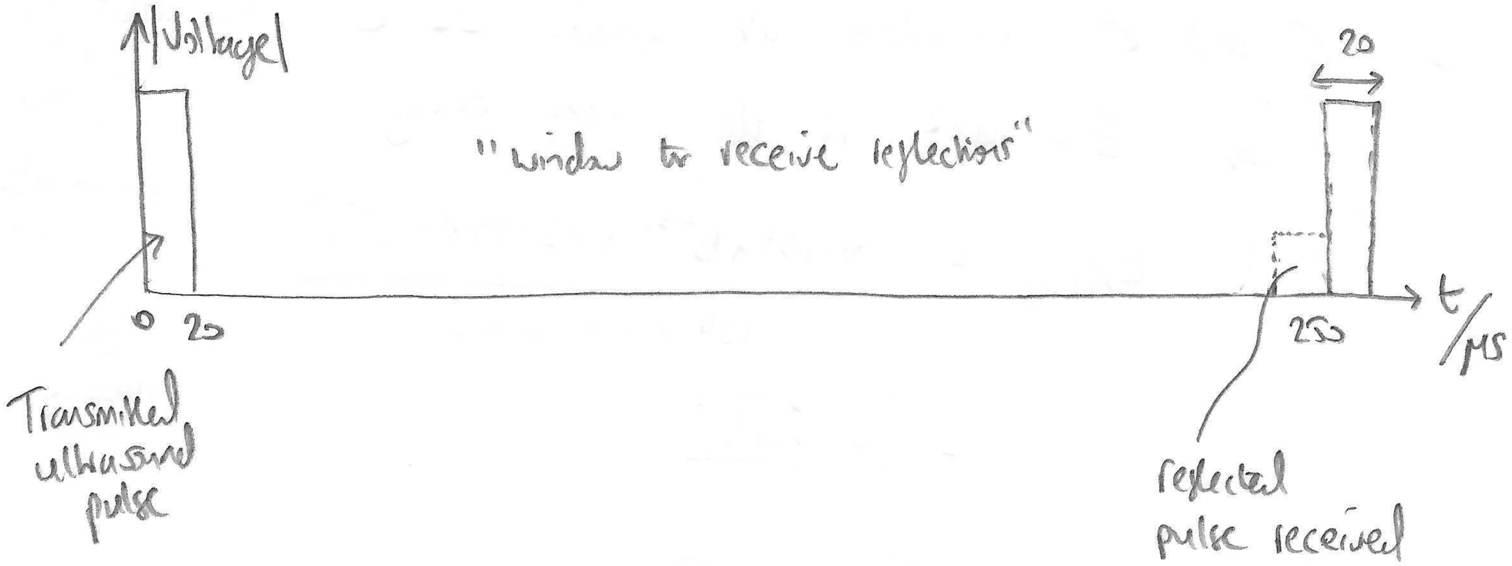
A_{60} be " after the (30 minute) scan

$$A_{30}/A_0 = 2^{-30/110} = \boxed{0.828}$$

$$A_{60}/A_0 = 2^{-60/110} = \boxed{0.685}$$

(v) Ultrasonic transducer produces 4000 pulse per second

$\therefore$ interpulse delay is $\frac{1}{4000}$ s $= \boxed{250\ \mu s}$



a) Max thickness of tissue is:

$\boxed{\Delta x = \frac{1}{2}c\,(250-20)\,\mu s}$ {Note "there and back distance"}

$\Delta x_{bone} = 4000\ m/s \times 230\times10^{-6}\ s \times \frac{1}{2}$

$= \boxed{0.46 m}$

$\Delta x_{fat} = \frac{1}{2}\times 1450 \times 230\times10^{-6}$ (m)

$= \boxed{0.17 m}$

b) Minimum thickness of soft tissue is $\frac{1}{2}\times 1540 \times 20\times10^{-6}$

c for soft tissue

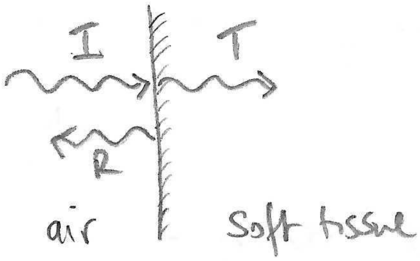
$= \boxed{0.015 m}$

(15.4 mm).

Note in $20\mu s$ there are $\dfrac{20\times10^{-6}}{\frac{1}{5.00\times10^{6}}}$ pulse oscillation periods, since ultrasound frequency is 5MHz.

ie $20\times5 = \boxed{100 \text{ periods}}$

(vi)



Reflection Coefficient: $\boxed{R = \dfrac{Z_1 - Z_2}{Z_1 + Z_2}}$

↑
Amplitude

Intensity reflection coefficient $= R^2$

Acoustic impedance ⟶ $\boxed{Z = \rho c}$ (density × speed)

a) For an air – soft tissue boundary:

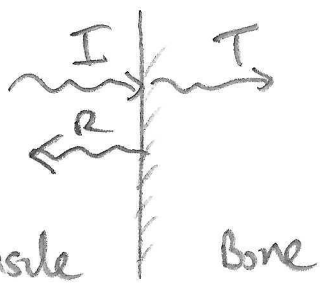
$$R^2 = \left(\frac{Z_{air} - Z_{st}}{Z_{air} + Z_{st}}\right)^2$$

$$= \left(\frac{1.3 \times 340 - 1060 \times 1540}{1.3 \times 340 + 1060 \times 1540}\right)^2$$

$$= \boxed{0.999}$$

So if you don't use a coupling gel between an ultrasonic transducer and soft tissue, you will get hardly any ultrasound transmitted into the soft tissue – most will be reflected off the air / soft tissue boundary.

b)



$$R^2 = \left(\frac{Z_M - Z_b}{Z_m + Z_b}\right)^2$$

$$= \boxed{0.405}$$

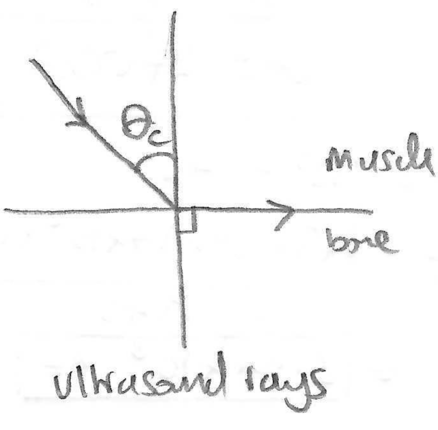
Reflected intensity fraction.

$$T^2 = 1 - R^2$$

$$= \boxed{0.595}$$

Transmitted intensity fraction.

c)

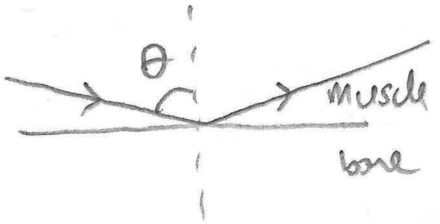


Snell's law: $\boxed{\dfrac{\sin\theta_c}{c_M} = \dfrac{\sin 90^\circ}{c_b}}$

$$\therefore \theta_c = \sin^{-1}\left(\frac{c_m}{c_b}\right) = \sin^{-1}\left(\frac{1580}{4000}\right)$$

$$= \boxed{23.3^\circ}$$

when $\theta > \theta_c$, $\boxed{R \to -1}$ ie 100% of ultrasound is reflected (and inverted).

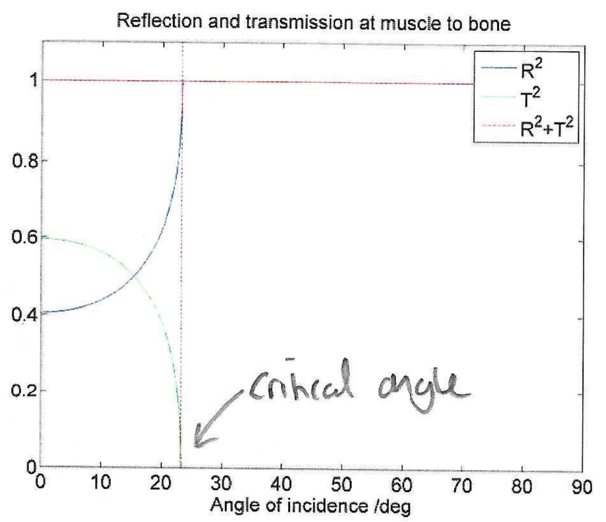


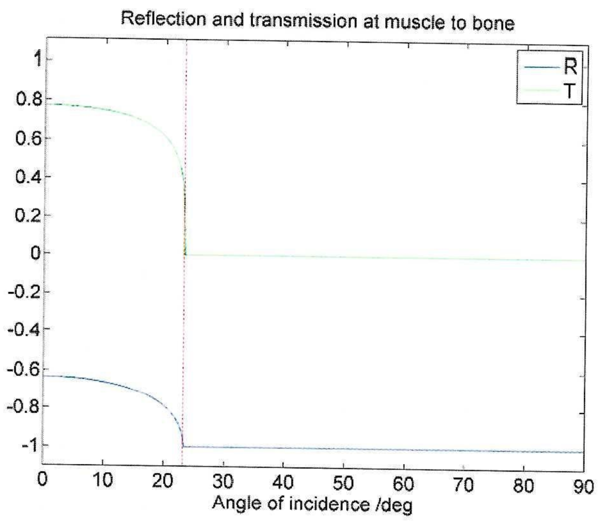
ie reflected intensity = incident intensity if $\theta > \theta_c$

Note: $R = \dfrac{Z_1 - Z_2}{Z_1 + Z_2}$ is <u>only true</u> at normal incidence

In general: $\boxed{R = \dfrac{-Z_2\cos\theta_1 + Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2}}$ where: $\boxed{\dfrac{\sin\theta_1}{c_1} = \dfrac{\sin\theta_2}{c_2}}$

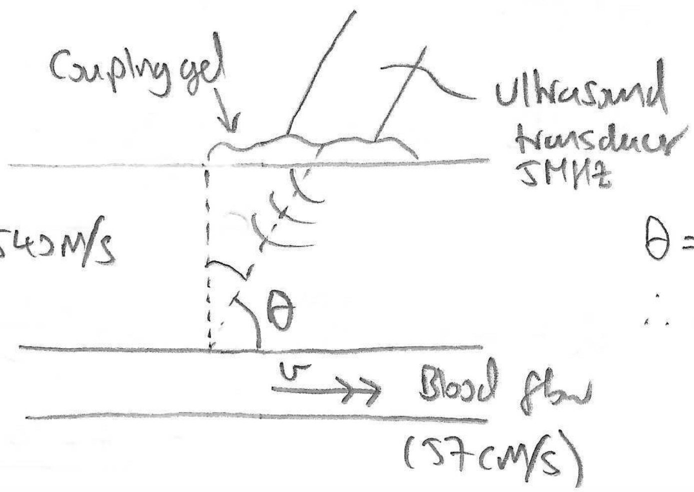
(<u>"Fresnel equation"</u> for longitudinal waves → should be similar to S polarized EM waves).





$T^2 = 1 - R^2 \Rightarrow T = \sqrt{1 - R^2}$

(vii)



Doppler shift is

$\boxed{\Delta f = 2 f v \cos\theta / c}$

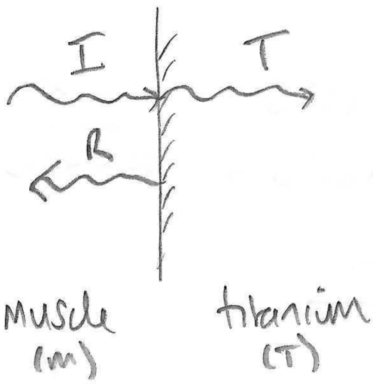
$\theta = 90^\circ - 50^\circ = 40^\circ$

$\therefore \Delta f = \dfrac{2 \times 5 \times 10^6 \times 57 \times 10^{-2} \times \cos 40^\circ}{1540}$

$= \boxed{2.84 \times 10^3 \text{ Hz}}$

$\therefore \quad \frac{\delta f}{f} = \frac{2.84 \times 10^3}{5.0 \times 10^6} = \boxed{0.057\ \%}$

(viii)



$\boxed{R^2 = 0.78}$

ie ultrasound intensity is 78% reflected at a muscle to titanium boundary

$$R^2 = \left(\frac{Z_M - Z_T}{Z_M + Z_T}\right)^2$$

$$\boxed{R = \frac{Z_M - Z_T}{Z_M + Z_T}}$$

Note inverted reflections
hence $\boxed{R = -\sqrt{0.78}}$

$\therefore \quad (Z_M + Z_T) R = Z_M - Z_T$

$\therefore \quad Z_T (R+1) = Z_M - Z_M R$

$$\boxed{Z_T = \frac{(1-R) Z_M}{1+R}}$$

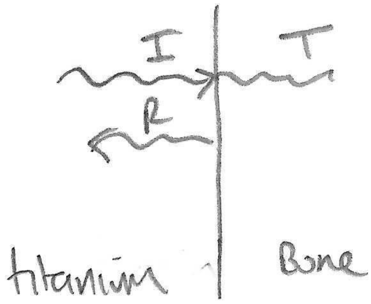
$Z_T = \rho_T c_T \quad \therefore \quad c_T = Z_T / \rho_T$

$$\therefore \quad c_T = \frac{1 + \sqrt{0.78}}{1 - \sqrt{0.78}} \times \frac{1070 \times 1580}{4500}$$

$= \boxed{6100\ m/s}$ to 2 s.f.

(6056 m/s to 4 s.f. given this data).

↑ it is actually 6070 m/s, so slightly more than 78% reflected.



$$R = \frac{Z_T - Z_B}{Z_T + Z_B}$$

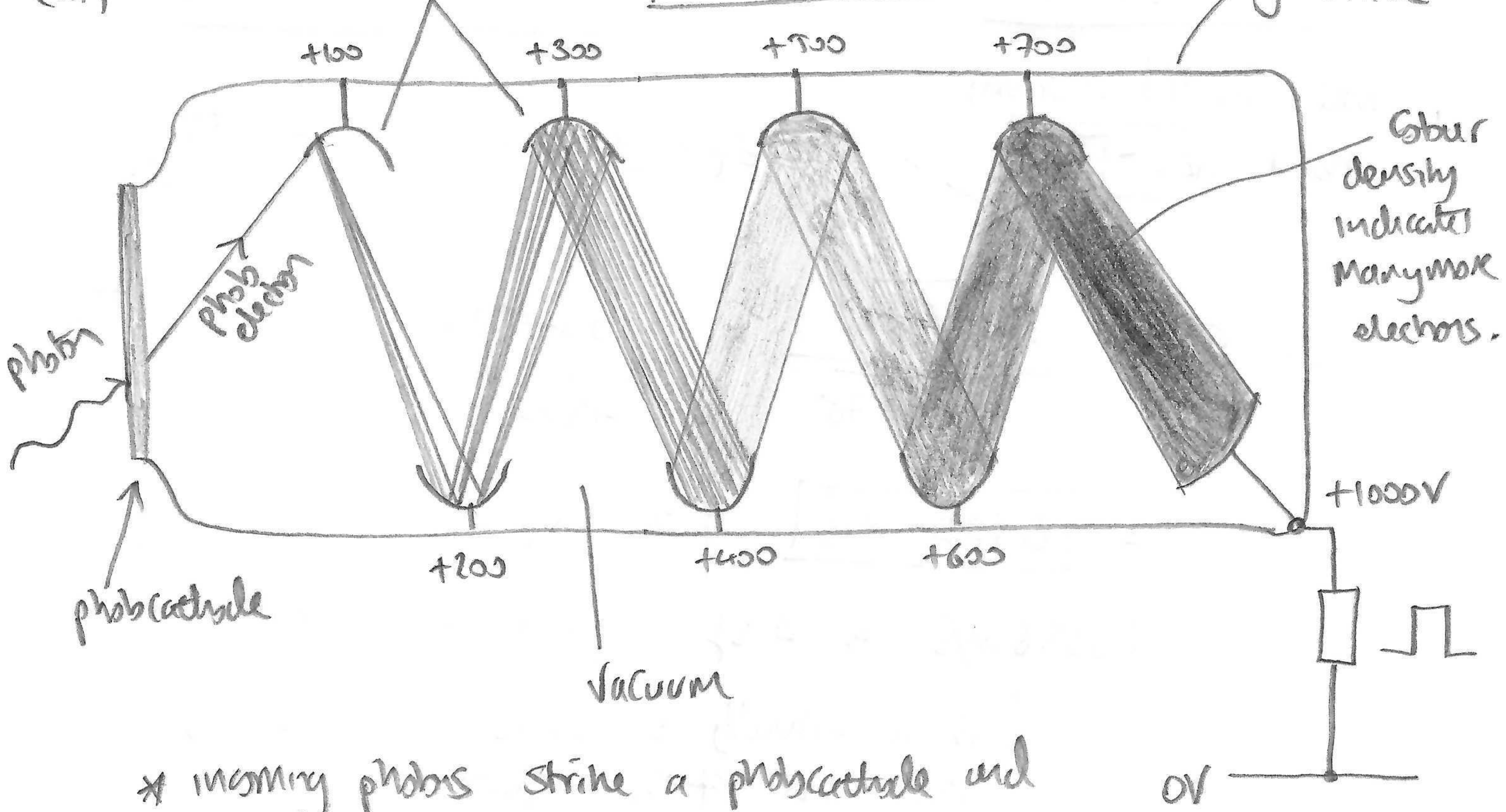
$$= \frac{4500 \times 6056 - 1900 \times 4000}{4500 \times 6056 + 1900 \times 4000}$$

$$= \boxed{0.564}$$

So $\boxed{\text{no inversion}}$ (high to low impedance)

and fraction of incident ultrasound power transmitted is $T^2 = 1 - R^2 = \boxed{68.2\%}$

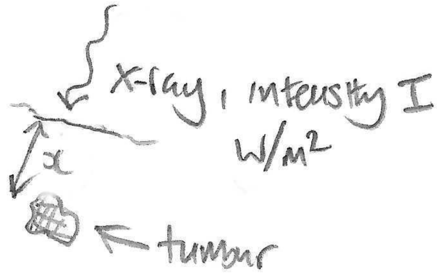
(ix) $\boxed{\text{PHOTOMULTIPLIER TUBE}}$



* incoming photons strike a photocathode and produce electrons
* Electrons are accelerated between terminals (dynodes) of increasing large voltage (but same 100V PD between dynodes)
* Each time an electron strikes a dynode, more electrons are emitted.
* Amplification factors could be $\simeq 10^5$ or higher → So small light intensities can result in measurable voltages

2/ → See separate document (table format)

3/ * let intensity of x-ray beam be intensity I_0



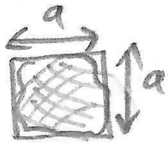
* Intensity at tumour is $I_0 e^{-\mu x}$, assuming negligible $\frac{1}{r^2}$ spreading of the beam over distance x.

depth of tumour is $\boxed{x = 15.2\,mm}$ in soft tissue

$\mu = \ln 2 / x_{\frac{1}{2}}$ and $x_{\frac{1}{2}} = 54.3\,mm$. "Half value thickness" (HVT).

* let $E = 234\,J$ be tumour destruction energy

* $E = I_0 e^{-\mu x} a^2 k t$ where a^2 is tumour cross section. $a = 2.2\,mm$.



$k = 0.11$ (absorbtion factor)

$t = 42 \times 60\,s$ (exposure time)

$$\therefore \quad I_0 = E / e^{-\mu x} a^2 k t$$

$$= \frac{234}{e^{-\frac{\ln 2 \times 15.2}{54.3}} \times (2.2 \times 10^{-3})^2 \times 0.11 \times 42 \times 60}$$

$$= \boxed{2.12 \times 10^5\ W/m^2}$$

Now if beam energy for max dose of X-rays in MeV ∝ 4 × distance to tumour in body/cm

$$\Rightarrow \quad E/_{MeV} = 4 \times 1.52 = \boxed{6.08}$$

∴ Since $E = \frac{hc}{\lambda}$

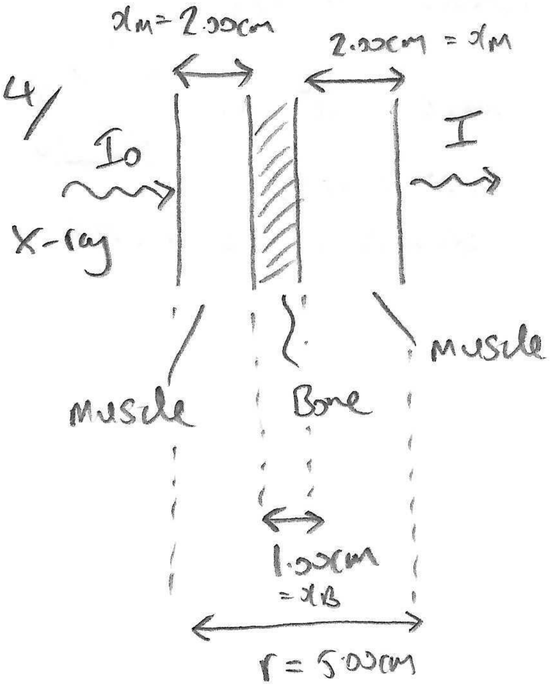
$\Rightarrow \quad \lambda_{min} = \frac{hc}{E}$

$= \frac{6.626\times10^{-34} \times 2.998\times10^{8}}{6.08\times10^{6}\times 1.602\times10^{-19}}$ (m)

$= \boxed{0.204 \text{ pm}}$

$(2.04\times10^{-13} \text{ m})$

4/



(Arm cross section)

$I = I_0 \times \frac{k}{r^2} e^{-(\mu_M x_M + \mu_b x_b + \mu_M x_M)}$ (Arm)

$I_* = I_0 \frac{k}{r^2}$ (no arm)

$I_* = 20 W/m^2$

∴ $I = 20 W/m^2 \times \exp(\; - * \;)$

$* = 0.21\times4.00 + 0.60\times1.00$
$\mu_M \times 2x_M + \mu_b \times x_b$

$\Rightarrow \boxed{I = 4.74 \text{ W/m}^2}$

Now repeat for leg of thickness $R = 14.0\,cm$. $x_b = 2.34\,cm$.

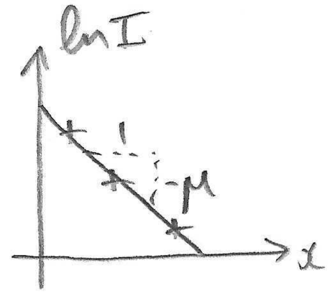
$$I = \frac{I_o k}{R^2} e^{-(2\mu_m x_m + \mu_b x_b)}$$

$$x_m = \frac{14 - 2.34}{2} = \boxed{5.83\,cm}$$

From above: $I_x = \frac{I_o k}{(5.0\,cm)^2}$ $\therefore I_o k = I_x (5.0\,cm)$

$$\therefore I = 20 \times \left(\frac{5.0\,cm}{14.0}\right)^2 e^{-(2 \times 0.21 \times 5.83 + 0.60 \times 2.34)} \quad (W/m^2)$$

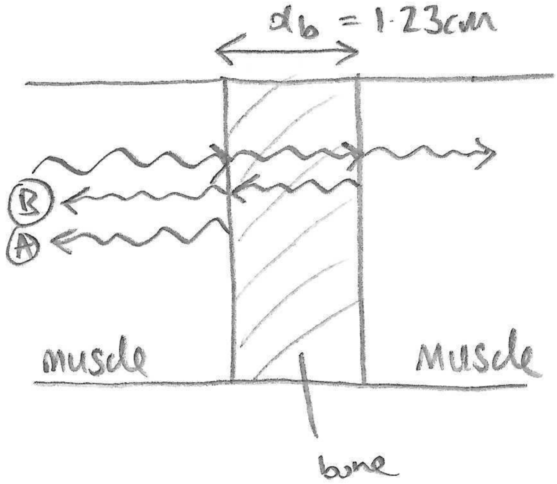
$$= \boxed{0.05\ W/m^2}$$



5/ See Excel sheet. $\ln I = \ln I_o - \mu x$

linearized equation.

6/



let A be reflection off muscle - bone boundary

Voltage peak is:

$$0.70 \times \frac{(Z_m - Z_b)}{Z_m + Z_b} = V_A$$

$$V_A = 0.70 \times \left(\frac{1070 \times 1580 - 1900 \times 4000}{1070 \times 1580 + 1900 \times 4000}\right)$$

$$V_A = 0.70 \times -0.6361$$

$$V_A = \boxed{-0.445\ \text{Volts}}$$

i.e. inverted

	ρ / kgm^{-3}	c / ms^{-1}
Muscle	1070	1580
Bone	1900	4000

"There and back time" for reflection B is:

$$\Delta t = \frac{x_b}{c_b} = \frac{1.23 \times 10^{-2}\,m}{4000\,m/s} \ (s)$$

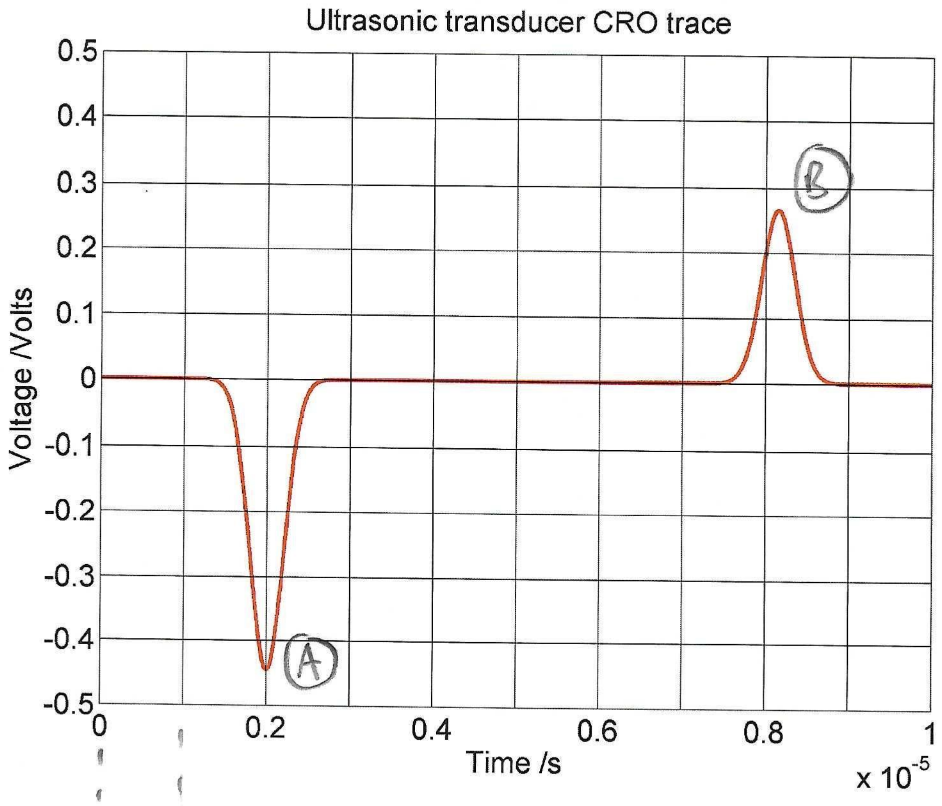
$$= \boxed{3.08\,\mu s}$$

Voltage peak of reflection B is:

$$V_B = \underbrace{0.70V \times \sqrt{1-0.6361^2}}_{\mathrm{I}} \times \underbrace{0.6361}_{\mathrm{II}} \times \underbrace{\sqrt{1-0.6361^2}}_{\mathrm{III}}$$

I transmission at muscle - bone boundary
II reflection at bone - muscle boundary
III transmission at bone - muscle boundary

$= \boxed{0.265 \text{ Volts}}$



1.0 μs = timebase

If higher frequency ultrasound is used, might expect $\boxed{\text{greater attenuation}}$, so perhaps reflection B would be lower in voltage, and A also. Pulse width might also be smaller, i.e. if waveform sticks to the same # periods in each pulse.

AF Dec 2020.