

How are numbers 'best' written down? Does it matter?

The **DECIMAL** system is when numbers are written right to left in powers of ten.

Only **ten symbols** are needed (0,1,2,3,4,5,6,7,8,9) plus a DECIMAL POINT to describe *any* number, of which there are *infinitely* many.

Not bad eh? In ancient cultures a *different* symbol is used for each integer, just like the way we say 'one', 'two', 'three' etc. This means as many symbols as numbers, not very nice for learning arithmetic or indeed printing mathematics!

So 1234.56 *means*

$$1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1} + 6 \times 10^{-2}$$

$$1000 + 200 + 30 + 4 + 0.5 + 0.06$$

Binary numbers 0, 1

We don't have to use the decimal system. In fact we can use any (integer!) number of symbols from two upwards.

A **two** symbol (0 or 1) system is **BINARY** (which is used to store and manipulate numbers by computers)

Decimal	Binary	
17	10001	$1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$ = $16 + 1 = 17$
1234	10011010010	Note $1024 + 128 + 64 + 16 + 2 = 1234$

1×2^{10}	0×2^9	0×2^8	1×2^7	1×2^6	0×2^5	1×2^4	0×2^3	0×2^2	1×2^1	0×2^0
1024	0	0	128	64	0	16	0	0	2	0

What are the following decimal integers in binary?

(a) 64 is **1000000** since $2^6 = 64$

(b) 73 is **1001001** since $64 + 8 + 1 = 73$

1×2^6	0×2^5	0×2^4	1×2^3	0×2^2	0×2^1	1×2^0
64	0	0	8	0	0	1

Hexadecimal numbers 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

A sixteen symbol system is **HEXADECIMAL**, which is typically used to describe computer memory addresses.

Decimal															
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Hexadecimal or 'base 16'

Decimal	Hexadecimal	
17	11	$1 \times 16^1 + 1 \times 16^0 = 16 + 1 = 17$
1234	4D2	$4 \times 16^2 + 13 \times 16^1 + 2 \times 16^0$ $= 4 \times 256 + 13 \times 16 + 2$ $= 1234$

Other 'popular' bases are:

What are the following in hexadecimal?

- (a) 31 is **1F** since $1 \times 16^1 + 15 \times 16^0 = 31$
 (b) 117 is **75** since $7 \times 16^1 + 5 \times 16^0 = 112 + 5 = 117$

12 Duodecimal **0,1,2,3,4,5,6,7,8,9,A,B**

60 Sexagesimal Used by the ancient Babylonians around 3000BC

𐎶 1	𐎶𐎵 11	𐎶𐎵𐎶 21	𐎶𐎵𐎶𐎵 31	𐎶𐎵𐎶𐎵𐎶 41	𐎶𐎵𐎶𐎵𐎶𐎵 51
𐎶𐎶 2	𐎶𐎶𐎵 12	𐎶𐎶𐎵𐎶 22	𐎶𐎶𐎵𐎶𐎵 32	𐎶𐎶𐎵𐎶𐎵𐎶 42	𐎶𐎶𐎵𐎶𐎵𐎶𐎵 52
𐎶𐎶𐎶 3	𐎶𐎶𐎶𐎵 13	𐎶𐎶𐎶𐎵𐎶 23	𐎶𐎶𐎶𐎵𐎶𐎵 33	𐎶𐎶𐎶𐎵𐎶𐎵𐎶 43	𐎶𐎶𐎶𐎵𐎶𐎵𐎶𐎵 53
𐎶𐎶𐎶𐎶 4	𐎶𐎶𐎶𐎶𐎵 14	𐎶𐎶𐎶𐎶𐎵𐎶 24	𐎶𐎶𐎶𐎶𐎵𐎶𐎵 34	𐎶𐎶𐎶𐎶𐎵𐎶𐎵𐎶 44	𐎶𐎶𐎶𐎶𐎵𐎶𐎵𐎶𐎵 54
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cuneiform digits

Note this wasn't a proper 'place value' system as there was no zero!

Although it did appear later as 