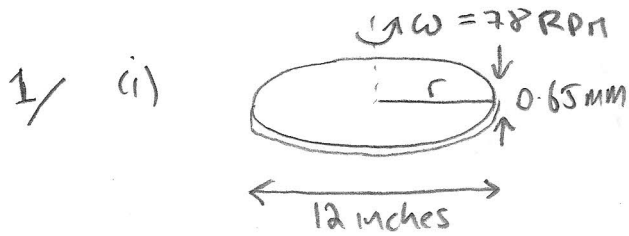


MOMENTS OF INERTIA AND ROTATIONAL DYNAMICS



Mass of vinyl is

$$M = \frac{\pi (6 \times 2.54)^2 \times 0.65 \times 0.925}{1000} \text{ kg}$$

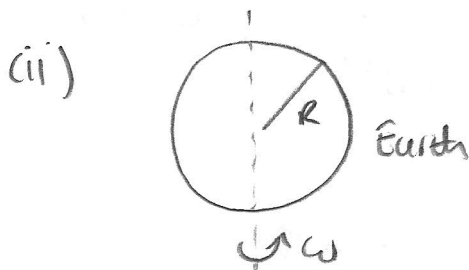
$$r = \frac{6 \times 2.54}{100} = 0.152 \text{ m}$$

$$M = 4.39 \times 10^{-2} \text{ kg} \quad (43.9 \text{ g})$$

$$I = \frac{1}{2} M r^2 = \frac{1}{2} (4.39 \times 10^{-2} \text{ kg}) (0.152)^2 = 5.09 \times 10^{-4} \text{ kg m}^2$$

$$L = I \omega = (5.09 \times 10^{-4}) (78 + 2\pi/60) = 4.16 \times 10^{-3} \text{ kg m}^2 \text{ s}^{-1}$$

$$E = \frac{1}{2} I \omega^2 = 1.70 \times 10^{-2} \text{ J}$$



$$I = \frac{2}{5} M R^2$$

$$\omega = \frac{2\pi}{T}$$

$$L = I \omega$$

$$E = \frac{1}{2} I \omega^2$$

$$\therefore L_{\oplus} = \frac{2}{5} M R^2 \times \frac{2\pi}{T}$$

$$L_{\oplus} = \frac{2}{5} \times 5.97 \times 10^{24} \times (6371 \times 10^3)^2 \times \frac{2\pi}{24 \times 3600}$$

$$L_{\oplus} = 7.05 \times 10^{23} \text{ kg m}^2 \text{ s}^{-1}$$

$$E = \frac{1}{2} I \omega^2 = \frac{1}{2} L \omega = \frac{1}{2} \times 7.05 \times 10^{23} \times \frac{2\pi}{24 \times 3600} = 2.56 \times 10^{29} \text{ J}$$

(iii) Angular momentum for Earth - Sun system

a)

$$L_{\oplus \odot} = m_{\oplus} r^2 \omega = 5.97 \times 10^{24} \times (1.496 \times 10^{11})^2 \times \frac{2\pi}{365 \times 24 \times 3600}$$

$$= 2.66 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$$

So $L_{\oplus \odot} / L_{\oplus} \approx 3.77 \times 10^6$

i.e. orbital angular momentum is about 4 million times larger than spin angular momentum for the Earth.

$$\begin{aligned}
 \text{b) } L_{M\oplus} &= M_M r_{M\oplus}^2 \omega \\
 &= 7.35 \times 10^{22} \times (394402 \times 10^3)^2 \times \frac{2\pi}{28 \times 24 \times 3600} \\
 &= \boxed{2.97 \times 10^{34}} \text{ kgm}^2\text{s}^{-1}
 \end{aligned}$$

$$L_{M\oplus} / L_{\oplus} \approx \boxed{4.2}$$

This explains why, due to tidal forces, the orbit of the moon should affect the rotation of the earth. [Apparently, billions of years from now, the moon will be so far away from Earth that the Earth's rotation axis may not be stable \Rightarrow 'seasonal chaos!']

(iv) Bohr theory of Hydrogen



$$L = m_e r v$$

$$L = n \hbar$$

$$\text{so } v = \frac{n \hbar}{m_e r}$$

$$\text{let } n=1.$$

$$\therefore \frac{c}{v} = \frac{m_e r c}{\hbar} = \frac{9.109 \times 10^{-31} \times 5.292 \times 10^{-11} \times 2.998 \times 10^8}{\frac{1}{2\pi} \times 6.63 \times 10^{-34}}$$

$$\approx \boxed{137}$$

[$\frac{1}{137} = \frac{e^2}{4\pi\epsilon_0 \hbar c}$ is called the FINE STRUCTURE constant

\Rightarrow special relativity effects may be small, but not negligible for the motion of the electron in the hydrogen atom.]

