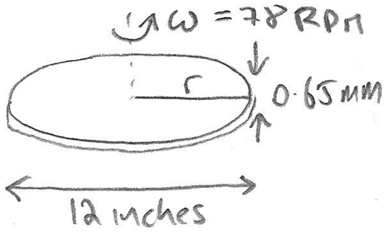
MOMENTS OF INERTIA AND ROTATIONAL DYNAMICS

1/ (i)



Mass of vinyl is

$$M = \frac{\pi (6 \times 2.54)^2 \times \frac{0.65}{10} \times 0.925}{1000} \text{ kg}$$

$$r = \frac{6 \times 2.54}{100} = \boxed{0.152 \text{ m}}$$

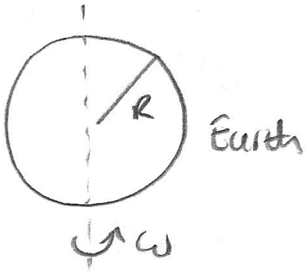
$$\boxed{M = 4.39 \times 10^{-2} \text{ kg}} \quad (43.9 \text{g})$$

$$I = \tfrac{1}{2} M r^2 = \tfrac{1}{2}(4.39 \times 10^{-2} \text{ kg})(0.152)^2 = \boxed{5.09 \times 10^{-4} \text{ kg m}^2}$$

$$L = I\omega = (5.09 \times 10^{-4})(78 \times 2\pi / 60) = \boxed{4.16 \times 10^{-3} \text{ kgm}^2\text{s}^{-1}}$$

$$E = \tfrac{1}{2} I \omega^2 = \boxed{1.70 \times 10^{-2} \text{ J}}$$

(ii)



$$I = \tfrac{2}{5} M R^2 \qquad \omega = 2\pi / T$$

$$L = I\omega \qquad E = \tfrac{1}{2} I \omega^2$$

$$\therefore \; L_\oplus = \tfrac{2}{5} M R^2 \times 2\pi / T$$

$$L_\oplus = \frac{\tfrac{2}{5} \times 5.97 \times 10^{24} \times (6371 \times 10^3)^2 \times 2\pi}{24 \times 3600}$$

$$\boxed{L_\oplus = 7.05 \times 10^{33} \text{ kgm}^2\text{s}^{-1}}$$

$$E = \tfrac{1}{2} I \omega^2 = \tfrac{1}{2} L\omega = \tfrac{1}{2} \times 7.05 \times 10^{33} \times \frac{2\pi}{24 \times 3600} = \boxed{2.56 \times 10^{29} \text{ J}}$$

(iii) Angular momentum for Earth - Sun system

a) $$L_{\oplus\odot} = M_\oplus r^2 \omega = \frac{5.97 \times 10^{24} \times (1.496 \times 10^{11})^2 \times 2\pi}{365 \times 24 \times 3600}$$

$$= \boxed{2.66 \times 10^{40} \text{ kgm}^2\text{s}^{-1}}$$

So $L_{\oplus\odot} / L_\oplus \approx \boxed{3.77 \times 10^6}$ ie orbital angular momentum is about 4 million times larger than spin angular momentum for the Earth.

b) $L_{M\oplus} = M_M r_{M\oplus}^2 \omega$

$= 7.35\times10^{22} \times (384402\times10^3)^2 \times \frac{2\pi}{28\times24\times3600}$

$= \boxed{2.97\times10^{34}}\ \mathrm{kg\,m^2\,s^{-1}}$

$L_{M\oplus}/L_{\oplus} \approx \boxed{4.2}$

This explains why, due to <u>tidal forces</u>, the orbit of the Moon should affect the rotation of the earth. [Apparently, billions of years from now, the moon will be so far away from Earth that the Earth's rotation axis may not be stable ⇒ 'Seasonal chaos!!']

(iv) Bohr theory of Hydrogen

$L = m_e r v \qquad L = n\hbar$

So $v = \frac{n\hbar}{m_e r}$ let $n=1$.

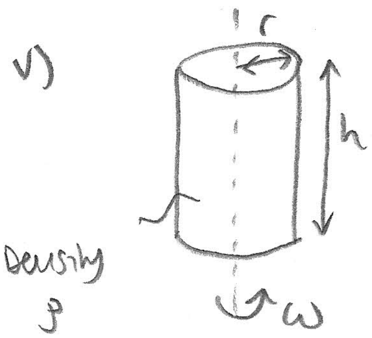
$\therefore \frac{c}{v} = \frac{m_e r c}{\hbar} = \frac{9.109\times10^{-31}\times5.292\times10^{-11}\times2.998\times10^{8}}{\frac{1}{2\pi}\times6.63\times10^{-34}}$

$\simeq \boxed{137}$

[$\frac{1}{137} = \frac{e^2}{4\pi\varepsilon_0\hbar c}$ is called the FINE STRUCTURE constant

⇒ special relativity effects may be small, but <u>not negligible</u> for the motion of the electron in the Hydrogen atom].

v)



Mass M of rotating cylinder is

$$M = \pi r^2 h \rho$$

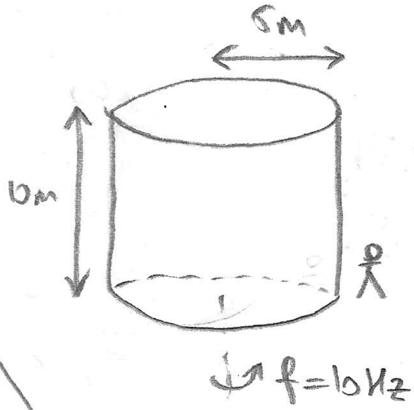
So $I = \frac{1}{2} M r^2 = \frac{1}{2}\pi r^4 h \rho$

$\therefore$ $E = \frac{1}{2} I \omega^2 = \frac{1}{4}\pi r^4 h \rho (2\pi f)^2$

$$\boxed{E = \pi^3 \rho h f^2 r^4}$$

$\therefore$ # Iron cylinders $= \dfrac{33 \times 10^{12}}{\pi^3 \times 7850 \times 10 \times 10^2 \times 5^4}$

$= \boxed{217}$



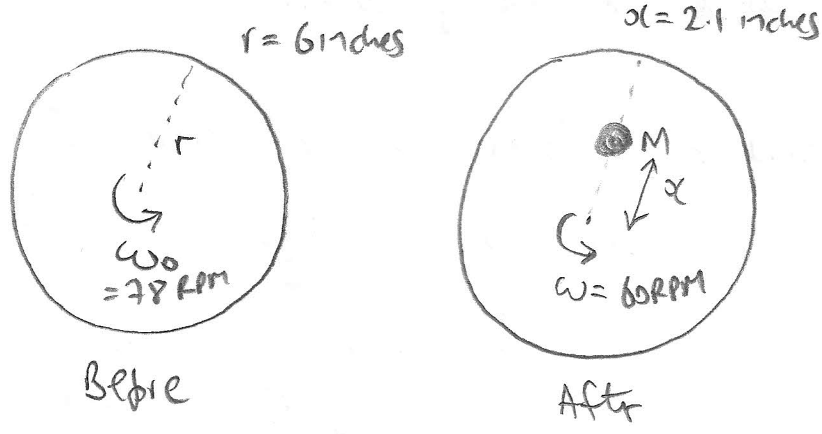
This is a <u>HUGE</u> feat of engineering!

In reality, there are a few rotating cylinders in Dinorwig, but none of this size. Most of the stored energy is in the GPE of the lake $\approx$ 900m above llanberis.

However, the 'flywheel' energy storage idea is certainly in use, from racing cars to NASA.

Friction with air and on bearings significant!

vi)



Conservation of angular momentum

$$I\omega_0 = I\omega + Mx^2\omega$$

$$\Rightarrow \boxed{M = \frac{I(\omega_0 - \omega)}{x^2 \omega}}$$

$I = \frac{1}{2} m r^2$

$M = \pi r^2 t \rho \longrightarrow$ So $I = \frac{1}{2}\pi r^4 t \rho$

$$\therefore \quad M = \frac{\frac{1}{2}\pi t \rho r^4}{x^2}\left(\frac{\omega_0}{\omega} - 1\right)$$

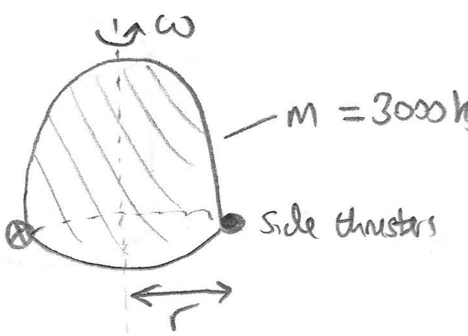
$$m = \underbrace{\pi r^2 t\rho}_{\text{disc mass}} \frac{1}{2}\left(\frac{r}{x}\right)^2\left(\frac{\omega_0}{\omega} - 1\right)$$

From (i) disc mass $= \boxed{43.9g}$

so $M = 43.9g \times \frac{1}{2}\left(\frac{6.0}{2.1}\right)^2\left(\frac{78}{60} - 1\right)$

$\boxed{M = 53.8g}$ (ie mass of mince pie)

vii)



r = 2.0M

$I = \frac{1}{3}mr^2$

$E = \frac{1}{2}I\omega^2$

so $E = \frac{1}{2}\left(\frac{1}{3}mr^2\right)\omega^2$

$E = \frac{1}{6} \times 3000 \times 2.0^2 \times \left(\frac{30 \times 2\pi}{60}\right)^2$

$E = \boxed{1.97 \times 10^4 J}$

This is the energy that equates to the work done by the side thrusters to cancel the roll of 30 RPM.

viii) Assume each thruster produces constant thrust T

$\therefore -2Tr = I\dot{\omega}$ so $\dot{\omega} = \frac{2Tr}{I}$

↑ opposes ω

$\dot{\omega} = \frac{2Tr}{\frac{1}{3}mr^2} = \frac{6T}{mr}$

$\therefore \boxed{\omega(t) = \omega_0 - \frac{6T}{mr}t}$ $\quad\therefore$ Since $\omega = d\theta/dt$

$\boxed{\theta = \omega_0 t - \frac{3T}{mr}t^2}$

$\omega = 0$ when $t_* = 5.0\,s$.

$$\therefore \quad \omega_0 - \frac{6T t_*}{Mr} = 0 \quad \Rightarrow \quad T = \frac{\omega_0 M r}{6 t_*}$$

$$T = \left(\frac{30 \times 2\pi}{60}\right) \times \frac{3000 \times 2.0}{6 \times 5.0}$$

$$\boxed{T = 628\,N}$$

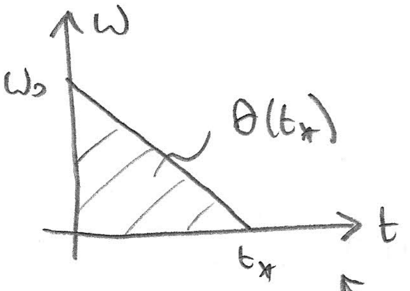
$$\therefore \quad \theta(t = 5.0s) = \left(\frac{30 \times 2\pi}{60}\right)(5.0) - \frac{3(628)}{3000 \times 2.0} \times (5.0)^2$$

$$= 7.85 \text{ radians} = \boxed{450^\circ}$$

[Note since $T = \frac{\omega_0 M r}{6 t_*}$, $\theta(t_*) = \omega_0 t_* - \frac{3 t_*^2}{\cancel{Mr}} \frac{\omega_0 \cancel{Mr}}{6 t_*}$

$$= \omega_0 t_* - \tfrac{1}{2}\omega_0 t_*$$

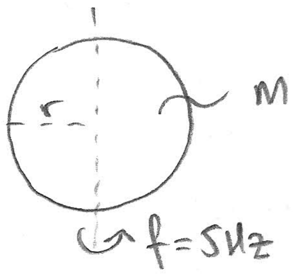
$$= \tfrac{1}{2}\omega_0 t_* \;]$$



which you can easily see from a graph of ω vs t

(ix)

Sphere

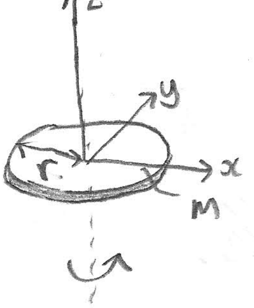


$$E = \tfrac{1}{2}\left(\tfrac{2}{5} M r^2\right)(2\pi f)^2$$

$$E = \tfrac{1}{5} M r^2 \times 4\pi^2 f^2$$

$$\boxed{E = \tfrac{4}{5}\pi^2 M r^2 f^2}$$

Coin



$$I_z = \tfrac{1}{2} M r^2$$

$$I_z = I_x + I_y$$

$I_x = I_y$ by symmetry

$$\therefore \quad I_x = \tfrac{1}{4} M r^2$$

"Flip axis"

Now $E = \tfrac{1}{2} I_x (2\pi F)^2$

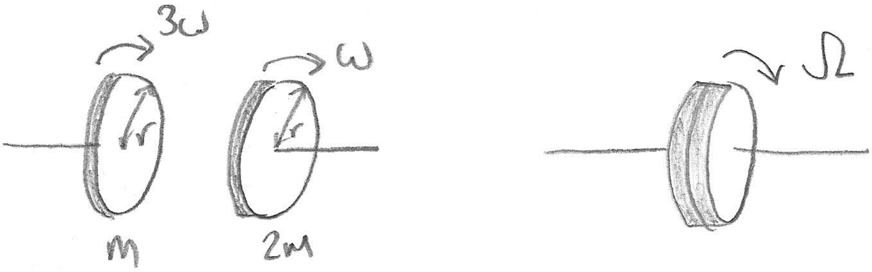
If coin and sphere have same KE rotationally.

$$\therefore \quad \tfrac{4}{5}\cancel{\pi^2}\cancel{M r^2} f^2 = \tfrac{1}{2}\,\tfrac{1}{4}\cancel{M r^2}\,\cancel{4\pi^2} F^2$$

$$\tfrac{8}{5} f^2 = F^2 \Rightarrow \boxed{F = \sqrt{\tfrac{8}{5}}\, f}$$

So if $f = 5$ Hz, Gin flip is $\sqrt{\frac{8}{5}} \times 5 = \boxed{6.3 \text{ Hz}}$

(x)



BEFORE

After

[Cylinder I is "$\frac{1}{2}mr^2$"]

Conservation of angular momentum:

$$\left(\tfrac{1}{2}mr^2\right)(3\omega) + \left(\tfrac{1}{2}2mr^2\right)\omega = \tfrac{1}{2}(3m)r^2\,\Omega$$

$$\omega\left(\tfrac{3}{2}+1\right) = \Omega\left(\tfrac{3}{2}\right)$$

$$\therefore \quad \Omega = \omega\left(1+\tfrac{2}{3}\right)$$

$$\therefore \quad \boxed{\Omega = \tfrac{5}{3}\omega}$$

Energy dissipated: $\Delta E = \frac{1}{2}\left(\frac{1}{2}mr^2\right)(3\omega)^2 + \frac{1}{2}\left(\frac{1}{2}2mr^2\right)\omega^2 - \frac{1}{2}\left(\frac{1}{2}(3m)r^2\right)\left(\frac{5}{3}\omega\right)^2$

$$\Delta E = \tfrac{1}{2}mr^2\omega^2\left(\tfrac{1}{2}\times 9 + 1 - \tfrac{3}{2}\times\left(\tfrac{5}{3}\right)^2\right)$$

$$= \tfrac{1}{2}mr^2\omega^2\left(\tfrac{4}{3}\right) = \boxed{\tfrac{2}{3}mr^2\omega^2}$$

Now initial KE is $\frac{1}{2}mr^2\omega^2\left(\frac{1}{2}\times 9 + 1\right) = \frac{11}{4}mr^2\omega^2$

So ΔE is a fraction of $\dfrac{2/3}{11/4} = 8/33 \simeq \boxed{24.2\%}$