Chemical processes, which involve the exchange and interactions between electrons in atomic and molecular structures, have energies similar to that of the ionization energy of Hydrogen, which is 13.6eV. **Nuclear** processes, which involve *changes to the atomic nucleus*, are about a *million* times more energetic. *Radioactivity* (alpha, beta, gamma) decay, *fission* and *fusion* have energies of the order of MeV. $1\text{eV} = 1.602 \times 10^{-19}\,\text{J}$.

Nuclear fission: Fragmentation of large nuclei such as uranium, resulting from neutron bombardment.

e.g. ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3 \times {}^{1}_{0}\text{n}$. Energy released is: $\Delta E = 173.28\,\text{MeV}$. Uranium density is 19.1 g/cm^3.

This reaction produces two more neutrons, so if uncontrolled can result in a chain reaction, and potentially a nuclear explosion. This process is utilized in atomic weapons. If neutron flux is controlled using non-fissile neutron absorbers (e.g. graphite moderator rods), the fission process can result in a sustained source of heat i.e. the basis of a nuclear reactor.

Nuclear fusion: Fusion of light nuclei into heavier nuclei, releasing energy due to a gain in net binding energy. Process that powers stars such as sun, and hence the ultimate engine of all life on Earth.

e.g. ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$. Energy released is: $\Delta E = 17.59\,\text{MeV}$. Deuterium mass = 2.0141u, tritium mass = 3.016u.

Note via the stellar fusion mechanism *nucleosynthesis,* all heavier elements (i.e. beyond H, He etc) are formed. We are but stardust, fired in the crucible of Supernovae!

Binding energy B is the energy required to break an atomic nucleus into constituent protons p and neutrons n.

${}^{A}_{Z}\text{X} \xrightarrow{B} Z \times {}^{1}_{1}\text{p} + (A - Z) \times {}^{1}_{0}\text{n}$. $M_X c^2 + B = Z m_p c^2 + (A - Z) m_n c^2$. M_X is the mass of element X.

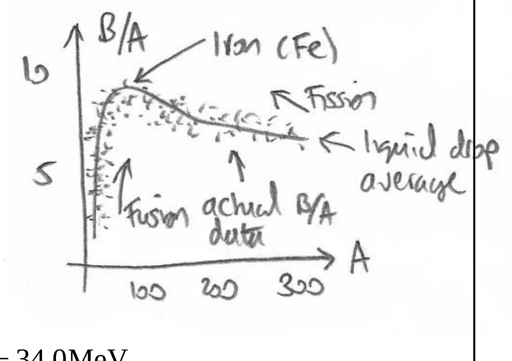
Speed of light $c = 2.998 \times 10^8\,\text{ms}^{-1}$; Proton mass $m_p = 1.6726 \times 10^{-27}\,\text{kg}$; Neutron mass $m_n = 1.6749 \times 10^{-27}\,\text{kg}$.

Atomic mass unit : $u = 1.6605 \times 10^{-27}\,\text{kg}$; $1\,\text{GeV}/c^2 = 1.7827 \times 10^{-27}\,\text{kg} = 1.0735u$. Atomic mass unit is $\frac{1}{12}$ of the mass of the nucleus of Carbon-12.

Electron mass: $m_e = 9.109 \times 10^{-31}\,\text{kg}$. $m_p = 0.9383\,\text{GeV}/c^2$; $m_n = 0.9396\,\text{GeV}/c^2$; $u = 0.9315\,\text{GeV}/c^2$

For a nuclear reaction: $\sum_i {}^{A_i}_{Z_i}X_i \rightarrow \sum_j {}^{A_j}_{Z_j}Y_j$, energy released is the difference in sums of binding energies:

$$\Delta E = \sum_j B_{Y_j} - \sum_i B_{X_i}$$



'Liquid Drop Model' or **'Semi-Empirical Mass Formula'** for binding energy:

$$B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A - 2Z)^2}{A} + \delta(A, Z)$$

$$\delta(A,Z) = \begin{cases} a_P A^{-3/4} & Z, A - Z \text{ even} \\ -a_P A^{-3/4} & Z, A - Z \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

$a_V = 15.76\,\text{MeV};\ a_S = 17.81\,\text{MeV};\ a_C = 0.711\,\text{MeV};\ a_A = 23.702\,\text{MeV};\ a_P = 34.0\,\text{MeV}$

Volume term, **S**urface term, **C**oulomb term, **A**symmetry term, **P**airing term. ** *TOP TIP:* Make yourself a SEMF calculator in a spreadsheet (with inputs A, Z) *before* your start answering the questions. Outputs are B, B/A in MeV. **

Question 1

(i) Calculate the mass of Krypton-92 in kg, and then in u, if the binding energy of Kr-92 is 764.77MeV.

(ii) If the binding energy of Ba-141 is 1145.36MeV, determine the binding energy of U-235, given the fission of U-235 to Kr-92 and Ba-141 releases 173.28MeV.

(iii) (a) Calculate the energy released per kg of U-235 in the fission reaction described in (ii) and in the notes.

(b) Natural gas has an energy density of $4.9 \times 10^{7}\,\text{J/kg}$. How much more energy dense is U-235?

(c) If the UK's total annual energy consumption is about $10^{19}\,\text{J}$, what is the equivalent *volume* of U-235 fuel?

(iv) ${}_{1}^{2}\text{H} + {}_{1}^{3}\text{H} \rightarrow {}_{2}^{4}\text{He} + {}_{0}^{1}\text{n}$ releases $\Delta E = 17.59\,\text{MeV}$. Calculate the binding energy of tritium (H-3) (in MeV) if the binding energy of Helium-4 is 27.27MeV and the mass of deuterium (H-2) is 2.0141u.

(v) Calculate the energy density (in J/kg) for the fusion reaction ${}_{1}^{2}\text{H} + {}_{1}^{3}\text{H} \rightarrow {}_{2}^{4}\text{He} + {}_{0}^{1}\text{n}$ in terms of deuterium and tritium fuel. What is this in terms of tonnes of fuel to supply the UK for a year?

(vi) (a) Assume deuterium-tritium fusion is spontaneous when the potential energy of two protons (of mean separation about $r \approx 10^{-15}\,\text{m}$,i.e. nuclear distances) equates to the thermal energy i.e. kinetic energy due to random motion. Hence estimate the temperature required to initiate fusion in a reactor on Earth.

$\varepsilon_0 = 8.85 \times 10^{-12}\,\text{m}^{-3}\text{kg}^{-1}\text{s}^{4}\text{A}^{2}$, $k_B = 1.38 \times 10^{-23}\,\text{JK}^{-1}$, $e = 1.602 \times 10^{-19}\,\text{C}$.

(b) The core temperature of the Sun is about 15 million Kelvin. How does this compare to your answer in (a)? Explain why fusion is still the main energy source of solar radiation.

(vii) Use the Semi-Empirical Mass Formula (SEMF) to predict the binding energy (in MeV) for Plutonium ${}_{94}^{239}\text{Pu}$.

(viii) An example Pu-239 fission reaction is: ${}_{94}^{239}\text{Pu} + {}_{0}^{1}\text{n} \rightarrow {}_{54}^{134}\text{Xe} + {}_{40}^{103}\text{Zr} + 3 \times {}_{0}^{1}\text{n}$. If the binding energy per nucleon B/A for Xe-134 is 8.4137MeV, use the SEMF to predict the binding energy of Zr-103, and the answer to (vii), to predict the energy released (in MeV) in this fission reaction. What is this in terms of energy /kg of Pu-235?

(ix) An example of *aneutronic* fusion, i.e. without any release of potentially hazardous neutron radiation, is the fusion of deuterium and lithium-6: ${}_{1}^{2}\text{H} + {}_{3}^{6}\text{Li} \rightarrow 2 \times {}_{2}^{4}\text{He}$. See (iv) for the binding energies of helium-4 and deuterium.

(a) Use the SEMF to predict the binding energy of Lithium-6, and hence calculate the energy released. Compare this to the actual amount, which is 22.4MeV.

(b) Explain why this reaction might be even more challenging to manifest in a fusion reactor (such as *tokomak*) than deuterium + tritium.

(x) Use the SEMF and determine the Z value which maximizes B/A, assuming that $A = 2Z$ i.e. an equal number of protons and neutrons. Ignore the pairing term. What element is this?

Question 2 ${}_{94}^{239}\text{Pu} + {}_{0}^{1}\text{n} \rightarrow {}_{94}^{240}\text{Pu} + \gamma$ occurs 27% of the time as does the Pu-239 fission described in Q1 (viii). This nuclear reaction produces gamma rays instead of nuclear fragments and more neutrons. Using the SEMF to determine the binding energy changes, calculate the smallest possible wavelength of the gamma rays produced. Planck's constant $h = 6.63 \times 10^{-34}\,\text{Js}$.

Question 3 Construct a spreadsheet model of the SEMF and plot B/A vs Z for (i) $A = 2.1Z$, (ii) $A = 2Z$, (iii) $A = 1.9Z$. Round A values to the nearest integers. Overlay these curves on the same graphs, and colour-code each one with reference to a legend. The largest $Z = 118$ (the element Oganesson).

Question 4 In pretty much every science fiction movie, from *Guardians of The Galaxy* to *Star Wars,* some form of plasma-based thruster is used to propel spacecraft at modest speeds. Whereas faster-than-light 'warp drive' travel is deemed impossible by Special Relativity (unless via some form of 'wormhole' you can take a short-cut between regions of the Universe), 'ion-thrusters' are very much science fact, although an engineering challenge at present. One idea is to use nuclear fusion to produce sufficient energy to create high velocity ions. These can be redirected by electric and magnetic fields, and ejected from the spacecraft, producing thrust. Assume a deuterium-tritium reaction produces helium-4, and $\Delta E = 17.59\,\text{MeV}$ of heat, which we shall assume is the kinetic energy of the helium-4 nuclei. Assume all the helium nuclei are ionized and fired out the back of the ship. By determining the average speed of the helium nuclei, calculate the rate of use of deuterium + tritium fuel (in kg/s) to provide an equivalent thrust to the 43.7kN of the AJ10 rocket engines used in the Apollo spacecraft. Ideally perform a relativistic calculation for the average ejection speed of helium nuclei.