NON UNIFORM CIRCULAR MOTION AF December 2019

Angular speed $\dot{\theta} = \omega = \dfrac{d\theta}{dt}$ Angular acceleration $\ddot{\theta} = \dot{\omega} = \dfrac{d^2\theta}{dt^2}$

Polar coordinates $\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{y}}\sin\theta$, $\hat{\boldsymbol{\theta}} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{y}}\cos\theta$, $\dfrac{d\hat{\mathbf{r}}}{dt} = \dot{\theta}\hat{\boldsymbol{\theta}}$, $\dfrac{d\hat{\boldsymbol{\theta}}}{dt} = -\dot{\theta}\hat{\mathbf{r}}$

Displacement, velocity and acceleration in polar coordinates $\mathbf{r} = r\hat{\mathbf{r}}$, $\mathbf{v} = r\dot{\theta}\hat{\boldsymbol{\theta}} + \dot{r}\hat{\mathbf{r}}$, $\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{\theta}}$

If motion is constrained to a circle i.e. $r = \text{constant}$: $\mathbf{v} = r\dot{\theta}\hat{\boldsymbol{\theta}}$ (i.e. velocity is purely tangential to the circle)
Acceleration radically inward: $a = r\omega^2$ Tangential acceleration: $a = r\dot{\omega}$

Question1

(i) The Ames research centre 20-G centrifuge is used by NASA to test the physiological response of pilots and astronauts to high levels of acceleration. A person is whirled round in a horizontal circle of radius $r = 8.8\text{m}$.

(a) Calculate ω in RPM if the radially inward acceleration is at the human limit of '12.5g'. $g = 9.81\text{ms}^{-2}$

(b) ω spins up from zero to 3 rads^{-1} in t seconds. If the tangential acceleration during this time is '0.5g', determine how long this takes.

(c) Calculate the total angle the centrifuge has turned (in degrees) in t seconds.

(ii) A Catherine wheel consists of a pair of fireworks mounted diametrically on a light wheel of radius 0.33m, pointing in opposing directions. The Catherine wheel spins up from rest to 600RPM in 5s.

(a) Calculate the angular acceleration $\dot{\omega}$ in rads^{-2} .

(b) Calculate the velocity of the fireworks (in m/s) after 5s.

(c) Calculate the total angle of turn of the wheel in 7s, assuming angular acceleration remains constant.

(iii) A gyroscope spins down from 15,000RPM to 2,000RPM with constant angular acceleration in 12.3s. Calculate the total angle of turn (in radians, and in degrees) during this time.

(iv) A *bolas* is a traditional throwing weapon used by *gauchos* (South American Cowboys) and earlier indigenous cultures. It consists of a weights on the end of a cord. A hunter on horseback throws the bolas at prey, and it wraps around their legs, restricting movement. A bolas of length 0.75m is spun up until the 0.2kg weight reaches a speed of 20m/s. The gaucho achieves this, accelerating the weight uniformly in a tangential sense, in 5.0s.

(a) Sketch a graph of angular speed of the bolas vs time, assuming it starts from rest.

(b) Sketch a graph of kinetic energy of the bolas vs time.

(c) How many complete rotations have been achieved in 5.0s?

(v) An ice-skater draws in her arms in order to reduce her *moment of inertia*. If her *angular momentum* is constant, this means she will *increase* her angular speed. The angular speed ω (in rad/s) of a particular skater varies with to time t via the equation $\omega = 1 + 5t^3$.

(a) Sketch a graph of angular acceleration vs time over the range $0 \le t \le 2.0\text{s}$

(b) Calculate how many spins she achieved in 2.0s.

(c) She applies a brake after 2.0s to spin down to 0 rad/s in 1.2s. Determine the average angular acceleration.

Question 2 A canyon-swing consists of a rope of length 10.0m connected to a rigid pier that hangs over the canyon. Jumpers step off the lip of the canyon with the rope initially held taut horizontally. Sophie's dad (mass 85kg) wishes to have a go, but Sophie spots the rating of the rope, which states 'maximum tension 2500N. Should her dad try out the swing?

Question 3 Boris is playing *Swingball* with his sister. After losing six games straight, Boris whacks the ball with speed u such that it spins in a vertical circle. *Swingball* consists of a tennis ball (mass $m = 58.0\text{g}$) connected via a light plastic tether to a pole that it is allowed to freely rotate about. The tether has a length of $r =$ 1.40m. Using Newton II in the radially inward direction of the vertical circular motion, and conservation of energy, show that the tension T in the tether vs angle of sweep θ of the tether from the vertical is given by: $T = mg\left(3\cos\theta - 2\right) + \frac{mu^2}{r}$. Assume Boris whacks the ball when $\theta = 0^\circ$. Hence calculate the minimum impulse that Boris must apply to the ball in order for it to spin in a vertical circle.

Question 4 Sybil the cat is mesmerized by a washing machine. She notices that a big ball of damp socks will sometimes make it all round the interior of the drum (or radius r) without slipping. The angular speed of the washing machine drum is ω, and is fixed for a particular spin-cycle.

(i) Show that in order for the damp socks to not lose contact with the drum, $\omega > \sqrt{g/r}$.

(ii) Show that in order for the damp socks to never slip along the drum surface, $\omega > \sqrt{\frac{g}{\mu r}}\left(1+\mu^2\right)^{\frac{1}{4}}$ where μ is the coefficient of friction between the socks and the washing machine drum.

(iii) Find the corresponding angular speeds (in RPM) to (i) and (ii) if $r = 0.3\text{m},\ \mu = \frac{1}{\sqrt{3}}$.

Question 5 Joe receives a model building kit for her birthday consisting of a car of mass m and a set of curved road pieces. She decides to build a 'loop-the-loop', consisting of a curved ramp of height h followed by a vertical circular loop section of radius r .

If the rotational kinetic energy of the wheels of the car can be ignored compared to that of bulk motion, the car is released from rest at the top of the ramp, and the car doesn't slip, show that :

(a) The speed of the car v at angle θ from the vertical is given by: $v = \sqrt{2gh}\sqrt{1 - \frac{r}{h}\left(1-\cos\theta\right)}$

(b) The normal contact force R on the car divided by its weight is: $\dfrac{R}{mg} = 3\cos\theta - 2 + \frac{2h}{r}$

(c) $h > \frac{5}{2}r$ in order for the car to complete a loop-the-loop without losing contact with the track.

(d) If $h = \frac{7}{4}r$, the car will leave the track at angle $\theta = \frac{\pi}{2} + \phi$ radians, and land at the base of the loop-the-loop. Show that $\phi = \frac{\pi}{6}$. Ignore air resistance. The base of the loop is where the car enters and exists*[1]

Question 6 N snails are distributed equally around a circle of radius R . They instinctively chase each other, but can only move with fixed speed u . Using a polar coordinate system, determine $\theta(t)$ and $r(t)$ for the snail with initial conditions $\theta = 0, r = R$, and plot the trajectory of the snail when $N = 5$. What is the acceleration?

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[1] Assume some form of shutter system allows the car to enter and exit the loop without having to change the plane of the track!