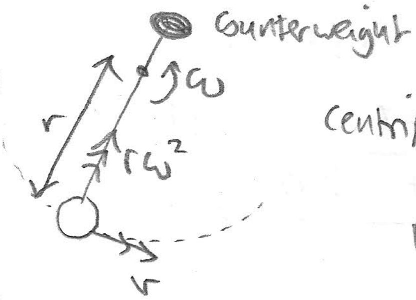
NON-UNIFORM CIRCULAR MOTION

1/ (i)



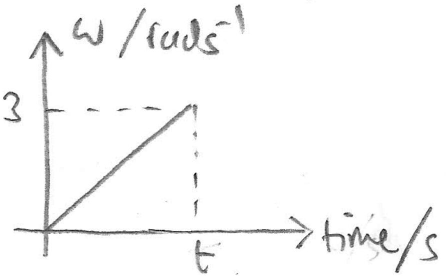
centrifuge at NASA Ames

$r\omega^2 = 12.5g$

a) $\omega = \sqrt{12.5g/r} = \frac{60}{2\pi}\sqrt{\frac{12.5 \times 9.81}{8.8}}$ RPM

$= \boxed{35.6 \text{ RPM}}$

b)



Tangential acceleration is $r\dot{\omega} = 0.5g$

$\dot{\omega} = 3/t$ and $\dot{\omega} = \frac{0.5g}{r}$

$\therefore \; 3/t = \frac{0.5g}{8.8}$

$\therefore \; \frac{3 \times 8.8}{0.5 \times 9.81} = t$

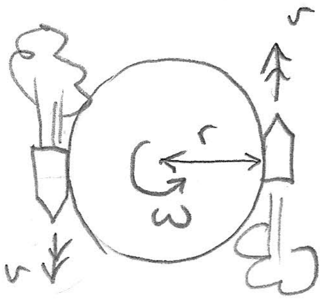
$\therefore \; \boxed{5.38\text{s} = t}$

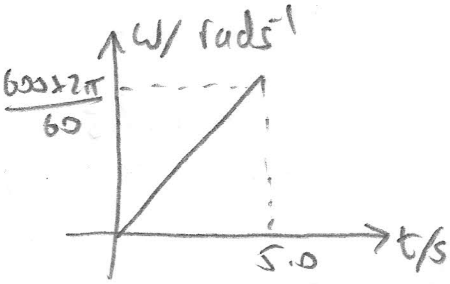
c) $\theta = \int \omega dt$ ie area undr $\omega(t)$ graph.

so $\theta = \frac{1}{2}(3)(t) = \frac{1}{2}(3)(5.38) = 8.07$ radians

$= \boxed{462.6^\circ}$ (1.28 rotations)

(ii)





a) $\dot{\omega} = \frac{600 \times 2\pi}{60 \times 5}$ rads^{-2}

$= 4\pi$ rads^{-2}

$\simeq \boxed{12.6 \text{ rads}^{-2}}$

$r = 0.33$m

b) At 5.0s, $v = r\omega = 0.33 \times \frac{600 \times 2\pi}{60}$

$= \boxed{20.7 \text{ m/s}}$

c) Total angle of turn is $\theta = \int_0^7 \omega \, dt$

Since $\dot{\omega}$ = constant, $\quad \theta = \frac{1}{2}(4\pi) \times (7)^2$

$\Rightarrow \boxed{\theta = \frac{1}{2}\dot{\omega} t^2}$ $\quad = 307.9$ radians or 17640°.

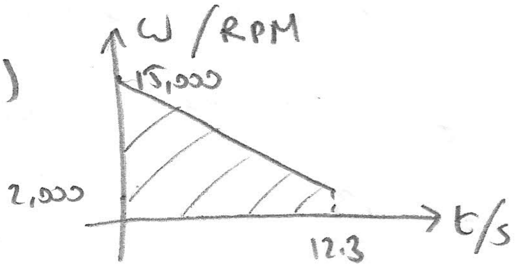
$= \boxed{49 \text{ rotations}}$

{ In our case, since $\dot{\omega} = 4\pi \text{ rads}^{-2}$

$\theta = \frac{1}{2}(4\pi)t^2$

$\frac{\theta}{2\pi} = t^2$ $\quad$ So $\boxed{\# \text{ rotations} = t^2}$ }

iii)



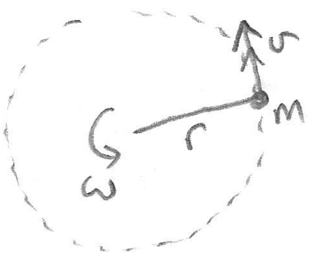
Total angle of turn is area under ω, t graph

$= \frac{1}{2}(12.3)(15{,}000 + 2{,}000) \times \frac{2\pi}{60}$ (rad)

$= 1.09 \times 10^4$ rad

$= \boxed{627{,}300^\circ}$

(iv)

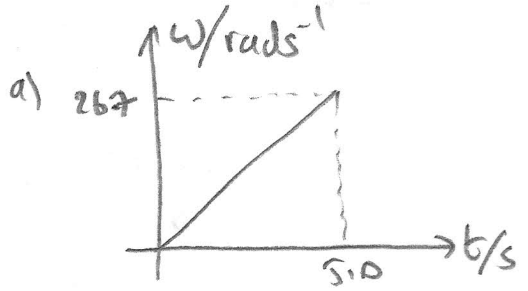


radius of bolus $r = 0.75$ M, $M = 0.20$ kg

$v = r\omega \quad \therefore \quad \omega = v/r$

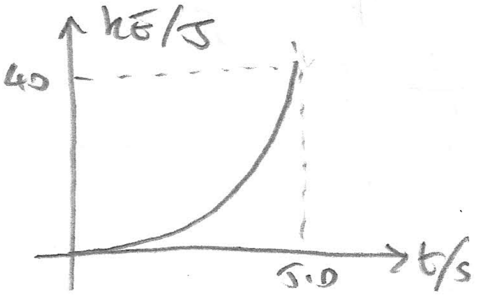
when $v = 20$ m/s, $\quad \omega = \frac{20}{0.75} = \boxed{26.7 \text{ rads}^{-1}}$

a)



b) $KE = \frac{1}{2}Mv^2$

$KE = \frac{1}{2}Mr^2\omega^2$



At 5.0s, $KE = \frac{1}{2}(0.2)(0.75)^2(26.7)^2$

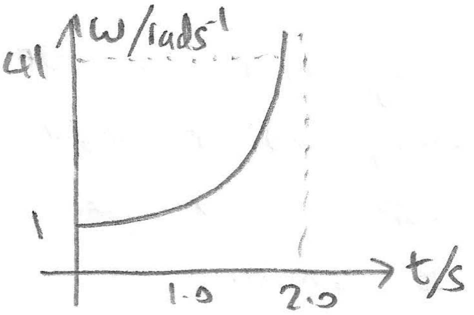
$= \boxed{40 \text{ J}}$

c) Angle of turn $\theta = \frac{1}{2}\left(\frac{20}{0.75}\right)(5)$

ie area under (t, ω) graph $=$ 66.7 radians

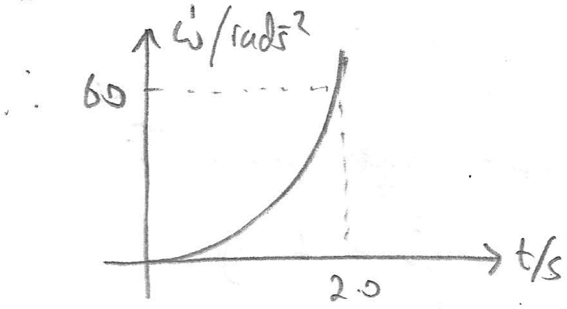
$=$ 10.6 rotations. So $\boxed{\text{10 complete rotations}}$

(v) a) $\boxed{\omega = 1 + 5t^3}$ for ice-skater.



when $t = 2.0\,s$, $\omega =$ rads^{-1}

$\boxed{\dot{\omega} = 15t^2}$ (ie $d\omega/dt$)

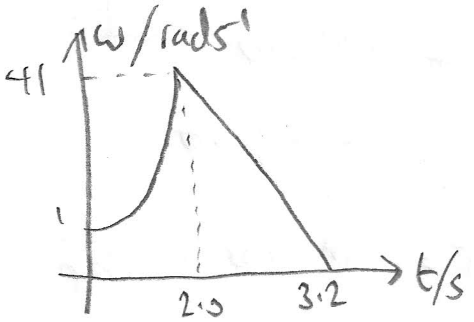


b) $\theta = \int_0^2 \omega\, dt = \left[t + \frac{5}{4}t^4\right]_0^2 = 2 + \left(\frac{5}{4}\right)(16) = \boxed{22\ \text{rad}}$

ie 3.50 rotations. (She lands backwards!)

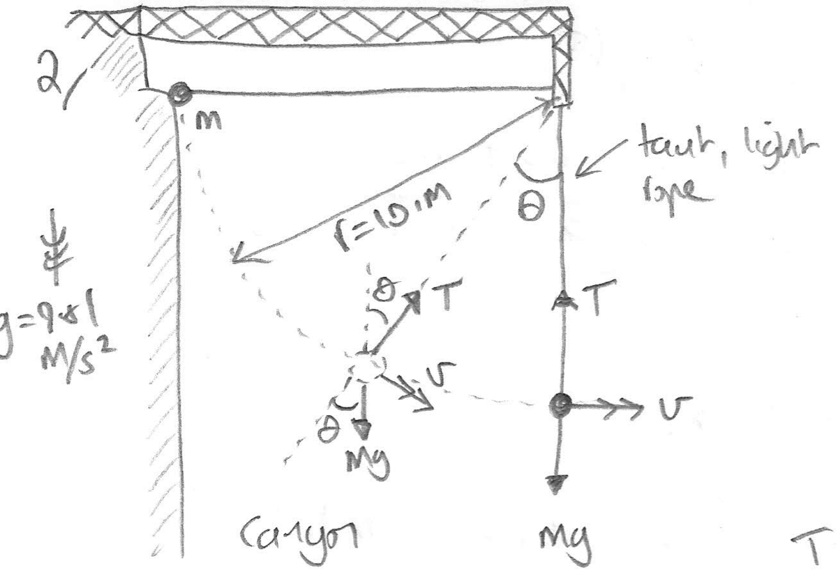
c) After 2.0s she brakes at constant angular acceleration $\dot{\omega}$

At 2.0s, $\omega = 41$ rads^{-1}



$\dot{\omega} = -\frac{41}{1.2} = \boxed{-34.2\ \text{rads}^{-2}}$

2/



Newton II radially inward

$$\frac{mv^2}{r} = T - mg\cos\theta$$

$$\therefore \boxed{T = mg\cos\theta + \frac{mv^2}{r}}$$

T is clearly largest when $\theta = 0$ (i.e. vertical) and v largest (when most GPE $\rightarrow \frac{1}{2}mv^2$) ↳ Also vertical.

So max rope tension T is when $\theta = 0$; $\frac{1}{2}mv^2 = mgr$

$$\Rightarrow \frac{mv^2}{r} = 2mg \quad \therefore \text{ max T is } \boxed{3mg}$$

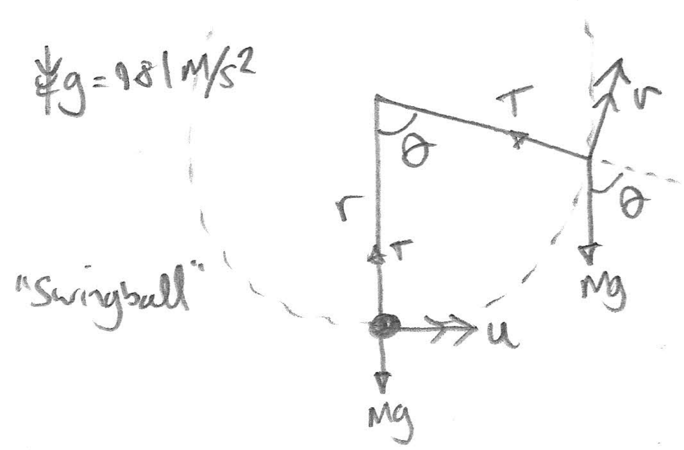
If $m = 85\,kg$, $r = 10.0\,m$;

Doesn't factor in max T

$$3mg = 3 \times 85 \times 9.81 = \boxed{2501.6\ N}$$

So if max tension is 2500 N, it might be unwise for Sophie's dad to try the swing!

3/



Newton II radially inward:

$$\frac{mv^2}{r} = T - mg\cos\theta$$

$$\therefore T = mg\cos\theta + \frac{mv^2}{r}$$

Conservation of energy:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$$

$$\therefore \frac{mv^2}{r} = \frac{mu^2}{r} - 2mg(1-\cos\theta)$$

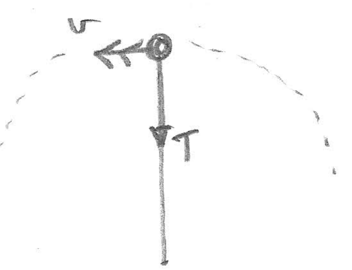
$\therefore \quad T = mg\cos\theta + \frac{mu^2}{r} - 2mg(1-\cos\theta)$

$\therefore \quad T = 3mg\cos\theta - 2mg + \frac{mu^2}{r}$

$\therefore \quad \boxed{T = mg(3\cos\theta - 2) + \frac{mu^2}{r}}$

Now ball spins in a vertical circle if $T > 0 \;\; \forall \theta$
in range $0 \le \theta \le 2\pi$

Smallest T is when $\cos\theta = -1$ ie $\theta = \pi$ (180°)



ie $mg(-3-2) + \frac{mu^2}{r} > 0$ for a full circle with taut rope.

$\therefore \quad u^2/r > 5g$

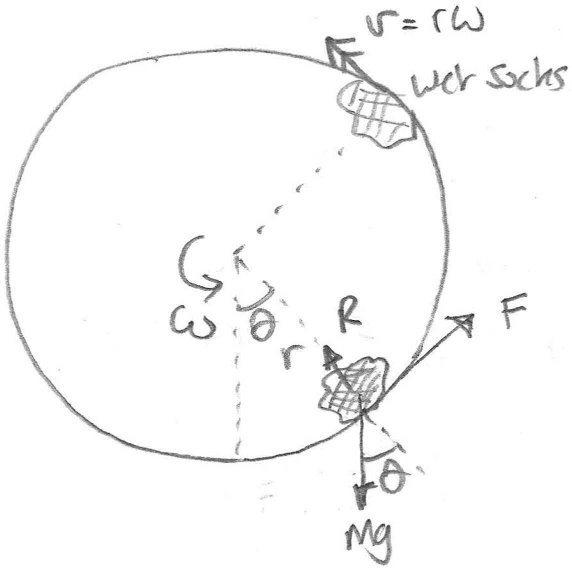
$\therefore \quad u^2 > 5gr$

$\therefore \quad \boxed{u > \sqrt{5gr}}$

So Boris' impulse mu must exceed:

$\frac{58.0}{1000} \times \sqrt{5 \times 9.81 \times 1.40} = \boxed{0.48 \text{ kg m/s}}$

4/



Assume ω = constant

Newton II radially inwards

$Mr\omega^2 = R - Mg\cos\theta$

$\therefore \quad \boxed{R = Mr\omega^2 + Mg\cos\theta}$

Newton II tangentially, assuming no slip of wet socks

$0 = F - Mg\sin\theta$

$\therefore \quad \boxed{F = Mg\sin\theta}$

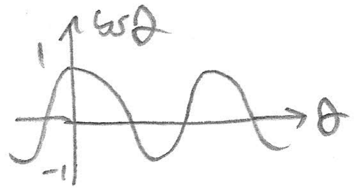
Also $\boxed{F \le \mu R}$

(i) To not lose contact with the drum, $R > 0$

$\therefore$ since $R = mr\omega^2 + mg\cos\theta$

$$\Rightarrow \quad r\omega^2 + g\cos\theta > 0$$

Now $\quad -1 < \cos\theta < 1$



So if $r\omega^2 + g\cos\theta > 0$ when $\cos\theta = -1$ (ie $\theta = 180°$) then it is true $\forall\theta$.

$\therefore \quad r\omega^2 - g > 0 \quad$ for socks to not lose contact

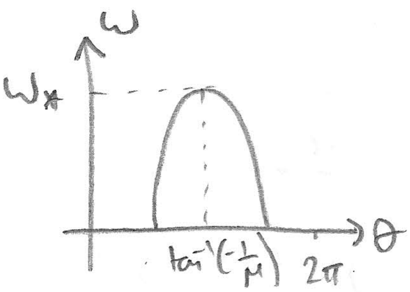
$$\Rightarrow \quad \boxed{\omega > \sqrt{\frac{g}{r}}}$$

(ii) For no slip $\quad F \leq \mu R$

$$\therefore \quad mg\sin\theta \leq \mu(mr\omega^2 + mg\cos\theta)$$

$$\therefore \quad \frac{g\sin\theta}{\mu r} \leq \omega^2 + \frac{g\cos\theta}{r}$$

$$\Rightarrow \quad \boxed{\omega \geqslant \sqrt{\frac{g}{\mu r}(\sin\theta - \mu\cos\theta)}}$$



For never any slip, $\omega > \omega_*$

$\omega = \omega_*$ when $\frac{d\omega}{d\theta} = 0$

$$\frac{d\omega}{d\theta} = \sqrt{\frac{g}{\mu r}}\frac{1}{2}(\sin\theta - \mu\cos\theta)^{-\frac{1}{2}}(\cos\theta + \mu\sin\theta)$$

So $\frac{d\omega}{d\theta} = 0$ when $\cos\theta = -\mu\sin\theta \Rightarrow \boxed{\tan\theta = -\frac{1}{\mu}}$

So $\omega_* = \sqrt{\frac{g}{\mu r}}\left(\sin + \mu^2\sin\theta\right)^{\frac{1}{2}}$.

Now $\cos^2\theta = \mu^2\sin^2\theta$

$1 - \sin^2\theta = \mu^2\sin^2\theta$

$1 = (1+\mu^2)\sin^2\theta$

So $\sin^2\theta = \dfrac{1}{1+\mu^2}$ $\therefore$ $\sqrt{\sin\theta} = (1+\mu^2)^{-\frac{1}{4}}$

$\therefore$ $\omega_* = \sqrt{\dfrac{g}{\mu r}}\,(1+\mu^2)^{\frac{1}{2}}\left(\dfrac{1}{1+\mu^2}\right)^{\frac{1}{4}}$

$\therefore$ for no slip $\boxed{\omega > \sqrt{\dfrac{g}{\mu r}}\,(1+\mu^2)^{\frac{1}{4}}}$

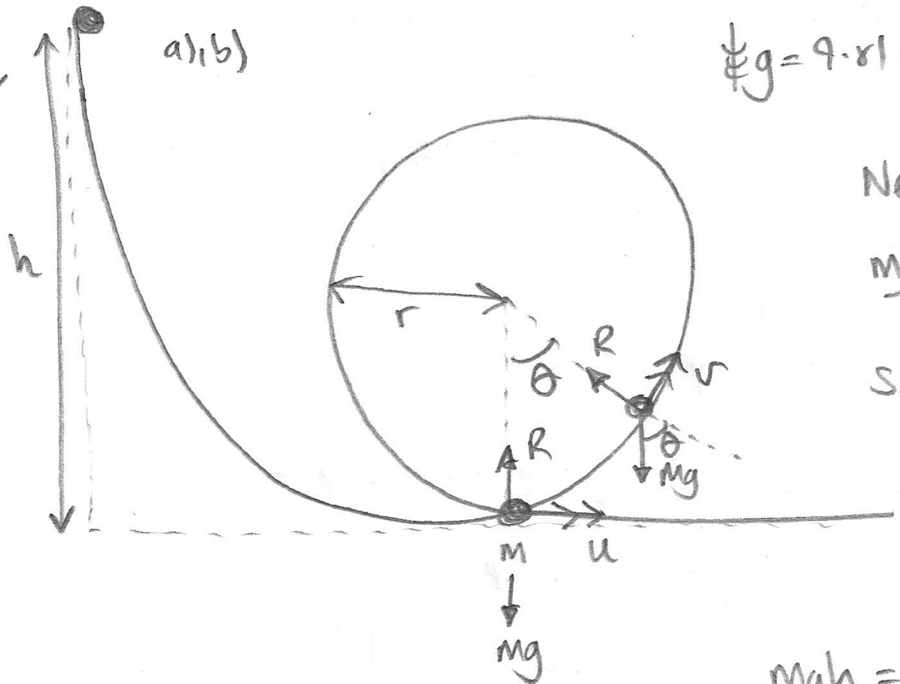
(iii) let $r = 0.3\text{m}$, $\mu = \frac{1}{\sqrt{3}}$

$\therefore$ $\sqrt{\dfrac{g}{r}} = \sqrt{\dfrac{9.81}{0.3}} = 5.72 \text{ rads}^{-1} = \boxed{54.6 \text{ RPM}}$

$\therefore$ $\sqrt{\dfrac{g}{\mu r}(1+\mu^2)^{\frac{1}{4}}} = \sqrt{\dfrac{9.81}{0.3}\sqrt{3}}\,\left(1+\frac{1}{3}\right)^{\frac{1}{4}}$

$= 8.09 \text{ rads}^{-1} = \boxed{77.2 \text{ RPM}}$

5) a)i b)



$g = 9.81 \text{ N/kg}$

Newton II radially inward:

$\dfrac{Mv^2}{r} = R - Mg\cos\theta$

So $\boxed{R = \dfrac{Mv^2}{r} + Mg\cos\theta}$

Now by conservation of energy:

$Mgh = \frac{1}{2}Mu^2 = \frac{1}{2}Mv^2 + Mgr(1-\cos\theta)$

$\therefore$ $\dfrac{Mv^2}{r} = \dfrac{2Mgh}{r} - 2Mg(1-\cos\theta)$ $\Rightarrow$ Also $2gh = v^2 + 2gr(1-\cos\theta)$

$\therefore$ $\boxed{R = \dfrac{2Mgh}{r} - 2Mg + 3Mg\cos\theta}$ and $\boxed{v = \sqrt{2gh}\left(1 - \frac{r}{h}(1-\cos\theta)\right)^{\frac{1}{2}}}$

c) Car is in contact with the loop if $R > 0$

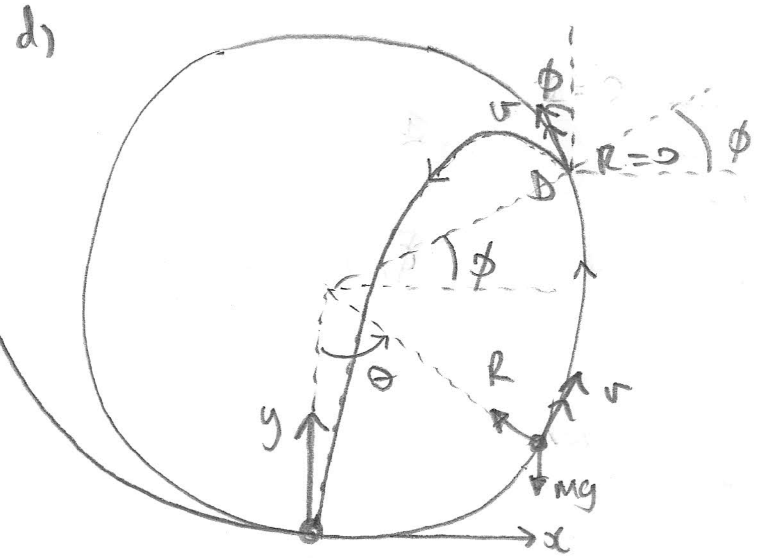
$$R/mg = 3\cos\theta - 2 + 2h/r$$

$$\Rightarrow \quad 3\cos\theta - 2 + 2h/r > 0 \quad \forall\, \theta$$

$1 < \cos\theta < 1$ so $-3 - 2 + 2h/r > 0$ for this to be true

$$\Rightarrow \quad \boxed{h > \tfrac{5}{2} r}$$

d)



$R = 0$ when $3\cos\theta - 2 + 2h/r = 0$

$$\Rightarrow \quad \boxed{\cos\theta = \tfrac{2}{3}(1 - h/r)}$$

let $k = h/r \Rightarrow \cos\theta = \tfrac{2}{3}(1-k)$

(we are trying to prove $(0,0)$ is on the projectile phase trajectory when $k = 7/4$)

Projectile phase:
($t=0$ at point of departure D)

$$\boxed{\begin{aligned} x &= r\cos\phi - v\sin\phi\, t \\ y &= r + r\sin\phi + v\cos\phi\, t - \tfrac{1}{2} g t^2 \end{aligned}}$$

Now $\theta = \pi/2 + \phi$ So $\cos\theta = \cos\frac{\pi}{2}\cos\phi - \sin\frac{\pi}{2}\sin\phi = -\sin\phi$

$$\therefore \boxed{\sin\phi = \tfrac{2}{3}(k-1)}$$

$$\cos^2\phi = 1 - \sin^2\phi$$

$$\therefore \boxed{\cos^2\phi = 1 - \tfrac{4}{9}(k-1)^2}$$

Now at $(0,0)$ $r\cos\phi = v\sin\phi\, t \Rightarrow t = r\cos\phi / v\sin\phi$

$\therefore$ using $y=0$:

$$0 = r(1+\sin\phi) + \frac{v\cos\phi\, r\cos\phi}{v\sin\phi} - \frac{1}{2} g \frac{r^2\cos^2\phi}{v^2\sin^2\phi}$$

$$\therefore\ 0 = r\left(1+\tfrac{2}{3}(k-1)\right) + r\,\frac{\left(1-\frac{4}{9}(k-1)^2\right)}{\frac{2}{3}(k-1)} - \frac{1}{2}\frac{gr^2}{v^2}\,\frac{\left(1-\frac{4}{9}(k-1)^2\right)}{\frac{4}{9}(k-1)^2}$$

Now $mgh = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$ (conservation of energy)

$$\Rightarrow \quad v^2 = 2gh - 2gr(1-\cos\theta)$$

So if $\cos\theta = \frac{2}{3}(1-h/r)$

$$\Rightarrow @\,D: \quad v^2 = 2gh - 2gr\left(1-\frac{2}{3}+\frac{2h}{3r}\right)$$

$$v^2 = 2gh - 2gr + 4gr/3 - 4gh/3$$

$$v^2 = \tfrac{2}{3}gh - 2gr/3$$

$$\boxed{v^2 = \tfrac{2}{3}g(h-r)}$$

So $$gr/v^2 = \frac{gr}{\frac{2}{3}gr\left(\frac{h}{r}-1\right)} = \frac{3}{2(k-1)}$$

$$\therefore\ 0 = 1+\tfrac{2}{3}(k-1) + \frac{1-\frac{4}{9}(k-1)^2}{\frac{2}{3}(k-1)} - \frac{3}{4}\,\frac{1}{k-1}\,\frac{\left(1-\frac{4}{9}(k-1)^2\right)}{\frac{4}{9}(k-1)^2}$$

let $z = \frac{2}{3}(k-1)$

$$\therefore\ 0 = 1+z + \frac{1-z^2}{z} - \frac{1}{2}\,\frac{1}{z}\,\frac{1-z^2}{z^2}$$

$$\therefore\ 0 = 2z^3 + 2z^4 + 2z^2(1-z^2) - 1 + z^2 \qquad (\times 2z^3)$$

$$\therefore\ 0 = 2z^3 + \cancel{2z^4} + 2z^2 - \cancel{2z^4} - 1 + z^2$$

$$\therefore\ 0 = 2z^3 + 3z^2 - 1 \qquad z=-1 \text{ is a factor}$$

so $$2z^3+3z^2-1 = (z+1)(2z^2+az+b)$$
$$= 2z^3 + (2+a)z^2 + z(a+b) + b$$

$a=1, b=-1$ $$\therefore\ 0 = (z+1)(2z^2+z-1) = (z+1)(2z-1)(z+1)$$

$$\therefore\ 0 = (2z-1)(z+1)^2$$

$\therefore$ if $(z+1)^2(2z-1)=0 \Rightarrow z=-1$ or $z=\frac{1}{2}$

$z=\frac{2}{3}(k-1)$ $\therefore \boxed{k=\frac{3z}{2}+1}$.

if $z=-1$, $k=-\frac{1}{2}$
which has no physical significance
Since $k=h/r$

So for our problem: $\boxed{z=\frac{1}{2}} \Rightarrow k=\frac{3}{4}+1 = \boxed{\frac{7}{4}}$ as required

Now $\sin\phi = \underbrace{\frac{2}{3}(k-1)}_{z}$ so when $z=\frac{1}{2}$, $\boxed{\phi = 30^\circ \text{ or } \pi/6}$

e) if projectile phase (from D to (0,0))

$$x = r\cos\phi - v\sin\phi\, t$$

So $x=0$ when $\boxed{t=\frac{r\cos\phi}{v\sin\phi}}$

$\phi=\pi/6$ So $t = \frac{r}{v}\frac{\cos 30^\circ}{\sin 30^\circ} = \frac{r}{v}\frac{\sqrt{3}/2}{1/2} = \frac{r}{v}\sqrt{3}$

Now $v^2 = \frac{2}{3}g(h-r)$ so if $h=\frac{7}{4}r$

$$\Rightarrow \quad v^2 = \frac{2}{3}gr\left(\frac{7}{4}-1\right) = \frac{1}{2}gr$$

$$\therefore \quad \boxed{v = \sqrt{\frac{gr}{2}}}$$

$$\therefore \quad t = \frac{r\sqrt{3}}{\sqrt{gr}/\sqrt{2}} = \boxed{\sqrt{\frac{r}{g}}\sqrt{6}}$$

Now in phase $(0,0) \to D$ in contact with track:

$$v = \sqrt{2gh}\left(1-\frac{r}{h}(1-\cos\theta)\right)^{\frac{1}{2}}$$

Now $v = r\dot{\theta}$

So $\dot{\theta} = \sqrt{\frac{2gh/r}{r}}\left(1 - \frac{r}{h}(1-\cos\theta)\right)^{\frac{1}{2}}$

$h/r = 7/4$ So $\dot{\theta} = \sqrt{\frac{2g}{r}\frac{7}{4}}\left(1 - \frac{4}{7}(1-\cos\theta)\right)^{\frac{1}{2}}$

$$\therefore \int_0^{\frac{\pi}{2}+\pi/6} \frac{d\theta}{1 - \frac{4}{7} + \frac{4}{7}\cos\theta} = \int_0^t dt \times \sqrt{\frac{g}{r}}\sqrt{\frac{7}{2}}$$

$$\int_0^{2\pi/3} \frac{7\,d\theta}{3 + 4\cos\theta} = t\sqrt{\frac{g}{r}}\sqrt{\frac{7}{2}}$$

Now $\int \frac{7\,d\theta}{3+4\cos\theta} = 2\sqrt{7}\tanh^{-1}\left(\frac{1}{\sqrt{7}}\tan\left(\frac{\theta}{2}\right)\right)$ (Wolfram Alpha!)

$\tan\left(\frac{2\pi}{3}\frac{1}{2}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

So $\int_0^{2\pi/3} \frac{7\,d\theta}{3+4\cos\theta} = 2\sqrt{7}\tanh^{-1}\left(\frac{\sqrt{3}}{\sqrt{7}}\right)$

Now $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ so $\tanh^{-1}\left(\frac{\sqrt{3}}{\sqrt{7}}\right)$

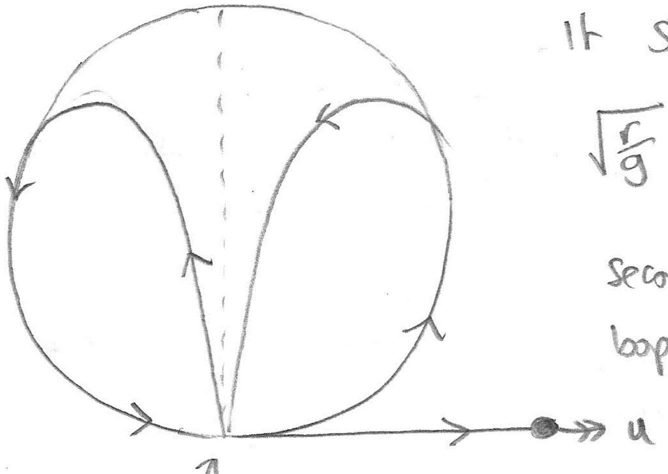
$$= \frac{1}{2}\ln\left(\frac{1+\sqrt{3}/\sqrt{7}}{1-\sqrt{3}/\sqrt{7}}\right) = \frac{1}{2}\ln\left(\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\right)$$

$$\therefore t\sqrt{\frac{g}{r}}\frac{\sqrt{7}}{\sqrt{2}} = \sqrt{7}\ln\left(\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\right)$$

$$\therefore \boxed{t = \sqrt{\frac{r}{g}}\sqrt{2}\ln\left(\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\right)}$$

$$t \simeq \sqrt{\frac{r}{g}} \times 2.216$$

Now by symmetry, if bounce (elastic) occurs at (0,0)



Assume special trapdoors / shutters in track here to allow entry, exit and elastic bounce!

It should take a further

$$\sqrt{\frac{r}{g}}\left(\sqrt{6}+\sqrt{2}\ln\left(\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\right)\right)$$

seconds before exiting the loop the loop at speed

$$\boxed{u=\sqrt{2gh}}$$

(ie the speed it entered)

$$h=\frac{7}{4}r \quad \text{so} \quad u=\sqrt{2g\tfrac{7}{4}r}$$

$$\boxed{u=\sqrt{\frac{7}{2}gr}}$$

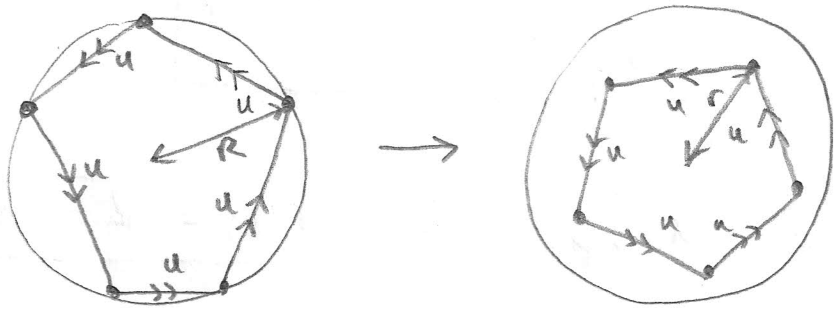
So total time inside circular track is

$$\boxed{t=\sqrt{\frac{r}{g}}\left(2\sqrt{6}+2\sqrt{2}\ln\left(\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\right)\right)}$$

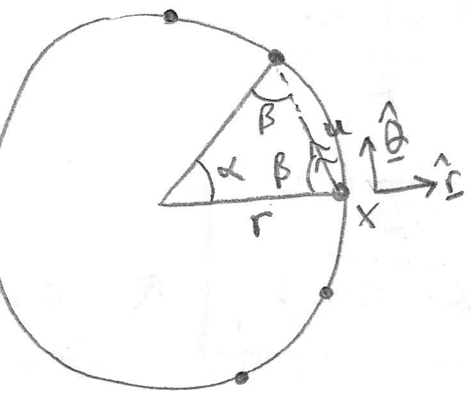
$$\approx 9.331\sqrt{\frac{r}{g}}$$

6/ The snails will preserve the N-gon shape, but the N-gon will rotate and shrink over time.

eg



$\alpha = \frac{2\pi}{N}$ $\qquad \beta = \frac{\pi - \alpha}{2} \Rightarrow \boxed{\beta = \frac{\pi}{2} - \frac{\alpha}{2}}$



Consider a polar coordinate system for snail X. $(t=0, \theta=0, r=R)$.

velocity $\boxed{\underline{v} = \dot{r}\,\hat{\underline{r}} + r\dot{\theta}\,\hat{\underline{\theta}}}$

Now $\boxed{\dot{r} = -u\cos\beta}$ and $\boxed{r\dot{\theta} = u\sin\beta}$

Now $\frac{\dot{r}}{\dot{\theta}} = \frac{dr}{dt} \Big/ \frac{d\theta}{dt} = \frac{dr}{d\theta}$

$$\frac{\dot{r}}{\dot{\theta}} = \frac{-u\cos\beta}{u\sin\beta / r} = -r\cot\beta$$

$\therefore \quad \frac{dr}{d\theta} = -r\cot\beta \Rightarrow \int_R^r \frac{1}{r}\,dr = -\cot\beta \int_0^\theta d\theta$

$\Rightarrow \quad [\ln r]_R^r = -\cot\beta\,\theta$

$\Rightarrow \quad \ln\left(\frac{r}{R}\right) = -\theta\cot\beta$

$\Rightarrow \quad \boxed{r = R e^{-\theta\cot\beta}}$

For other snails, starting at $\theta_n = (n-1) \times \frac{2\pi}{N}$

$$\boxed{r_n(\theta) = R e^{-(\theta - \theta_n)\cot\beta}}$$

Now $\dot{r} = -u\cos\beta \quad \therefore \quad \boxed{r(t) = R - ut\cos\beta}$

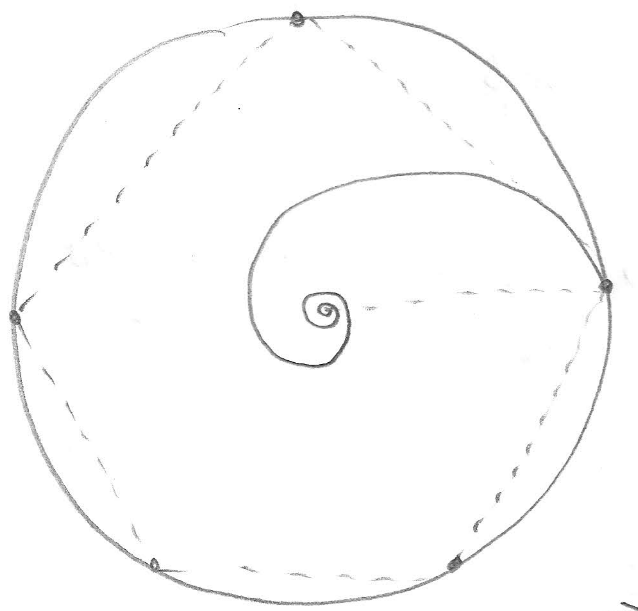
So since $\ln\left(\frac{r}{R}\right) = -\theta\cot\beta$

$$\Rightarrow \theta(t) = -\frac{1}{\cot\beta}\ln\left(\frac{R - ut\cos\beta}{R}\right)$$

$$\Rightarrow \boxed{\theta(t) = \tan\beta \ln\left(\frac{R}{R - ut\cos\beta}\right)}$$

add θ_n for snail n.

It takes the snails T seconds to reach $(0,0)$
where $0 = R - uT\cos\beta \Rightarrow \boxed{T = \frac{R}{u\cos\beta}}$



Extra: what is the acceleration of the snail?

$$\boxed{\underline{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\underline{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\underline{\theta}}}$$

$\dot{r} = -u\cos\beta$, $r\dot{\theta} = u\sin\beta$

so $\ddot{r} = 0 \quad \Rightarrow \dot{\theta}^2 = \frac{u^2\sin^2\beta}{r^2}$

$$\dot{\theta} = -\tan\beta \frac{R}{R - ut\cos\beta}(-u\cos\beta)/R$$

$$\Rightarrow \boxed{\dot{\theta} = \frac{u\sin\beta}{R - ut\cos\beta}}$$

i. $\underline{a} = -\frac{r u^2 \sin^2\beta}{r^2}\hat{\underline{r}} + \left(r\ddot{\theta} - 2u\cos\beta \, \frac{u\sin\beta}{R - ut\cos\beta}\right)\hat{\underline{\theta}}$

$$\ddot{\theta} = -\frac{u\sin\beta}{(R-ut\cos\beta)^2}(-u\cos\beta)$$

$$\ddot{\theta} = \frac{u^2\sin\beta\cos\beta}{(R-ut\cos\beta)^2} = \frac{u^2\sin\beta\cos\beta}{r^2}$$

So $\underline{a} = -\frac{u^2\sin^2\beta}{r}\hat{\underline{r}} + \left(\frac{u^2\sin\beta\cos\beta}{r} - \frac{2u^2\times\cos\beta\sin\beta}{r}\right)\hat{\underline{\theta}}$

$$\boxed{\underline{a} = -\frac{u^2\sin^2\beta}{r}\hat{\underline{r}} - \frac{u^2}{r}\sin\beta\cos\beta\,\hat{\underline{\theta}}}$$

where $r = R - ut\cos\beta$.

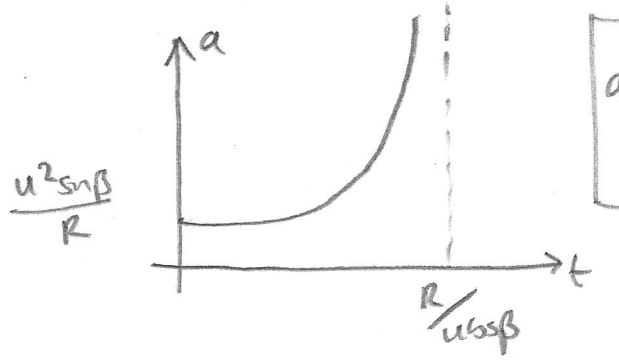
So $\boxed{\underline{a} = -\frac{u^2}{r}\sin\beta\left(\sin\beta\,\hat{\underline{r}} + \cos\beta\,\hat{\underline{\theta}}\right)}$

So the magnitude of acceleration $a = |\underline{a}|$

is: $\boxed{a = \frac{u^2}{r}\sin\beta}$ (sinβ will be +ve)

Since $\left|\sin\beta\,\hat{\underline{r}} + \cos\beta\,\hat{\underline{\theta}}\right|$

$$= \sqrt{\sin^2\beta + \cos^2\beta} = 1.$$



$$\boxed{a(t) = \frac{u^2\sin\beta}{R - ut\cos\beta}}$$

$$\boxed{\beta = \frac{\pi}{2} - \frac{\pi}{N}}$$